Electromagnetic Wave Propagation based upon Spectral-Element Method

CASIS Workshop

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Overview

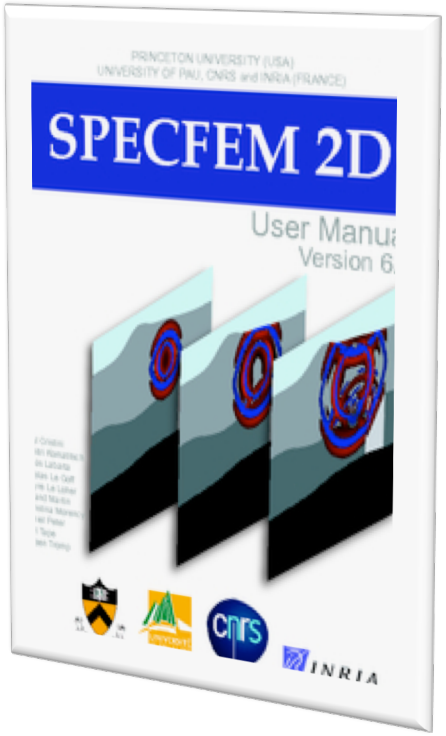
- SPECFEM seismic packages
- Seismic wave equations
- Spectral-element method
- Electromagnetic & seismic wave equations mathematical equivalences
- GPR examples

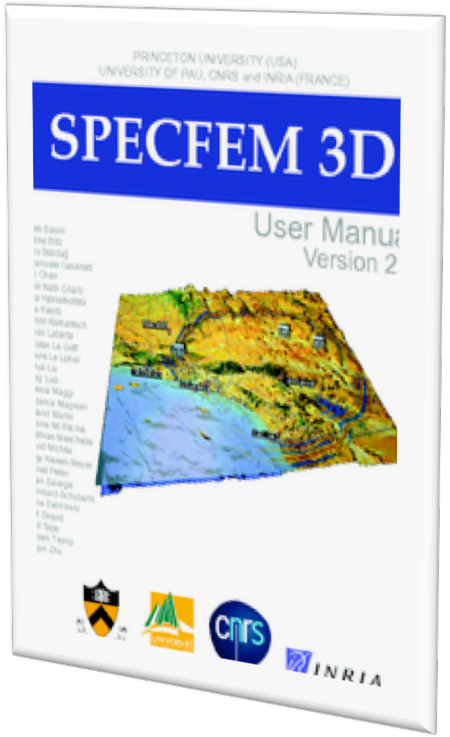
⇒ Maxwell's equations implemented in the spectral-element code SPECFEM, providing the ability to handle complex geometries and retaining the strength of exponential convergence

Morency, C. "Electromagnetic wave propagation based upon spectral-element methodology", (2018) submitted to Geophysical Journal International

SPECFEM Packages – open source, widely used by seismology community

- CUBIT compatible
- Modules: (an)elastic, acoustic, poroelastic
- Forward & Adjoint wave propagation
- Topography & Bathymetry
- Parallel computation (SCOTCH for mesh partitioning & load balancing) & GPU





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Seismic wave equations in SPECFEM

- **Acoustic media** ***[Chaljub & Valette, 2004; Komatitsch et al., 2005]***

$$\rho_a \partial_t^2 \chi = \kappa_a \nabla^2 \chi$$

and

fluid displacement is $\mathbf{u_f} = \nabla \chi$

acoustic pressure is $p_a = -\rho_a \partial_t^2 \chi$

- **Elastic media** ***[Komatitsch & Tromp, 1999]***

$$\rho \partial_t^2 \mathbf{s} = \nabla \cdot \mathbf{T}$$

$$\mathbf{T} = \mathbf{c} \ : \ \epsilon$$

where

$$c_{ijkl} = (\kappa - 2/3)\delta_{ij}\delta_{kl} + \mu(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$$

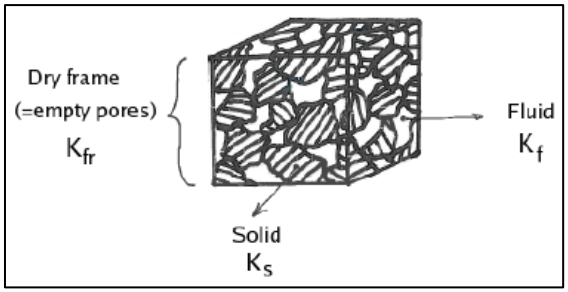
(e.g., isotropic case)

