



Capacity Limits for Spatially Multiplexed Free-Space Optical Communication

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Orbital Angular Momentum Multiplexing

ARTICLES

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nature
photonics

Terabit free-space data transmission employing orbital angular momentum multiplexing

Jian Wang^{1,2*}, Jeng-Yuan Yang¹, Irfan M. Fazal¹, Nisar Ahmed¹, Yan Yan¹, Hao Huang¹, Yongxiong Ren¹, Yang Yue¹, Samuel Dolinar³, Moshe Tur⁴ and Alan E. Willner^{1*}

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Maggie McKee

27 June 2013



ARTICLE

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OPEN

High-capacity millimetre-wave communications with orbital angular momentum multiplexing

Yan Yan^{1*}, Guodong Xie^{1*}, Martin P.J. Lavery^{2*}, Hao Huang^{1*}, Nisar Ahmed¹, Changjing Bao¹, Yongxiong Ren¹, Yinwen Cao¹, Long Li¹, Zhe Zhao¹, Andreas F. Molisch¹, Moshe Tur³, Miles J. Padgett² & Alan E. Willner¹

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Spiralling radio waves could revolutionize telecommunications.

Edwin Cartlidge

02 March 2012

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A Different Angle on Light Communications

Alan E. Willner¹, Jian Wang², Hao Huang¹

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Twisted light sends Mozart image over record distance

Vienna demonstration shows that the technology can boost data capacity of laser beam over long distances.

Mark Peplow

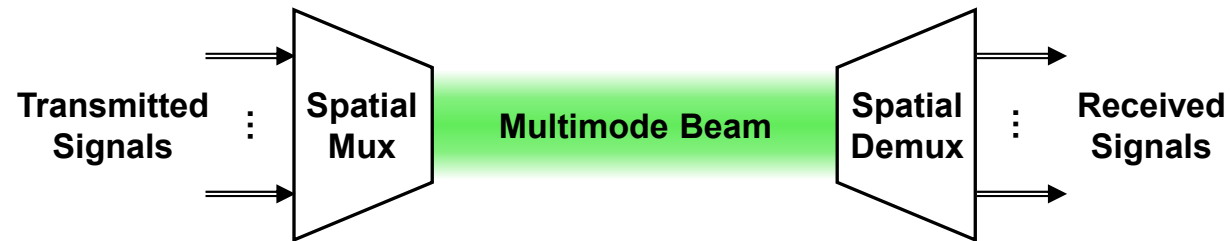
12 November 2014



Orbital Angular Momentum Multiplexing

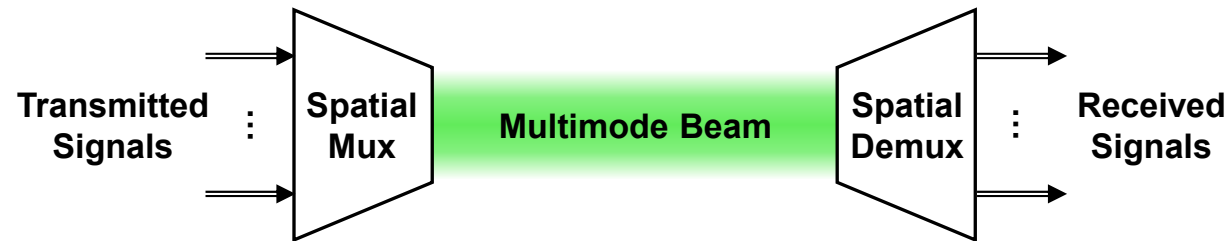
- Orbital angular momentum beam: an e-m wave with a helical wavefront.
- OAM has been used for sensing and manipulation.
- OAM has been proposed as a new degree of freedom for multiplexing information in free-space links.
- It has been suggested that OAM multiplexing offers infinite capacity.
- The capacity of OAM multiplexing has not been studied or compared to other spatial multiplexing methods.

Spatially Multiplexed Free-Space Links



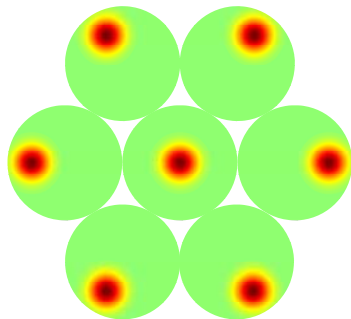
- Employ multiple spatial (and polarization) modes to increase capacity.

Spatially Multiplexed Free-Space Links

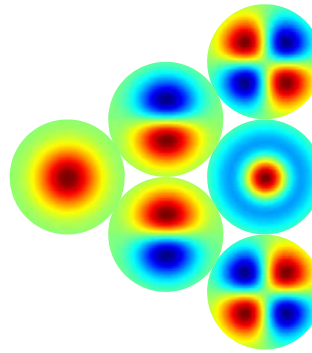


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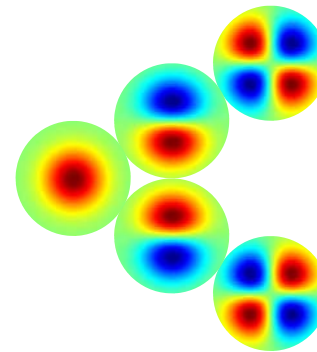
Candidate mode sets



**Parallel
Gaussian
Beams**



**Laguerre-
Gaussian
Modes**



**Orbital Angular
Momentum
Modes**



Outline

- (De)Multiplexers and Link Designs
- Physical Comparison: Counting Modes and Spatial Subchannels
- Information-Theoretic Comparison: Capacity and Degrees of Freedom
- Discussion



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Multiplexers and Demultiplexers

Most SDM FSO experiments have employed:

- Phase masks + beamsplitters.
- Overall loss (mux + demux) scales with square of number of modes.

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Multiplexers and Demultiplexers

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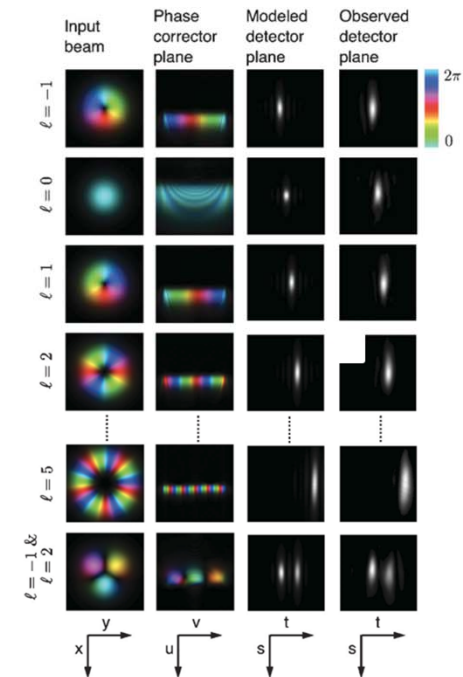
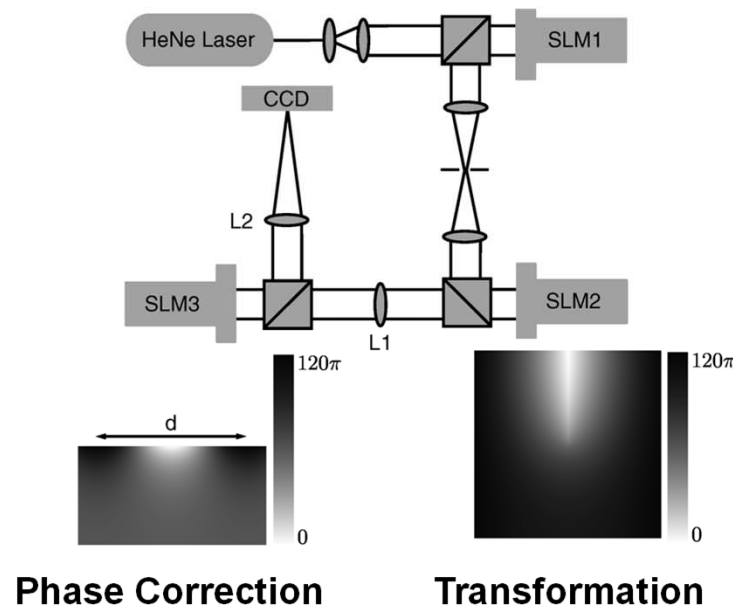
Designs presented here are:

- **Fundamentally lossless:** do not employ incoherent splitting/combining.
- **Reciprocal:** a demultiplexer is a multiplexer operated in reverse.

Some designs presented here are:

- **Fundamentally crosstalk-free:** provide a one-to-one mapping between inputs/outputs and modes (assuming precise alignment/orientation).

