



# Out of Band Acoustic Fields: Theory and Application



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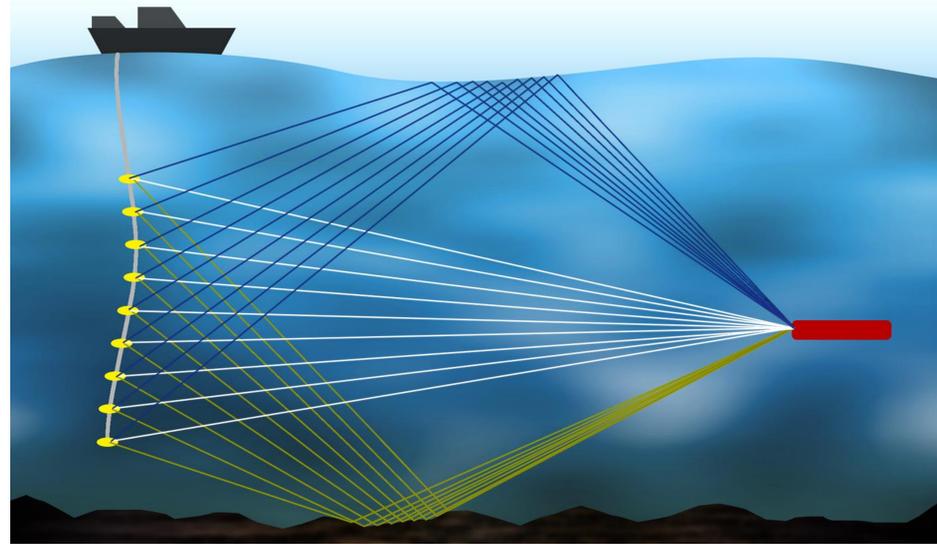
**ABSTRACT:** Acoustics is a branch of physics largely governed by linear field equations. Linearity carries with it the implication that only the frequencies broadcast by acoustic sources can be measured in the surrounding acoustic medium. However, nonlinearities introduced not in the physical world, but in the mathematical and signal processing realm, have the potential to change frequency content. In my dissertation, nonlinear mathematical constructions termed autoproductions were created which have the potential to shift frequencies from the measured, in-band frequencies to other higher or lower frequencies which may no longer be in-band. These out-of-band autoproduction fields did not physically propagate in the environment, and yet, this research has found that autoproductions can nonetheless mimic genuine out-of-band fields in a number of different acoustic environments. Data from experiments, as well as theoretical analyses not shown here, suggest that autoproduction-based source localization can make physics-based array signal processing robust at arbitrarily high frequencies – a novel and important contribution to existing literature.

## Passive Remote Sensing for Underwater Acoustics



Fig. 1 – Remote sensing flow chart, particularly for underwater acoustics

Fig. 2 – Schematic for shallow ocean wave propagation, with rough bottom, variable surface, spatially varying sound speed (given by color changes in the volume). The source is nominally given by the red rectangle, the vertical line array of underwater microphones is given in yellow, and a few example acoustic ray-paths are given in white, blue, and yellow. The aspect ratio here is not intended to be 1:1 – nominal shallow ocean depths are usually around ~100m, and nominal ranges of interest vary from 1 to 10km, depending on the frequency and amplitude of the source.



## Matched Field Processing's Environmental Mismatch Problem

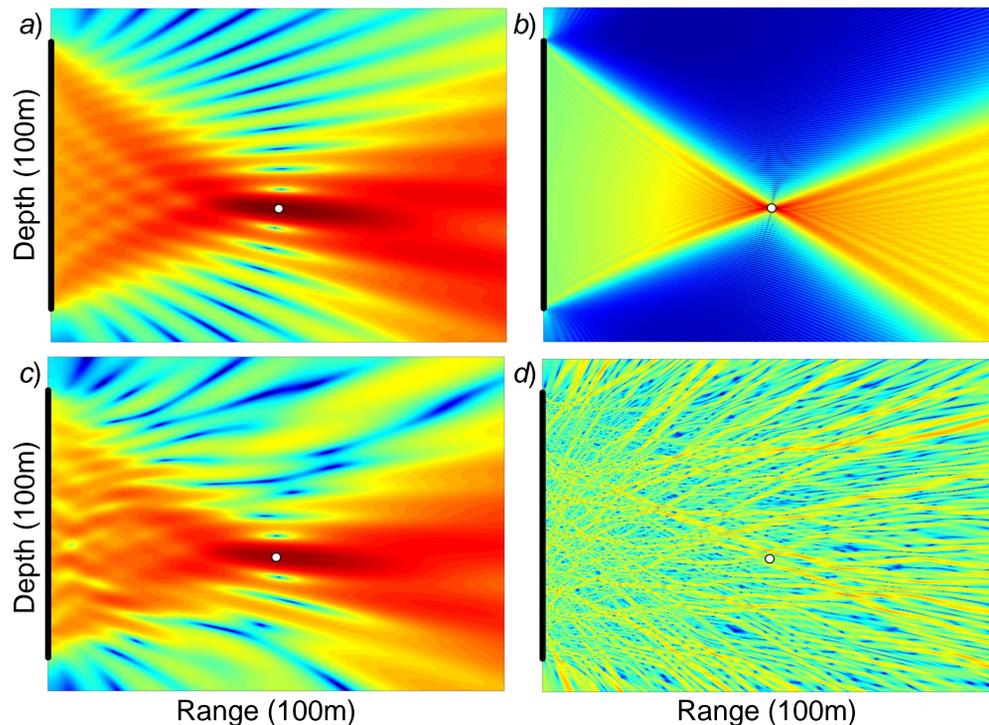
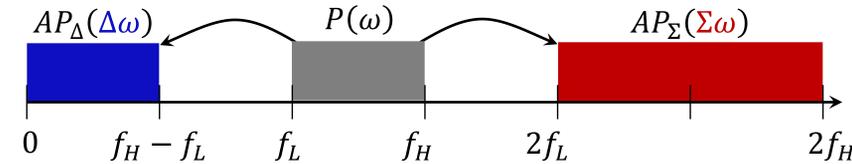