- **Poroleastic media** ***[Biot, 1962; Morency and Tromp, 2008]***

$$\rho \partial_t^2 \mathbf{u_s} + \rho_f \partial_t^2 \mathbf{w} = \nabla \cdot \mathbf{T}$$

$$\frac{\rho_f c}{\phi} \partial_t^2 \mathbf{w} + \rho_f \partial_t^2 \mathbf{u_s} + \mathbf{b}(t)(*.)\partial_t \mathbf{w} = \nabla \cdot \mathbf{T_f}$$

where
$$\begin{cases} \rho = (1-\phi)\rho_s + \phi\rho_f \\ \mathbf{w} = \phi(\mathbf{u_f} - \mathbf{u_s}) \end{cases}$$



$$\begin{cases} \mathbf{T} = (1-\phi)\mathbf{T_s} + \phi\mathbf{T_f} \\ \quad = (B\nabla \cdot \mathbf{u_s} + C\nabla \cdot \mathbf{w})\mathbf{I} + 2\mu_{fr}\mathbf{D_s} \\ \mathbf{T_f} = -p_f\mathbf{I} = (C\nabla \cdot \mathbf{u_s} + M\nabla \cdot \mathbf{w})\mathbf{I} \end{cases}$$

Spectral-Element Method

- Strong form: $\rho \partial_t^2 \mathbf{s} = \nabla \cdot \mathbf{T} + \mathbf{f}$

$$\mathbf{f} = -\mathbf{M} \cdot \nabla \delta(\mathbf{x} - \mathbf{x_s}) S(t)$$

- Weak form:

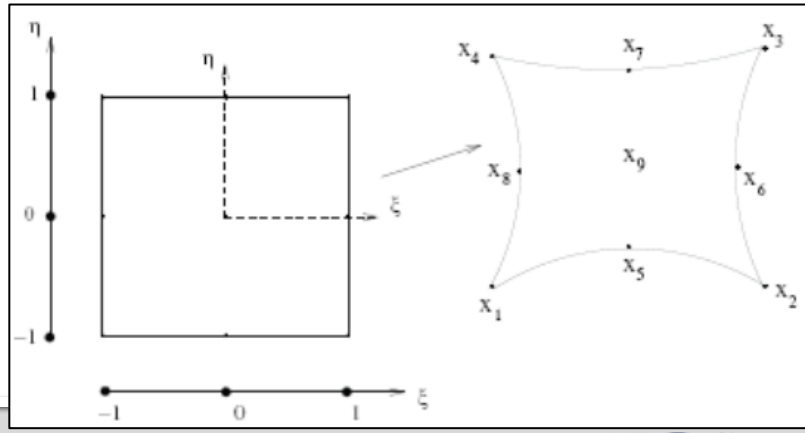
$$\int_\Omega \rho \mathbf{v} \cdot \partial_t^2 \mathbf{s} \, \mathrm{d}^3\mathbf{x} = -\int_\Omega \nabla \mathbf{v} \; : \; \mathbf{T} \mathrm{d}^3\mathbf{x} + \int_\Gamma \mathbf{n} \cdot \mathbf{T} \cdot \mathbf{v} \mathrm{d}^2\mathbf{x} + \mathbf{M} \; : \; \nabla \mathbf{v}(\mathbf{x_s}) S(t)$$

  - *Weak form valid for any test vector* $\mathbf{v}$
  - *Boundary integral naturally unfolds*

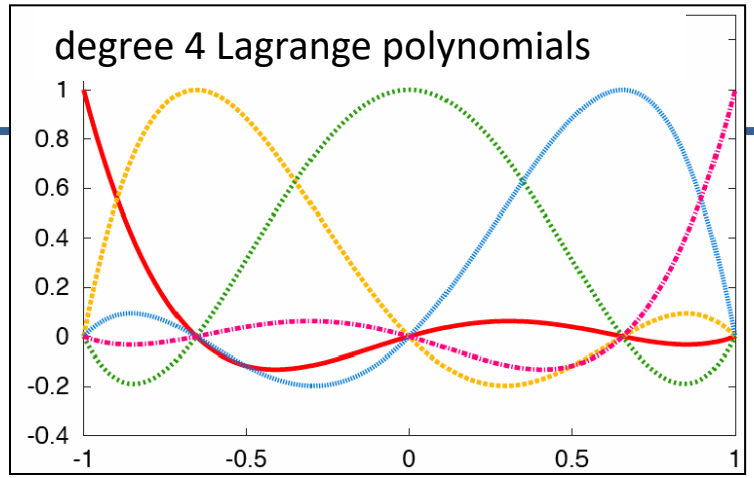
- Mapping from reference square (cube) to quadrilateral (hexahedral) element:

$$\mathbf{x}(\xi) = \sum_{a=1}^{M} \mathbf{x}_a N_a(\xi)$$

Jacobian $\quad J = \left| \dfrac{\partial(x, y, z)}{\partial(\xi, \eta, \zeta)} \right|$