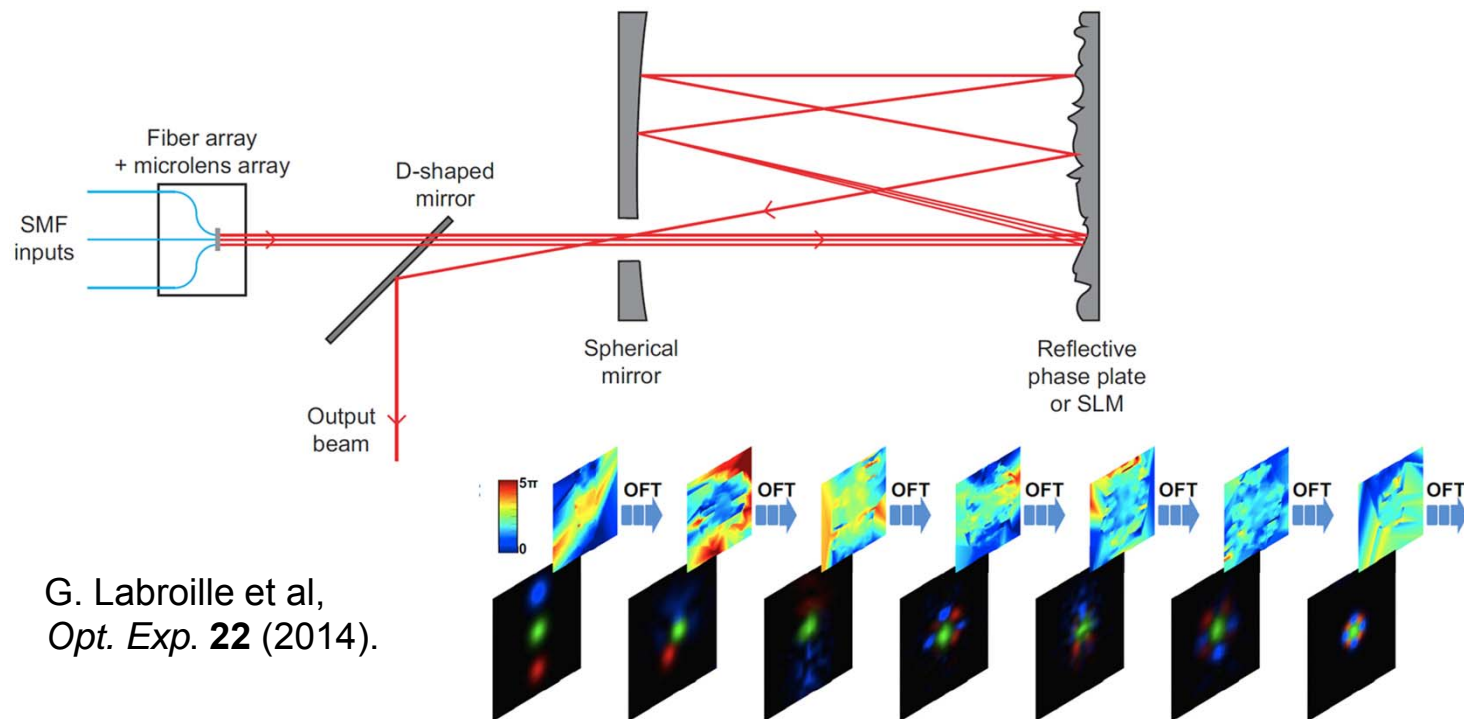
Polar-to-Cartesian Coordinate Conversion (OAM Modes, Low Crosstalk)



G. C. G. Berkhout et al, *Phys. Rev. Lett.* **105** (2010).
M. N. O'Sullivan et al, *Opt. Express* **20** (2012).

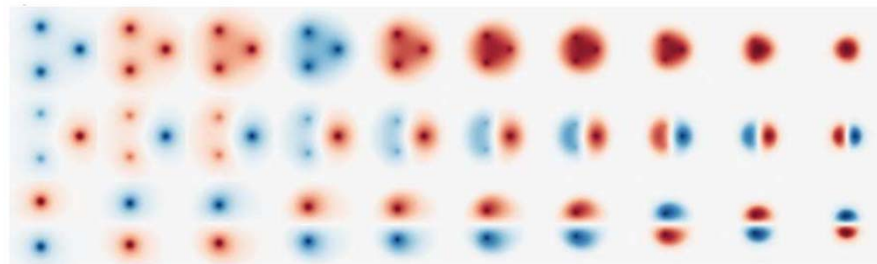
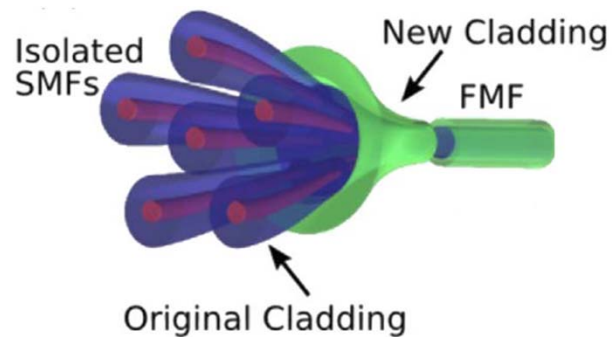
- Designed specifically for OAM modes.
- Can yield low crosstalk, and is independent of azimuthal orientation.

Multi-Plane Conversion (Any Modes, Low Crosstalk)



- Can be designed for L-G, H-G, OAM, or any mode set.
- Can yield low crosstalk.

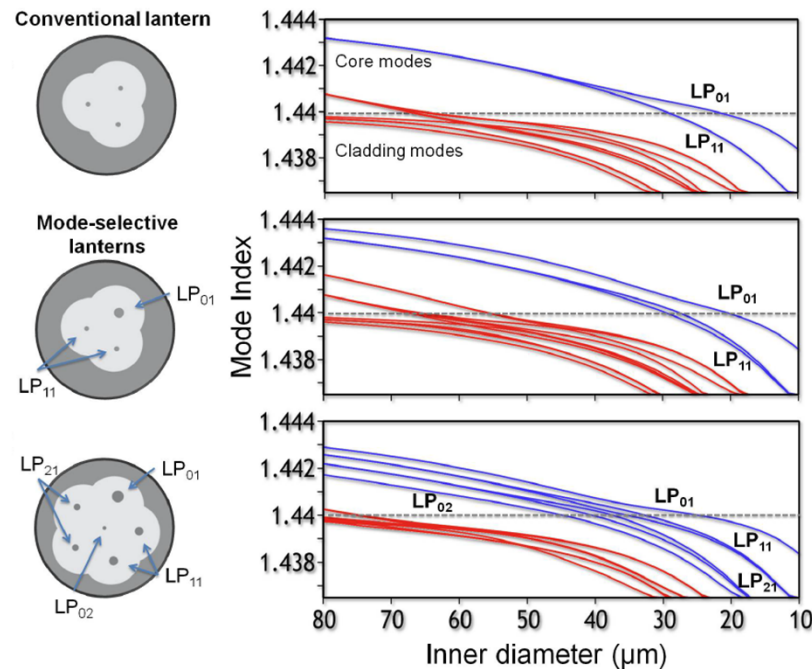
Photonic Lantern: Adiabatic Mode Conversion (LG or HG Modes, High Crosstalk)



S. G. Leon-Saval et al, *Opt. Lett.* **30** (2005).
N. K. Fontaine et al. *Opt. Express* **20** (2012).

- Suitable for L-G or H-G modes.
- Yields inter- and intra-group crosstalk, necessitating MIMO equalization.

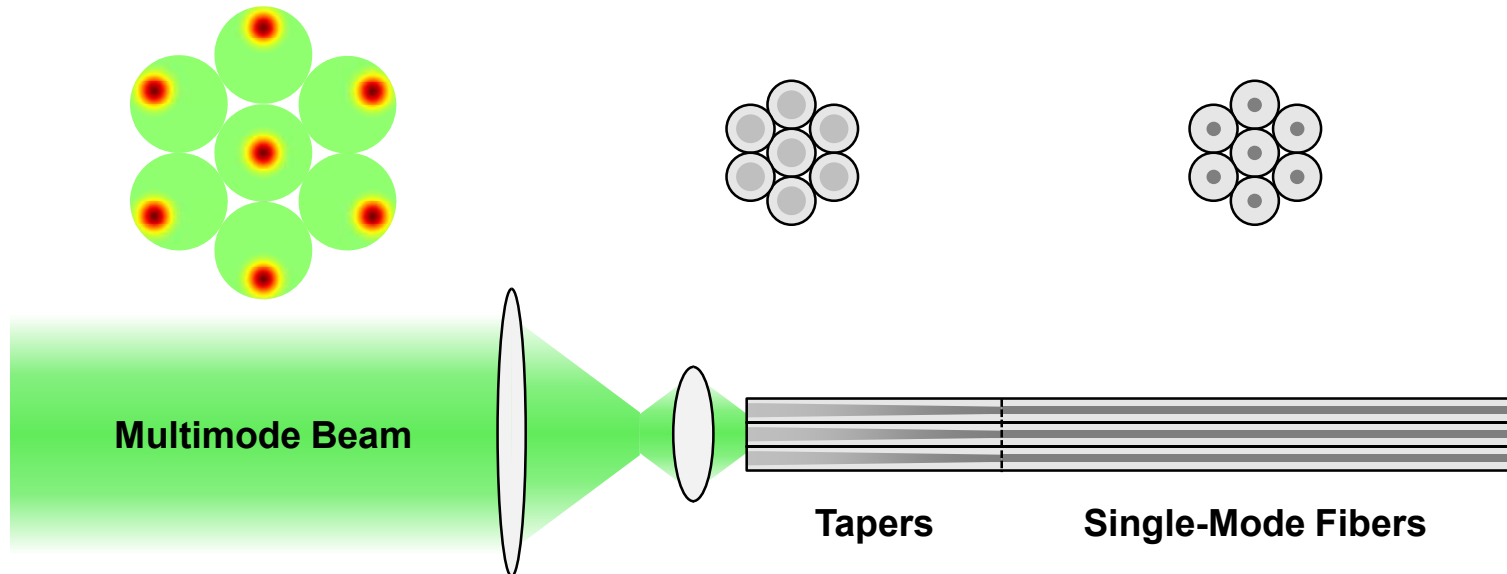
Mode-Selective Photonic Lantern (LG or HG Modes, Intra-Group Crosstalk Only)