Fig. 3 – Matched Field Processing (MFP) plots. The color scale, which is shown logarithmically from -50dB (blue) to 0dB (red), shows how well the measured and the modeled field correlate, for a given guessed source location, with red indicating likely source locations. Figs 3a and 3c show a 200Hz source, while Figs 3b and 3d show a 2kHz source. The top panels do not incorporate any mismatch – implying that the environment is perfectly known, which is practically not achievable. The lower two panels show the same amount of mismatch (0.5ms gaussian random time delays added to each path). At 200Hz, MFP is robust, but at 2kHz, MFP is wholly unable to localize the source, even with infinite signal-to-noise ratio.

## Autoproductions: Out of Band Acoustic Fields



### Frequency Difference Autoproduction

$$AP_{\Delta}(\Delta\omega) = \langle P(\omega_+)P^*(\omega_-) \rangle_{\Sigma\omega}$$
$$\Delta\omega = \omega_+ - \omega_-$$

### Frequency Sum Autoproduction

$$AP_{\Sigma}(\Sigma\omega) = \langle P(\omega_+)P(\omega_-) \rangle_{\Delta\omega}$$
$$\Sigma\omega = \omega_+ + \omega_-$$

Fig. 4 – Autoproductions are defined as a multiplication of measured, in-band fields at two different frequencies. To reduce cross-terms, which arise in a multipath environment, bandwidth averages are taken, either coherently as shown here (if the source waveform is known), or incoherently via a cross-spectral density matrix (if the source waveform is unknown).

## Kauai Acoustic Communications Experiment 2011

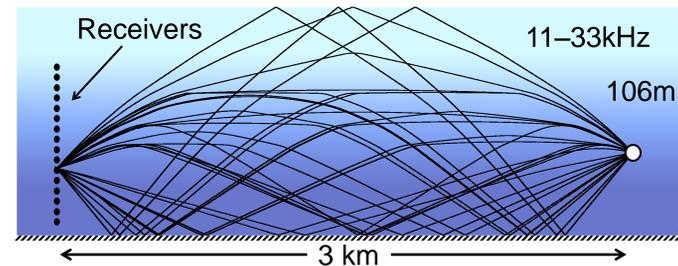


Fig. 5 – Geometry for the Kauai acoustic communications experiment in June of 2011, performed at the Pacific Missile Range Facility in Hawaii. There were 16 source depths available, all 3km away from a 16 element vertical line array. Each were broadcasting a 100ms linear-frequency-modulated pulse (i.e. chirp) between 11 and 33kHz.

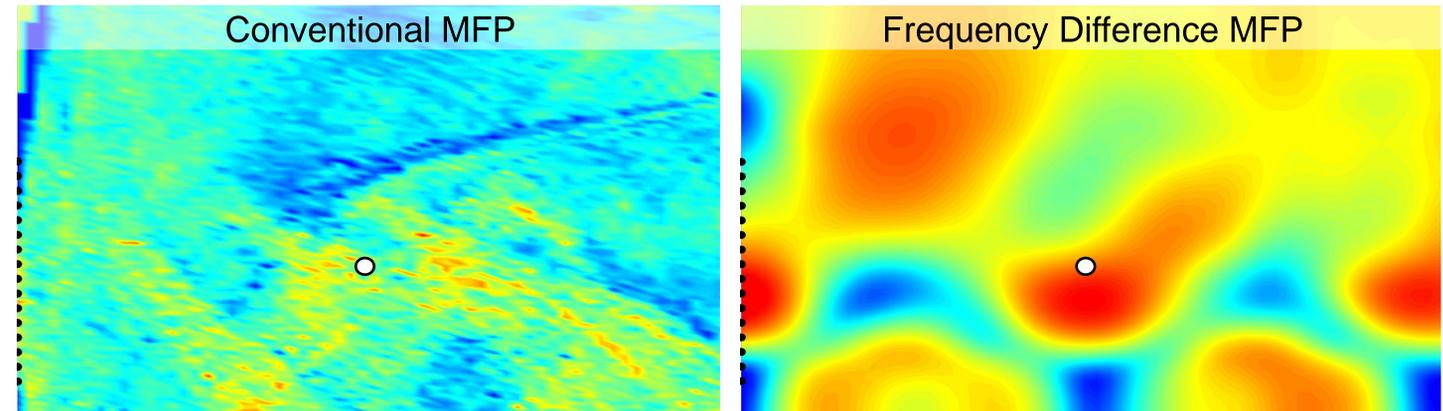


Fig. 6 – Comparison of MFP outputs, using the measured data from the KAM11 experiment. These plots have horizontal ranges of 6km, and vertical extents of 106m. The true source location is labeled with a white circle, and the array positions are given by the black dots on the left. The color scale spans 5 dB, and is normalized to the highest peak in the field. Conventional MFP fails to localize due to environmental mismatch, which was expected for source frequencies above 1 kHz. Frequency difference MFP is performed by shifting the in-band frequencies down to difference-frequencies between 50Hz to 500Hz (and incoherently averaging in difference frequency). This result is not perfect, as there are large sidelobes, and range/depth errors on the order of 10%. But, it is substantially better than conventional MFP, which is unable to provide any confident range or depth estimates.