Spectral-Element Method

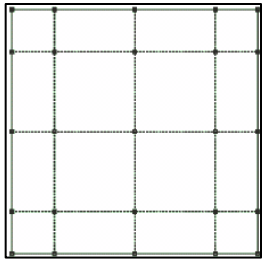


- Lagrange polynomials to expand field functions at Gauss-Lobatto-Legendre (GLL) integration points:

$$l_\alpha(\xi_{\alpha'}) = \delta_{\alpha\alpha'}$$

$$\Rightarrow f(\mathbf{x}(\xi,\eta,\zeta)) \sim \sum_{\alpha,\beta,\gamma}^{n_l} f^{\alpha\beta\gamma} l_\alpha(\xi) l_\beta(\eta) l_\gamma(\zeta)$$

degree 4 GLL points



- GLL quadrature integration rule:

$$\int_{\Omega_e} f(\mathbf{x})\mathrm{d}^3\mathbf{x} = \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} f(\mathbf{x}(\xi,\eta,\zeta)) J(\xi,\eta,\zeta)\mathrm{d}\xi\mathrm{d}\eta\mathrm{d}\zeta$$

$$\sim \sum_{\alpha,\beta,\gamma}^{n_l} w_\alpha w_\beta w_\gamma J^{\alpha\beta\gamma} f^{\alpha\beta\gamma}$$

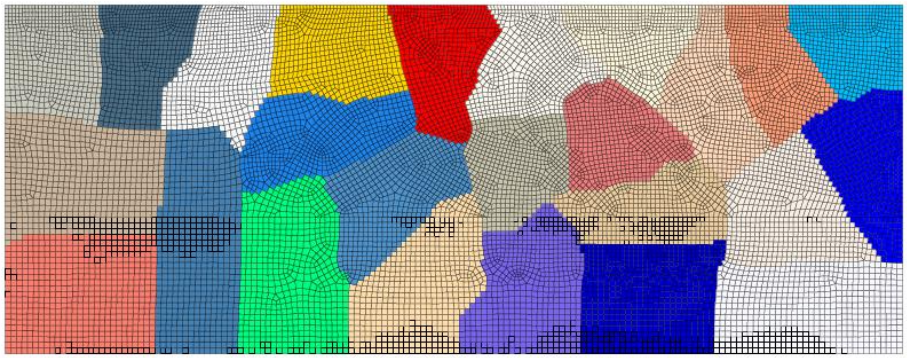
➡ diagonal mass matrix: explicit time marching

(Newmark time scheme)

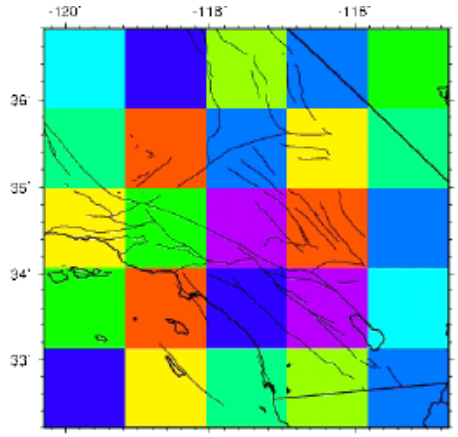
$$\mathbf{M}\ddot{\mathbf{S}} = -\mathbf{K}\mathbf{S} + \mathbf{F}$$

Spectral-Element Method

Globe partitioning
6 chunks of n*n mesh slices



Regional (SEG/EAGE) irregular partitioning of mesh slices



Regional (S. California) regular partitioning of n*m mesh slices

<u>Resolution & stability criteria</u>

- 5 points per shortest wavelength
- Courant number < 0.3

[Komatitsch & Vilotte, 1998]

Electromagnetic and seismic wave equivalence
1) Recasting the EM governing equations