S. G. Leon-Saval et al, *Opt. Express* **22** (2014).

- Suitable for L-G or H-G modes.
- Yields intra-group crosstalk, necessitating intra-group MIMO equalization.

Imaging and Mode-Size Conversion (Gaussian Beams, Low Crosstalk)

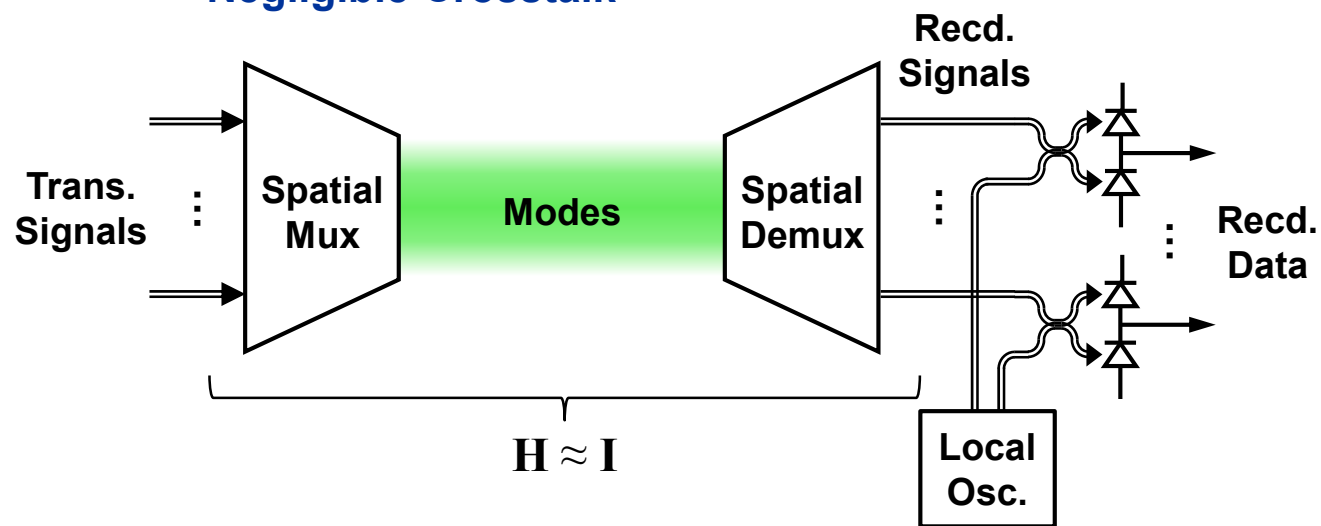


- Can yield low crosstalk for Gaussian beams.
- Can demultiplex other mode sets, but resulting crosstalk necessitates MIMO equalization. This demultiplexing strategy is considered in analysis below.

Coherent Detection Links

- Higher spectral efficiency.
- Higher receiver sensitivity.

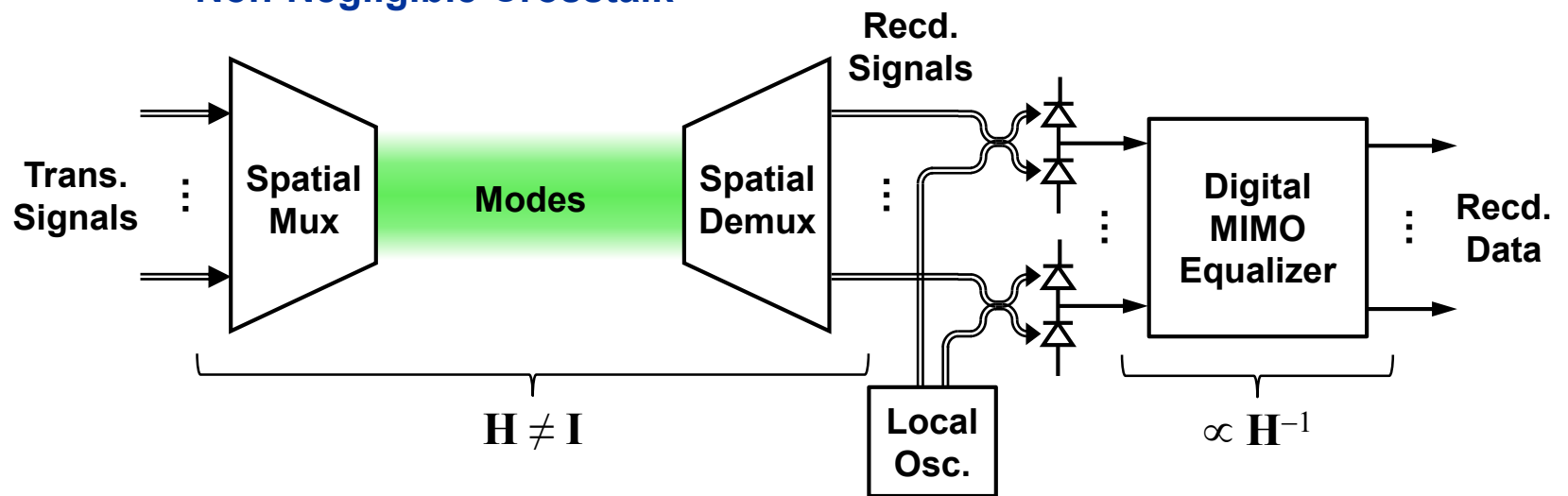
Negligible Crosstalk



Coherent Detection Links

- Higher spectral efficiency.
- Higher receiver sensitivity.

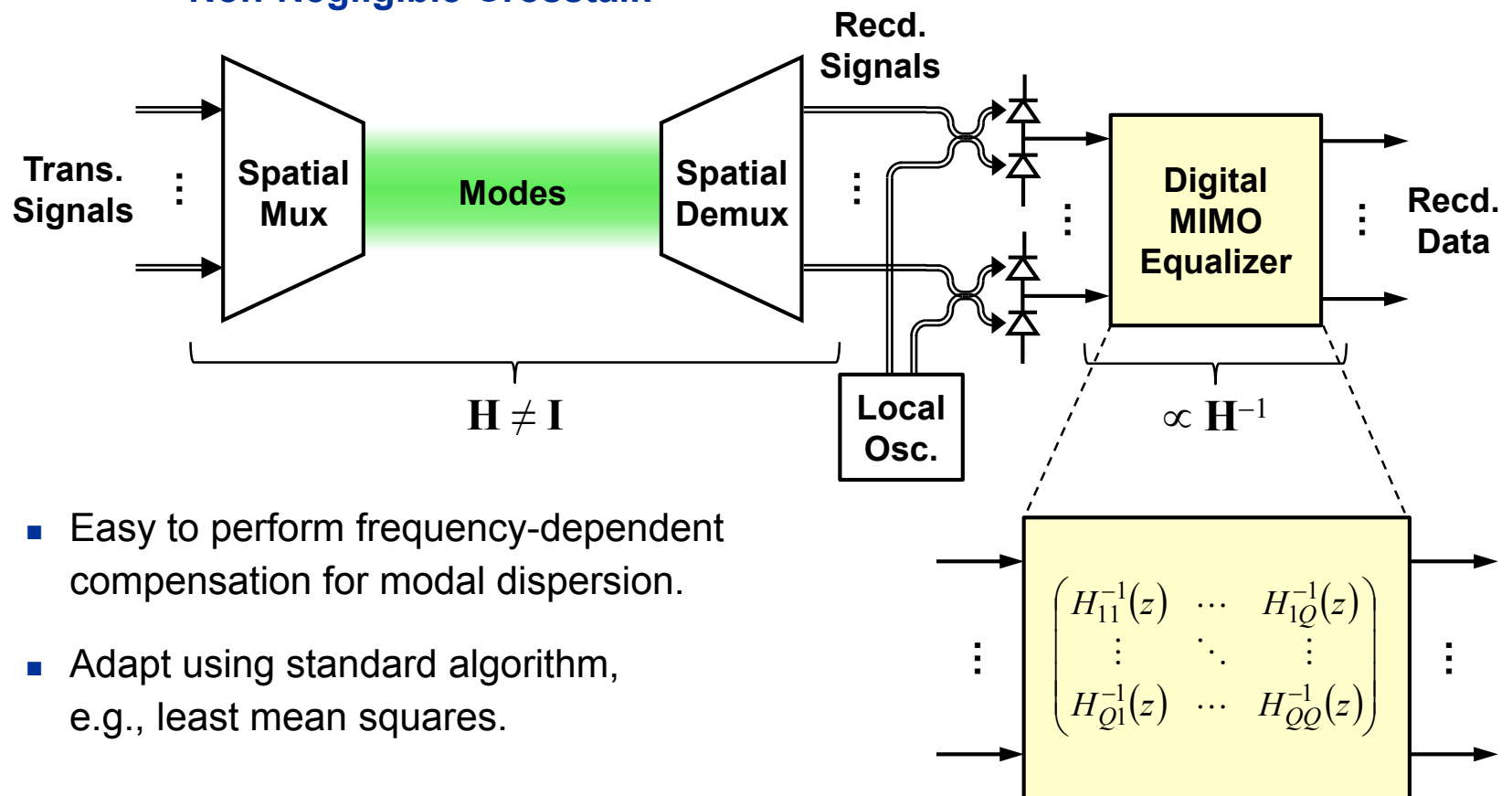
Non-Negligible Crosstalk



Coherent Detection Links

- Higher spectral efficiency.
- Higher receiver sensitivity.

Non-Negligible Crosstalk

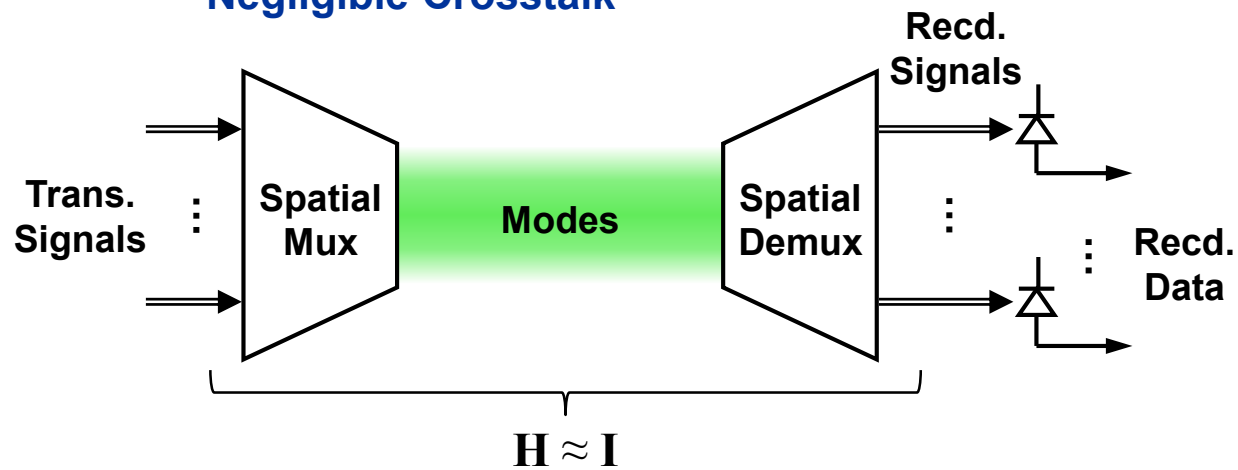


- Easy to perform frequency-dependent compensation for modal dispersion.
- Adapt using standard algorithm, e.g., least mean squares.

Direct Detection Links

- Lower spectral efficiency.
- Lower receiver sensitivity.

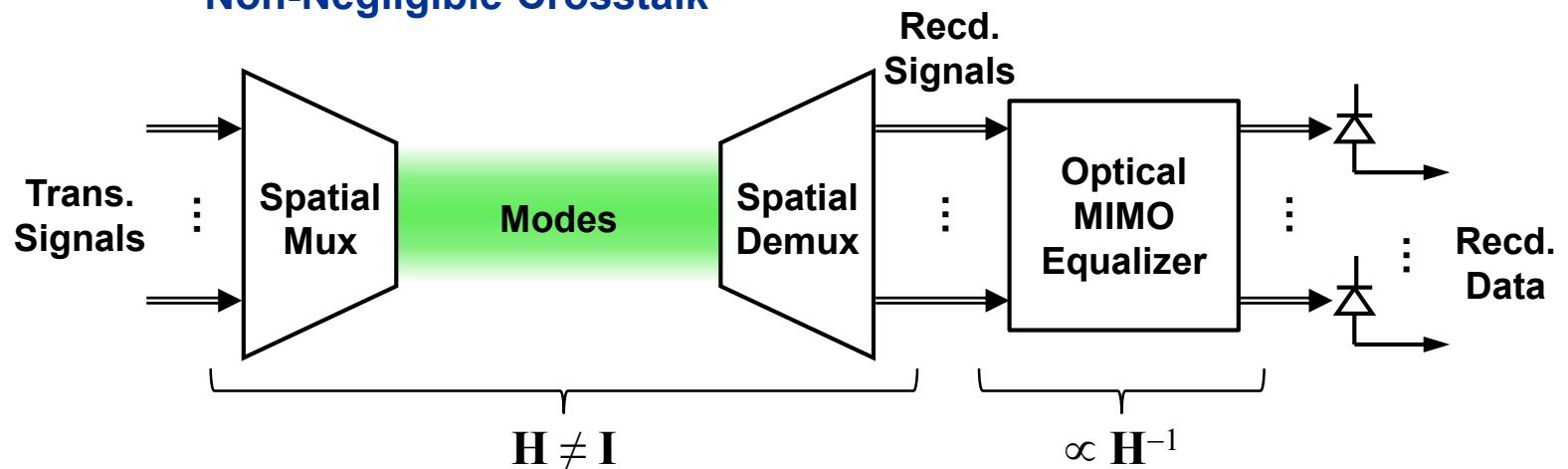
Negligible Crosstalk



Direct Detection Links

- Lower spectral efficiency.
- Lower receiver sensitivity.