Faraday's law:	$\nabla \times \mathbf{E} = -\partial_t \mathbf{B}$	$\frac{1}{2}[\nabla \mathbf{E} - (\nabla \mathbf{E})^T] = -\partial_t \tilde{\mathbf{B}}$
Ampère – Maxwell's law:	$\nabla \times \mathbf{H} = \mathbf{J} + \partial_t \mathbf{D}$	$\nabla \cdot \tilde{\mathbf{H}} = \mathbf{J} + \partial_t \mathbf{D}$
Constitutive equation 1:	$\mathbf{D} = \epsilon * \mathbf{E}$	$\mathbf{D} = \epsilon * \mathbf{E}$
Constitutive equation 2:	$\mathbf{B} = \mu \mathbf{H}$	$\tilde{\mathbf{B}} = -\tilde{\mu} : \tilde{\mathbf{H}}$
Current density:	$\mathbf{J} = \sigma * \mathbf{E} + \mathbf{j_s}$	$\mathbf{J} = \sigma * \mathbf{E} + \mathbf{j_s}$

$$\tilde{\mathbf{H}} = \begin{bmatrix} 0 & -H_3 & H_2 \\ H_3 & 0 & -H_1 \\ -H_2 & H_1 & 0 \end{bmatrix}$$

$$\tilde{\mathbf{B}} = \frac{1}{2} \begin{bmatrix} 0 & B_3 & -B_2 \\ -B_3 & 0 & B_1 \\ B_2 & -B_1 & 0 \end{bmatrix}$$

$\tilde{\mu}$ is a fourth order magnetic permeability

[funded under LDRD 17-LW-029]

[Ikelle, 2012; Morency, 2018]

Electromagnetic and seismic wave equivalence 2) Mathematical equivalence EM/Seismic wave equations

$$\epsilon * \partial_t^2 \mathbf{E} = \nabla \cdot (\partial_t \tilde{\mathbf{H}}) - \partial_t \mathbf{J}$$

$$\partial_t \tilde{\mathbf{H}} = \tilde{\mu}^{-1} \ : \ -\partial_\mathbf{t} \tilde{\mathbf{B}}$$

$$-\partial_\mathbf{t} \tilde{\mathbf{B}} = \frac{1}{2}[\nabla E - (\nabla E)^T]$$

$$\mathbf{J} = \sigma * \mathbf{E} + \mathbf{j_s}$$

$$\rho \partial_t^2 \mathbf{s} = \nabla \cdot \mathbf{T} + \mathbf{f}$$

$$\mathbf{T} = \mathbf{c} \ : \ \epsilon$$

$$\epsilon = \frac{1}{2}[\nabla s + (\nabla s)^T]$$

$$\mathbf{f} = -\mathbf{M} \cdot \nabla \delta(\mathbf{x} - \mathbf{x_s}) S(t)$$

Leverage the advantages of open source seismic software SPECFEM:

- Same architecture as seismic wave solution
- Handles complex geometry (finite element approach)
- Retains exponential convergence and accuracy (high-order polynomials interpolation)

Attenuation and dispersion processes included:

- dielectric permittivity relaxation function using a Zener model
- conductivity relaxation function using a Kelvin-Voigt model

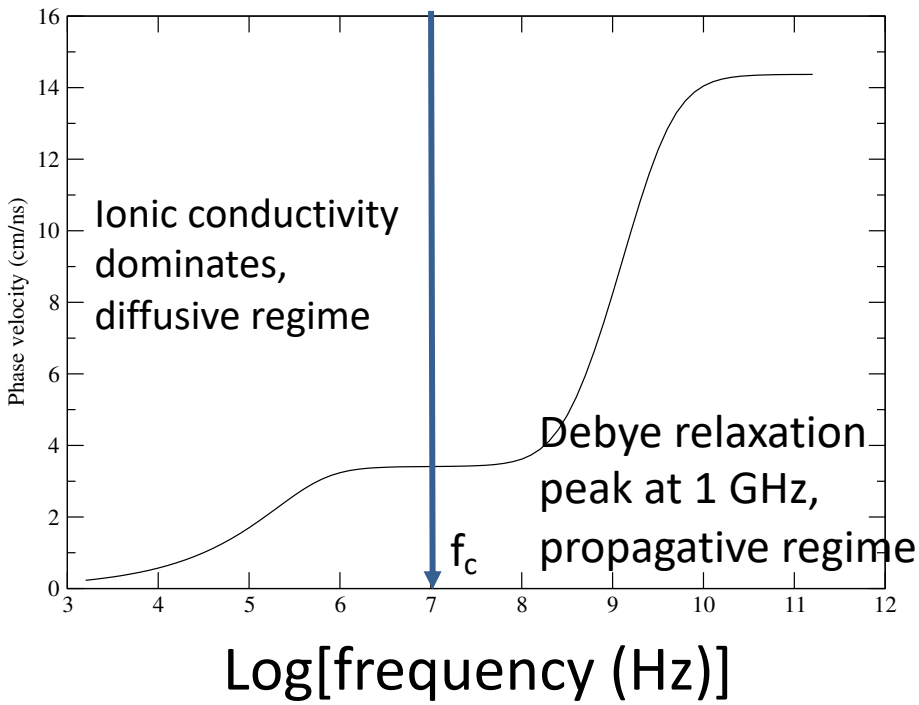
[Morency, 2018]