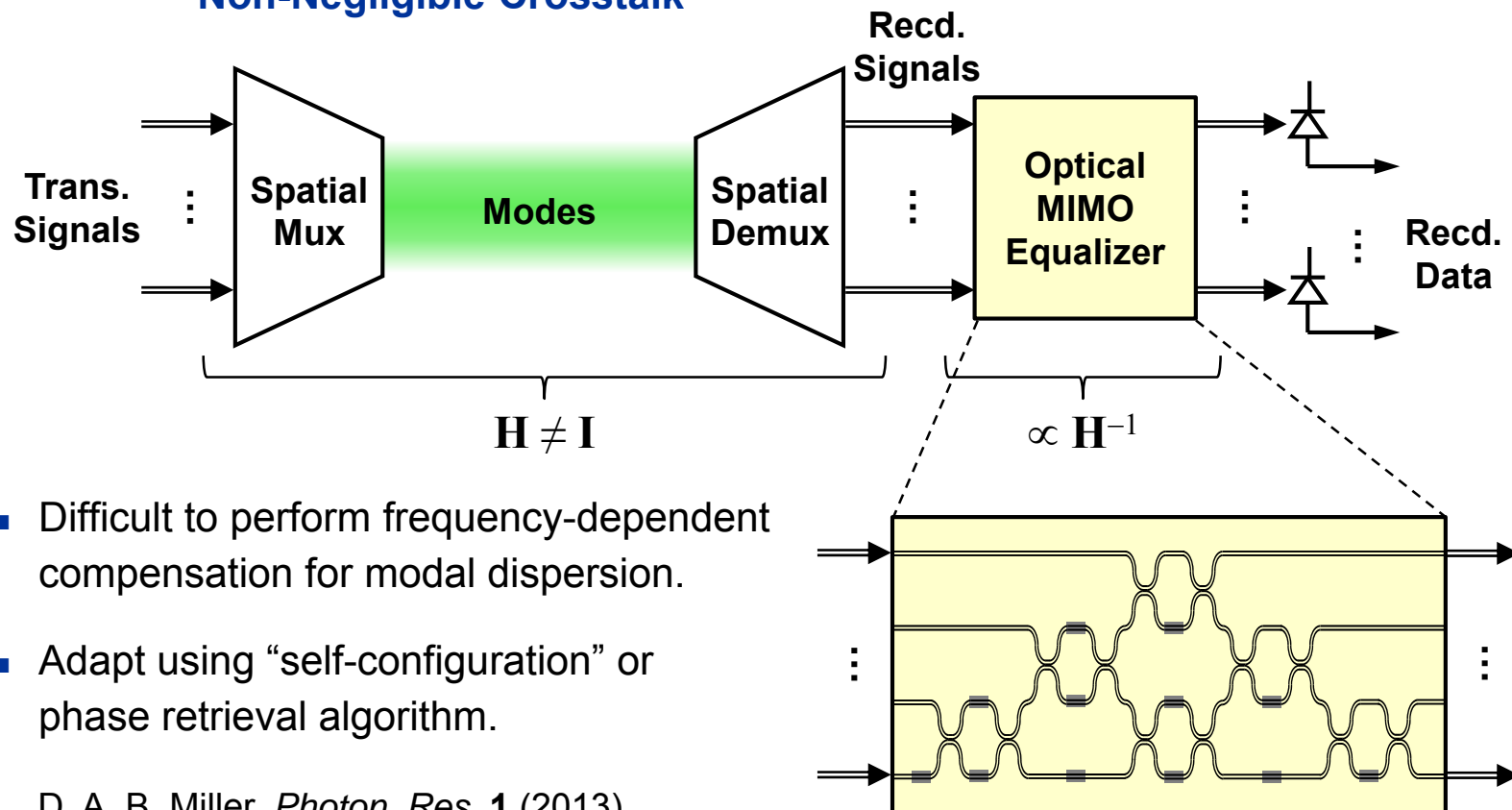
Non-Negligible Crosstalk



Direct Detection Links

- Lower spectral efficiency.
- Lower receiver sensitivity.

Non-Negligible Crosstalk



- Difficult to perform frequency-dependent compensation for modal dispersion.
- Adapt using “self-configuration” or phase retrieval algorithm.

D. A. B. Miller, *Photon. Res.* **1** (2013).

S. Ö. Arik and J. M. Kahn, *Opt. Lett.* **41** (2016).



Outline

- (De)Multiplexers and Link Designs
- **Physical Comparison: Counting Modes and Spatial Subchannels**
- Information-Theoretic Comparison: Capacity and Degrees of Freedom
- Discussion

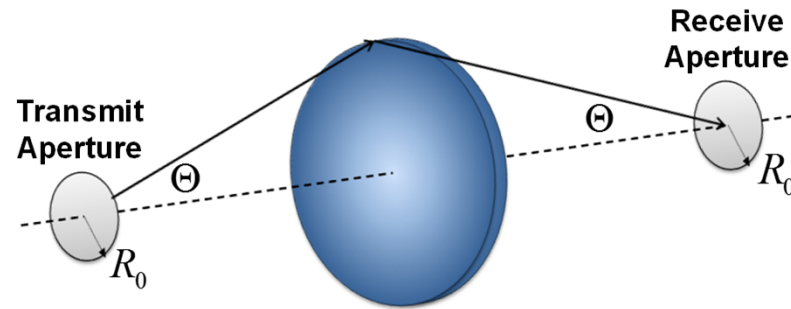
Capacity Limits of Spatially Multiplexed Links



- Use Q spatial modes to increase transmission capacity by a factor of Q . (Can use two polarizations to achieve $2Q$.)
- Design system so all Q modes pass with roughly equal gains (near-unitary transmission matrix).
- Constrain diameter \times numerical aperture (space-bandwidth product) and ask:

How does choice of mode set affect multiplexing gain Q ?

Canonical Symmetric One-Lens Link



- Given

R_0 radius of transmit and receive apertures

$NA = \sin\Theta$ numerical aperture of lens

the quantity

$$M = \frac{\pi R_0 NA}{\lambda} = \frac{V}{2}$$

is proportional to the link space-bandwidth product, and determines the number of modes that can propagate through the link.

- Provided no beam clipping occurs, the model can also describe:
 - Asymmetric one-lens link
 - Symmetric or asymmetric two-lens link
 - Ideal parabolic-index fiber

Counting Modes

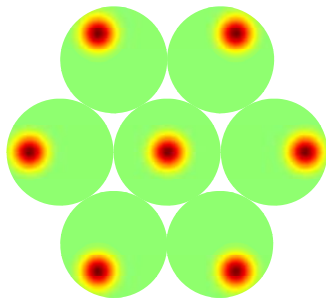
- How many modes Q can propagate through an optical system described by

$$M = \frac{\pi R_0 NA}{\lambda} = \frac{V}{2} \quad ?$$

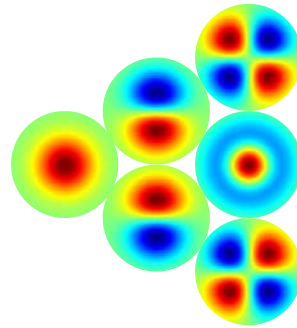
- This approach is approximate because:
 - The modes are not necessarily eigenfunctions of the optical system.
 - The optical system does not necessarily have a sharp cutoff.

Counting Modes (2)

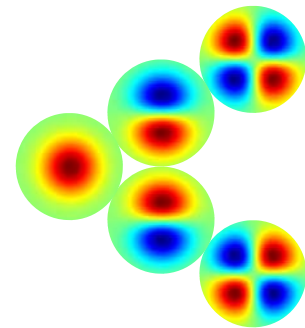
Very Low Space-BW Product: $M = 3$



Gaussian
 $Q \approx 7$



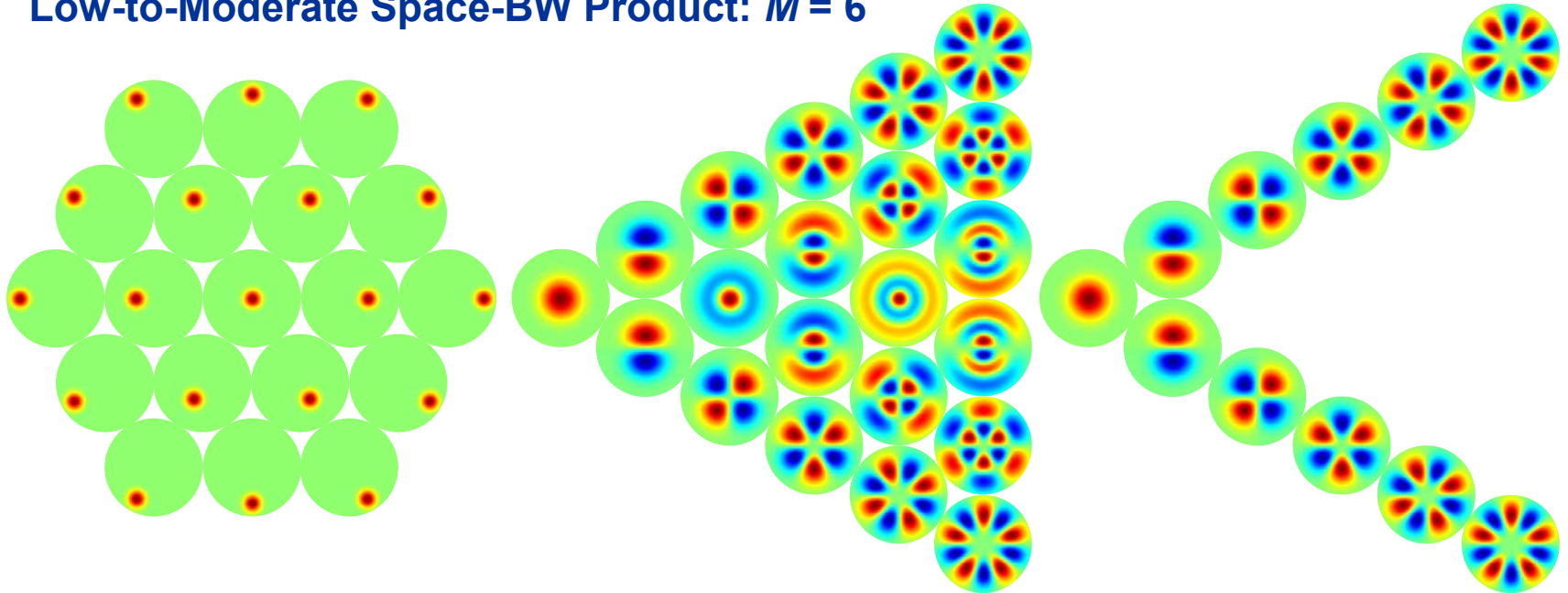
Laguerre-Gaussian
 $Q \approx 6$



Orbital Angular Momentum
 $Q \approx 5$

Counting Modes (2)

Low-to-Moderate Space-BW Product: $M = 6$



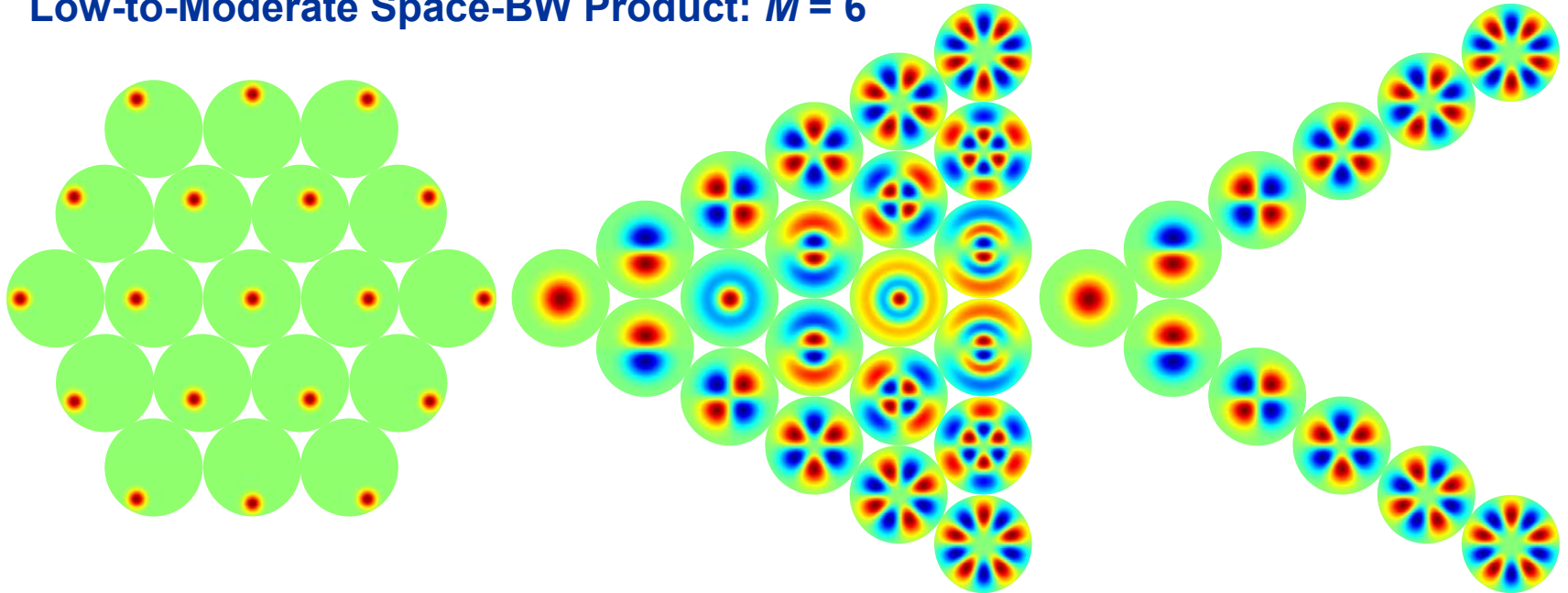
Gaussian
 $Q \approx 19$

Laguerre-Gaussian
 $Q \approx 21$

Orbital Angular Momentum
 $Q \approx 11$

Counting Modes (2)

Low-to-Moderate Space-BW Product: $M = 6$



Gaussian

$Q \approx 19$

$Q \sim M^2$

Laguerre-Gaussian

$Q \approx 21$

$Q \sim M^2$

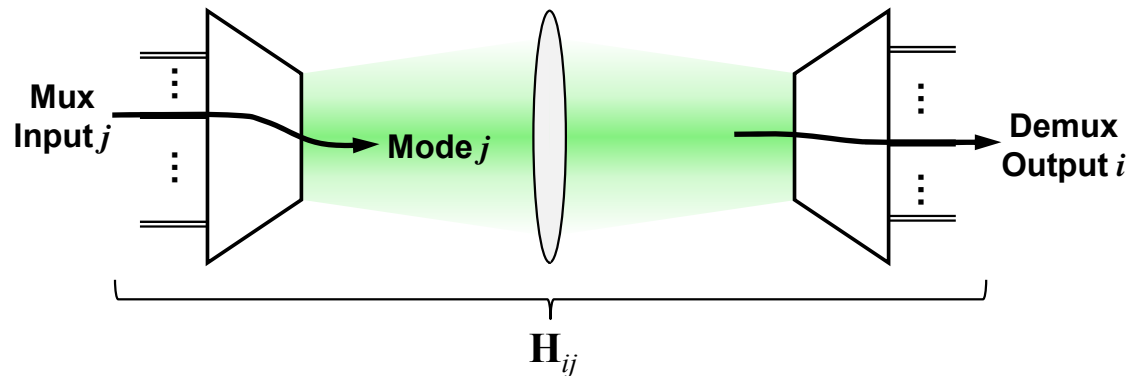
Orbital Angular Momentum

$Q \approx 11$

$Q \sim M$

- For details, see N. Zhao et al, *Nature Photonics* **9**, 822 (2015).

Transmission Matrix



- Assume ideal multiplexer and perfect alignment (for now).
- \mathbf{H}_{ij} is transmission coefficient between mode j and output i .
- \mathbf{H} includes modes far beyond nominal cutoff determined by mode counting.

L-G, OAM and Gaussian Modes

- Consider imaging demultiplexer designed for Gaussian beams.
- Diffraction loss + crosstalk $\rightarrow \mathbf{H}$ is non-diagonal and non-unitary.

OAM Modes Only

- Also consider ideal OAM demultiplexer (optimistic).
- Diffraction loss $\rightarrow \mathbf{H}$ is diagonal and non-unitary.



Counting Spatial Subchannels

- Perform a singular value decomposition of the transmission matrix:

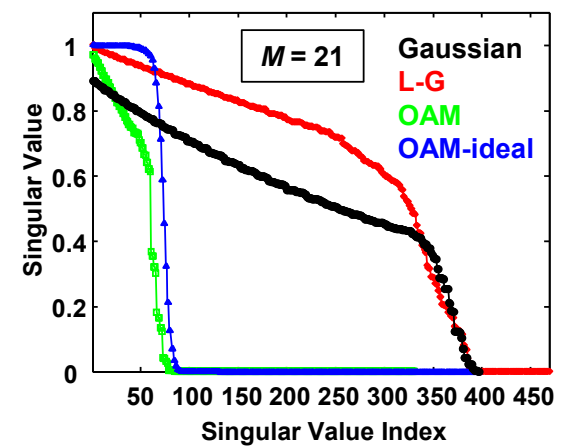
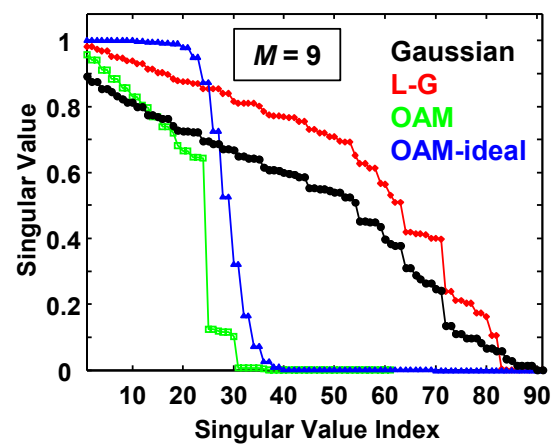
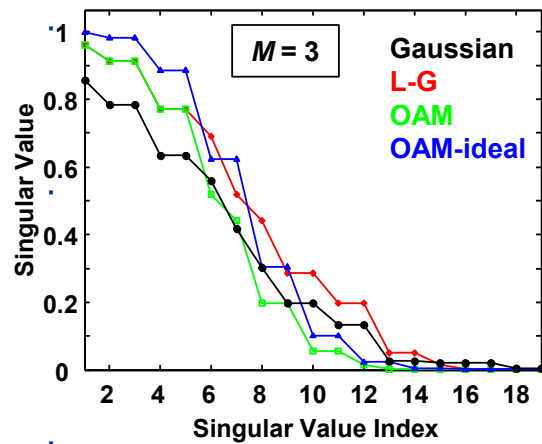
$$\mathbf{H} = \mathbf{V} \mathbf{D} \mathbf{U}^H$$

- \mathbf{U} and \mathbf{V} are unitary matrices. Their columns are transmit and receive bases that diagonalize \mathbf{H} into uncoupled spatial subchannels.
- \mathbf{D} is a diagonal matrix of the singular values:

$$\mathbf{D} = \text{diag}(\sqrt{\lambda_1}, \dots, \sqrt{\lambda_Q})$$

- $\{\lambda_1, \dots, \lambda_Q\}$ are eigenvalues of $\mathbf{H}\mathbf{H}^H$, representing power gains of spatial subchannels.

Counting Spatial Subchannels





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Channel Capacity

Capacity depends on:

- Operating SNR.
- Number of subchannels Q and their gains $\{\lambda_1, \dots, \lambda_Q\}$.