Electromagnetic and seismic wave equivalence
3) Summary

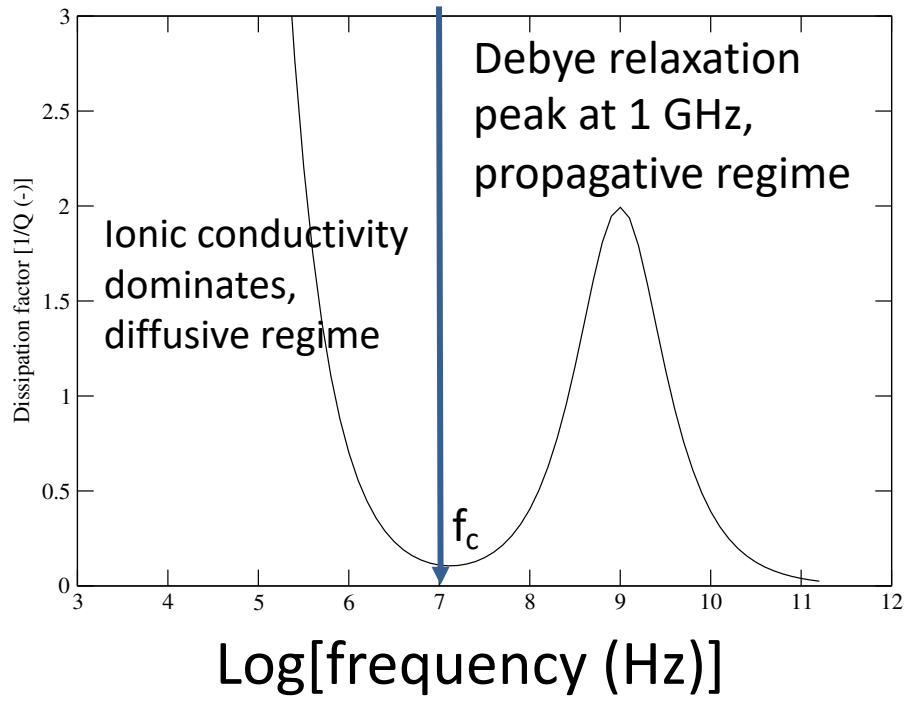
EM parameters		Seismic parameters	
Electric field	$\mathbf{E}$	$\mathbf{s}$	Displacement
Magnetic field	$\partial_t \tilde{\mathbf{H}}$	$\mathbf{T}$	Stress tensor
Fourth order magnetic permeability	$\tilde{\mu}^{-1}$	$\mathbf{c}$	Fourth order elastic tensor
Current density	$\partial_t \mathbf{J}$	$\mathbf{f}$	Source force
Magnetic induction	$-\partial_t \tilde{\mathbf{B}} = \frac{1}{2}[\nabla E - (\nabla E)^T]$	$\epsilon = \frac{1}{2}[\nabla s + (\nabla s)^T]$	Strain