Assume:

- Transmitter knows $\{\lambda_1, \dots, \lambda_Q\}$ and beamforming matrix \mathbf{U} .
- Equal noise power σ^2 per receiver.
- Constraint on total transmit power $P = \sum_{q=1}^Q P_q$.
- Total SNR is $SNR = \frac{1}{\sigma^2} \sum_{q=1}^Q P_q$.

Channel Capacity

Capacity depends on:

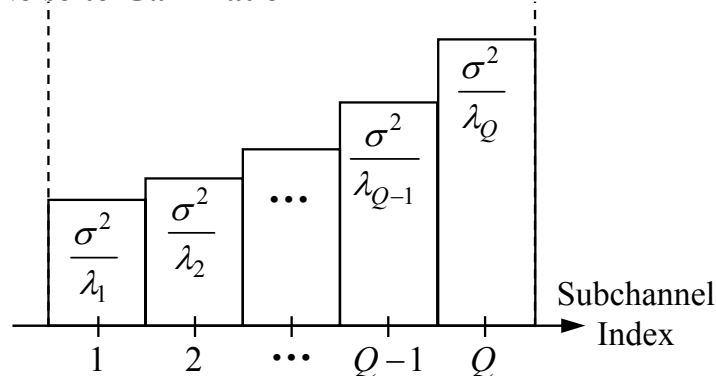
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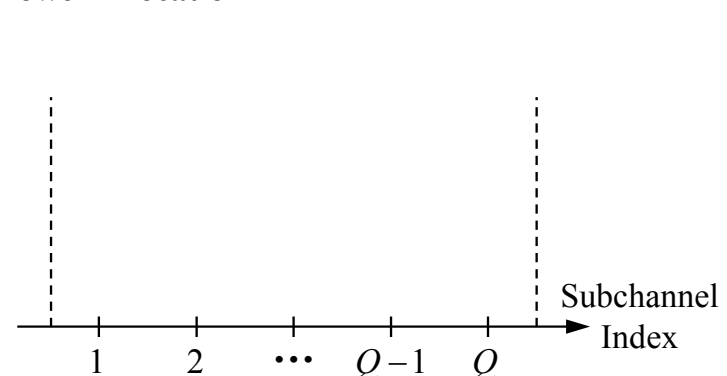
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Optimal Power Allocation

Noise-to-Gain Ratio



Power Allocation



Channel Capacity

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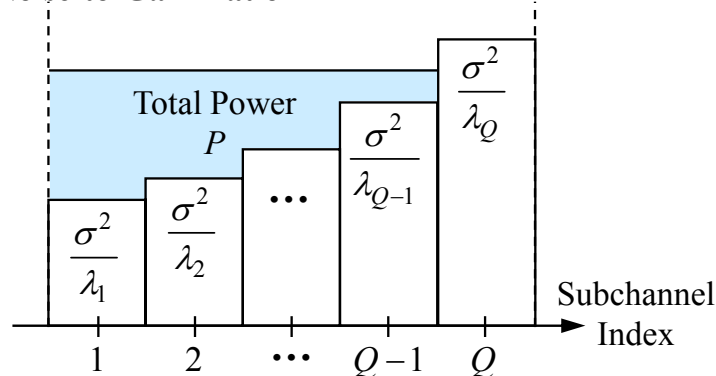
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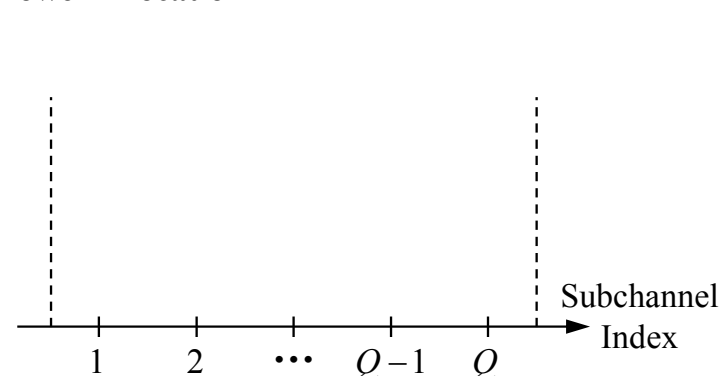
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Optimal Power Allocation

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Power Allocation



Channel Capacity

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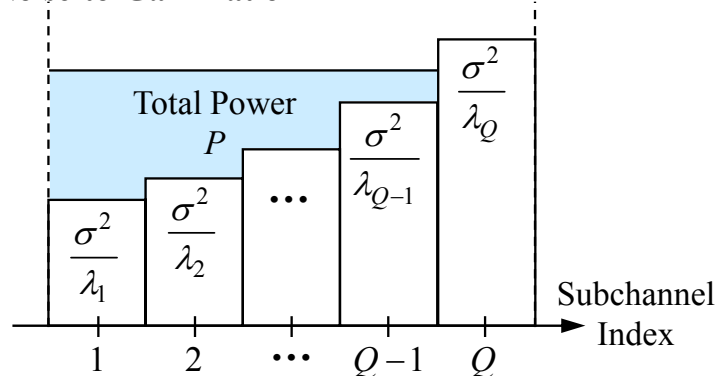
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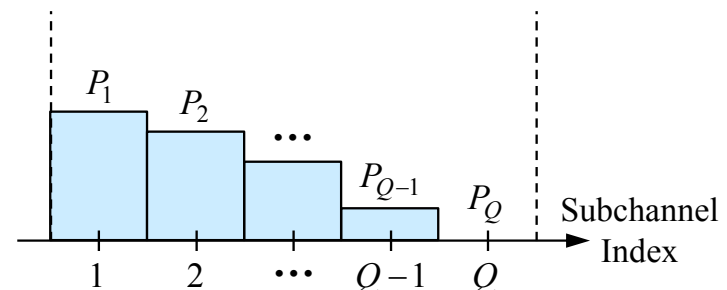
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Optimal Power Allocation

Noise-to-Gain Ratio

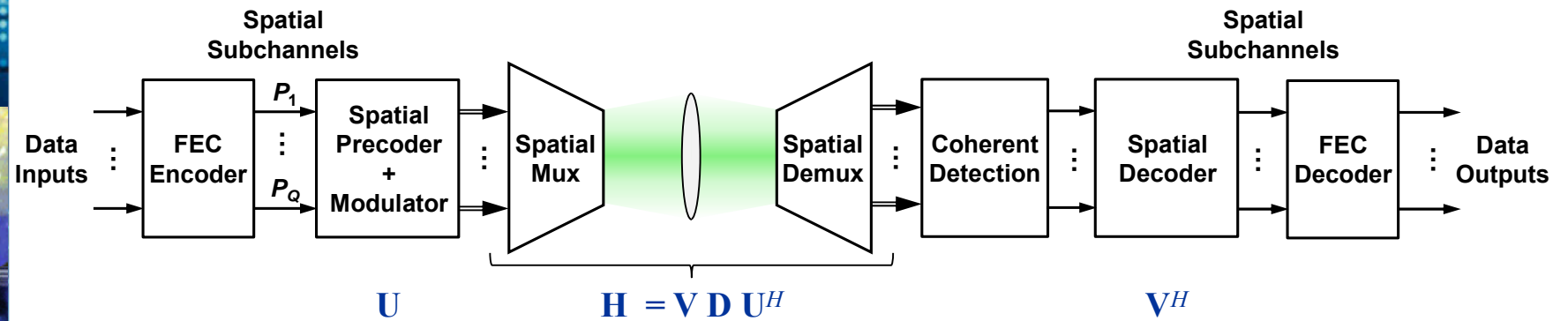


Power Allocation



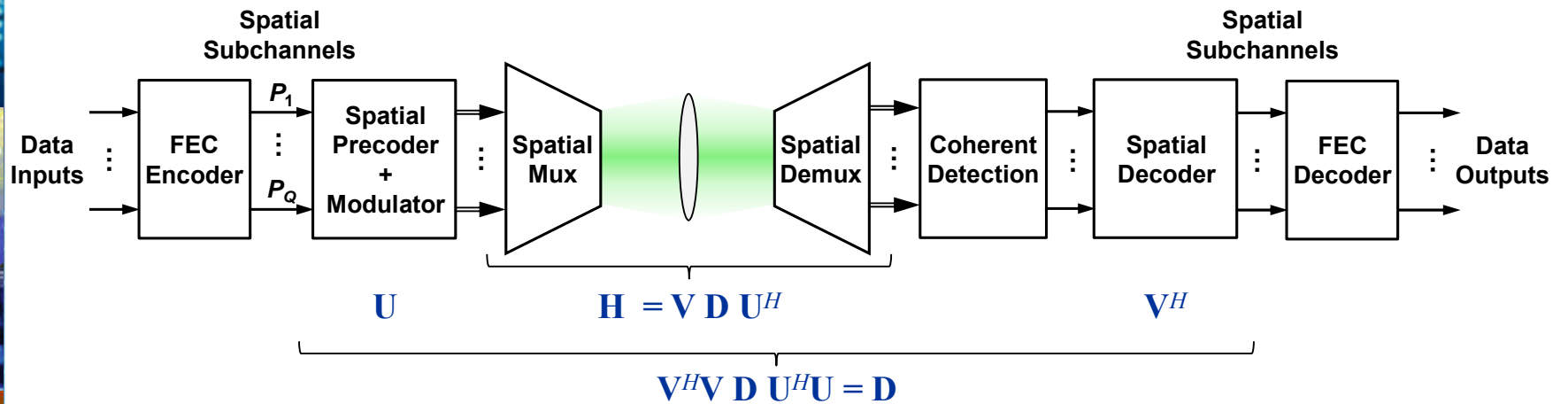
Channel Capacity (2)