Frequency dependence

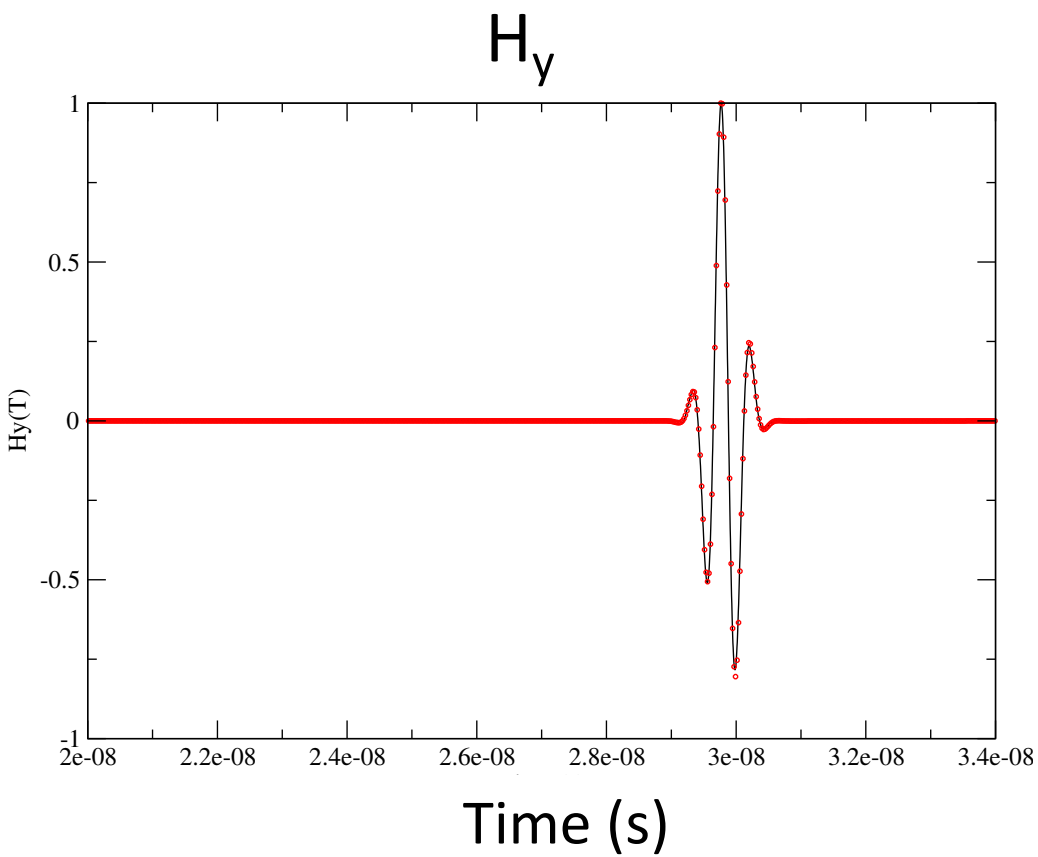
Phase Velocity



Dissipation factor



Validation of EM module



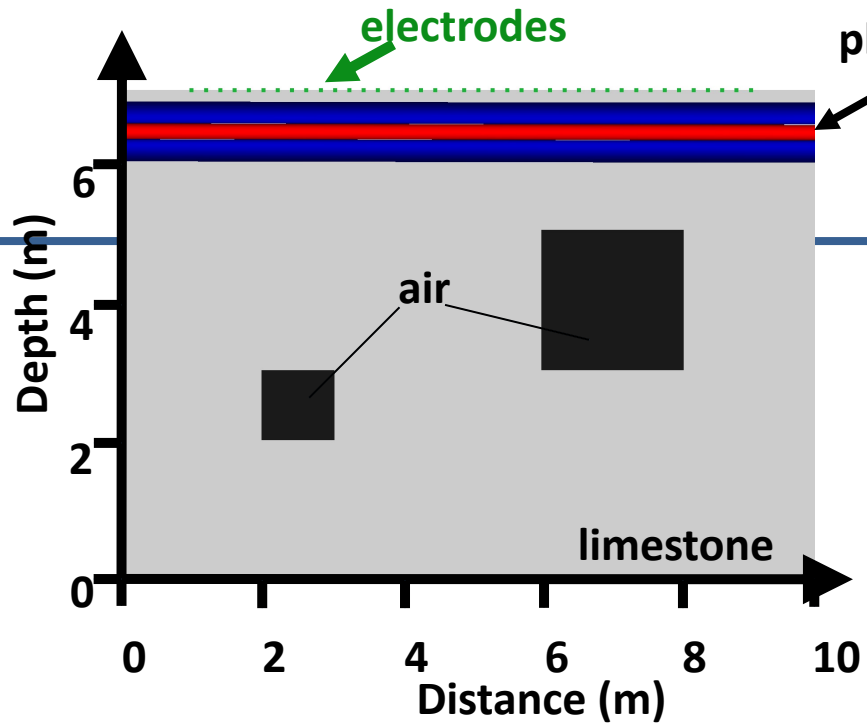
- Ricker wavelet 2 GHz source
- Source-Receiver distance = 2.8 m

Black = numerical model

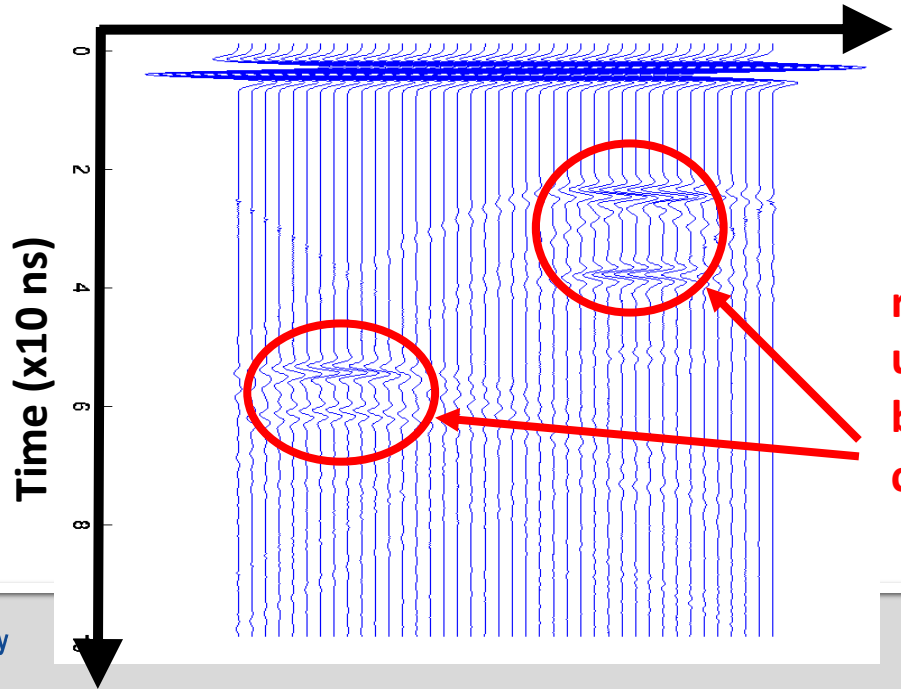
Red = analytical solution after Carcione & Cavallini (1995)

2D TE solution

Model geometry

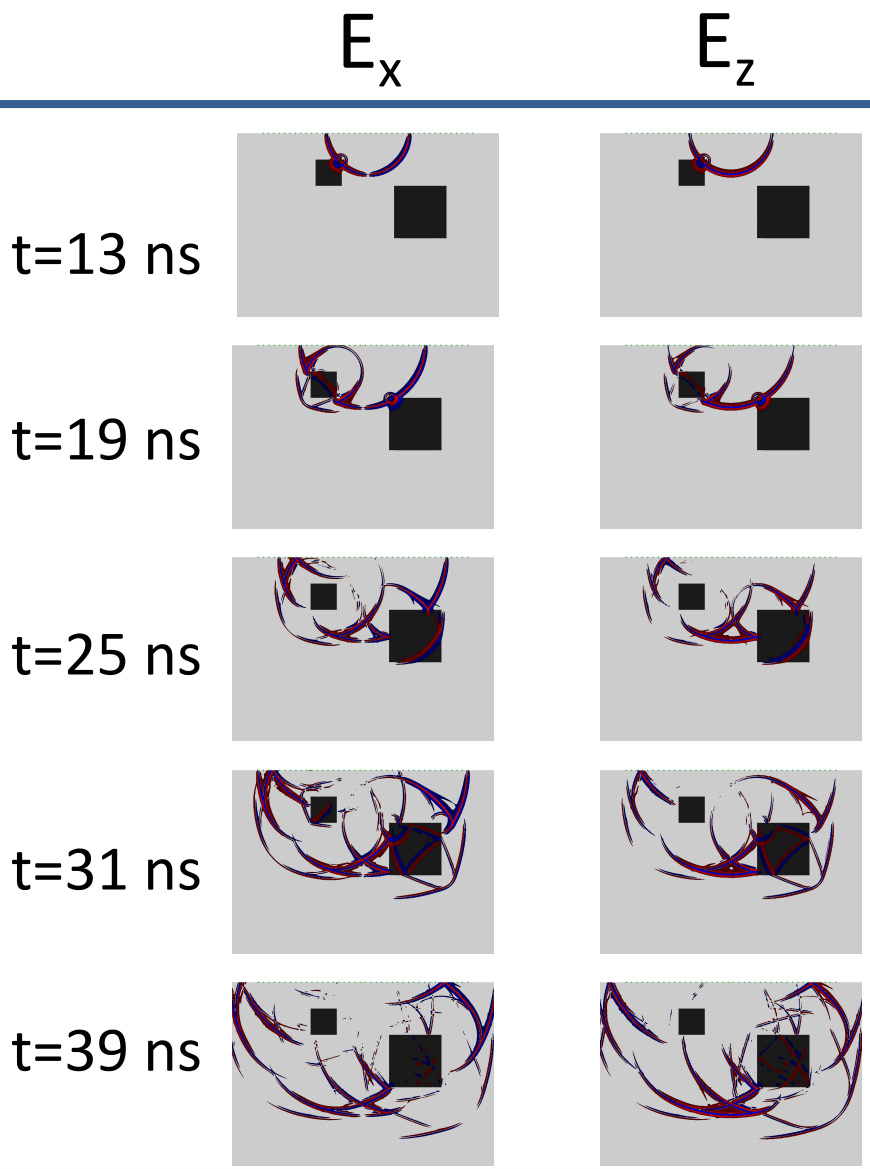


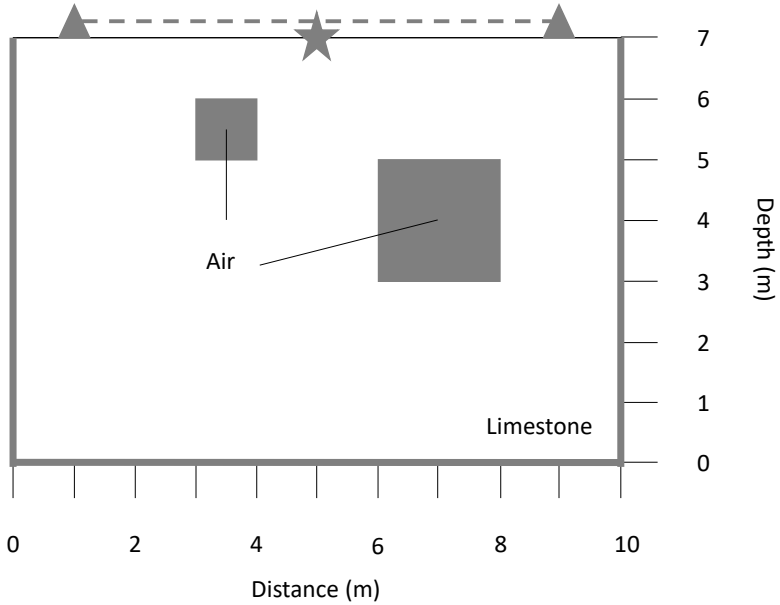
Radargrams (E_y)



reflections from the upper & lower boundaries of the cavities

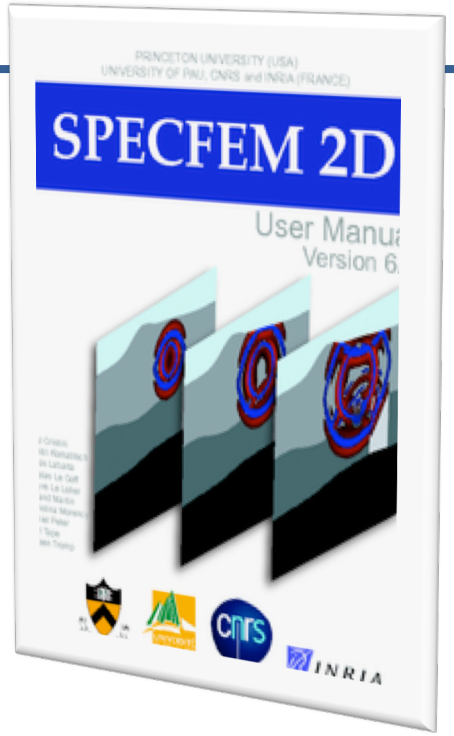
GPR example

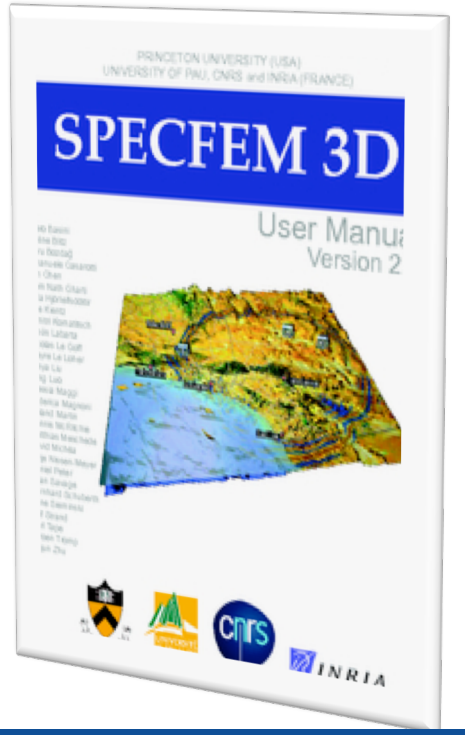

E_x
E_z
t=13 ns
t=19 ns
t=25 ns
t=31 ns
t=39 ns


Air
Limestone
0
1
2
3
4
5
6
7
Depth (m)
0
2
4
6
8
10
Distance (m)

Conclusion

- CUBIT compatible
- Modules: forward (an)elastic, acoustic, poroelastic seismic, and ***electromagnetic***
- Adjoint seismic wave propagation and ***adjoint electromagnetic***
  => S. Mukherjee's presentation
- Topography & Bathymetry
- Parallel computation (SCOTCH for mesh partitioning & load balancing) & GPU





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Seismic and EM forward & adjoint softwares for imaging and tomography