Capacity-Achieving Transmission System



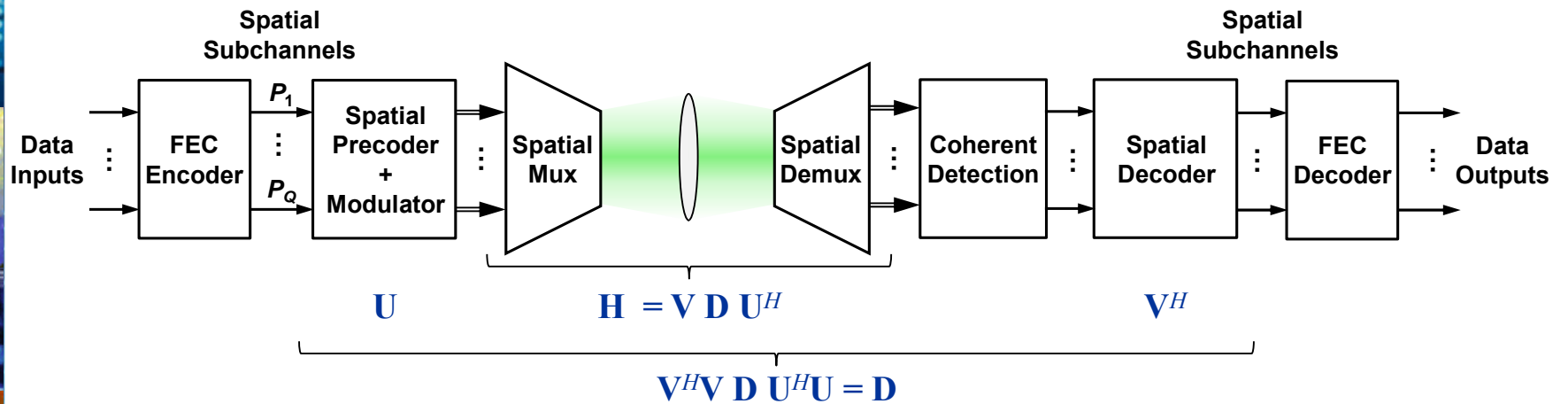
Channel Capacity (2)

Capacity-Achieving Transmission System



Channel Capacity (2)

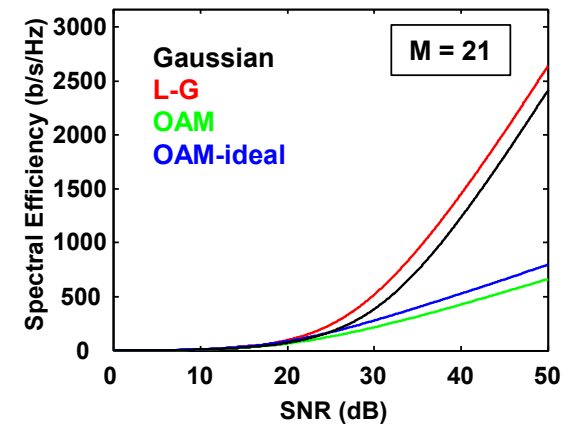
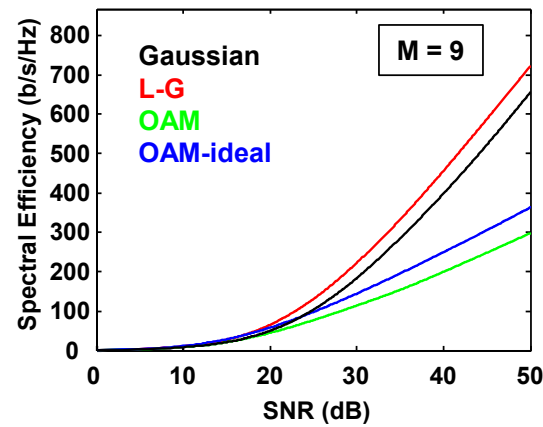
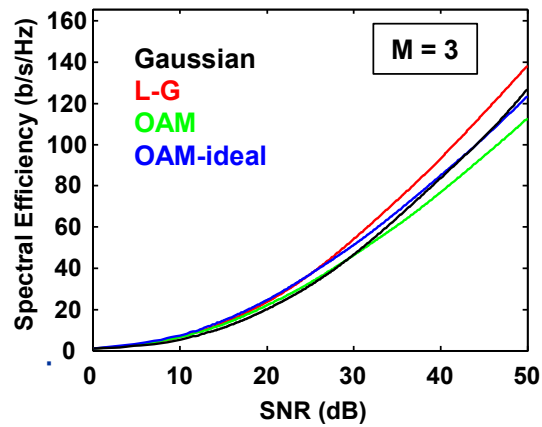
Capacity-Achieving Transmission System



- Capacity per unit bandwidth:

$$SE = \sum_{q=1}^Q \log_2 \left(1 + \frac{\lambda_q P_q}{\sigma^2} \right)$$

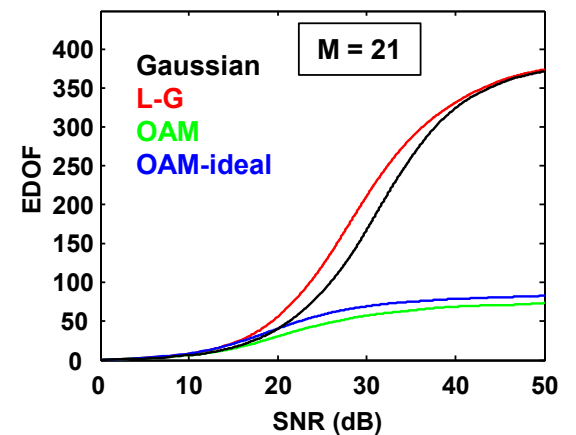
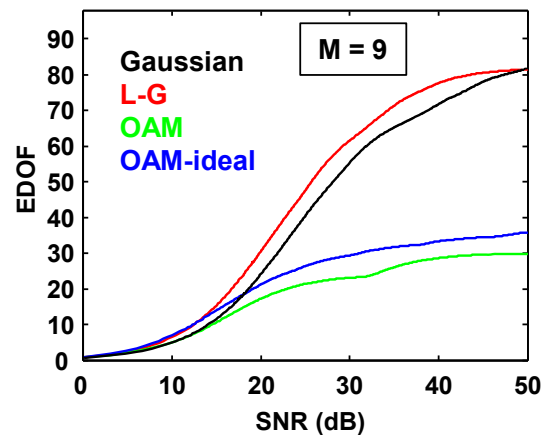
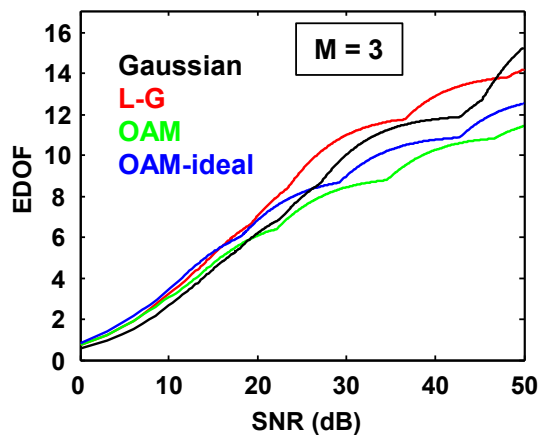
Channel Capacity (2)



Effective Degrees of Freedom

- Number of subchannels effectively conveying information:

$$EDOF = \left. \frac{d}{d\delta} SE(2^\delta P) \right|_{\delta=0}$$



- Low SNR: power-limited, $EDOF$ independent of Q .
- High SNR: mode-limited, $EDOF$ approaches Q .

D.-S. Shiu et al, *Trans. Commun.* **48**, 502 (2000).



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Discussion

- In choosing a mode set for spatial multiplexing, one should consider:
 - Completeness of the set
 - Ease of implementation

Whether or not the set includes OAM is irrelevant.

- Given a space-bandwidth product M , a system should:
 - Operate at a multiplexing gain below the maximum Q .
 - Demultiplex to a complete mode set (L-G, H-G or Gaussian)

to optimize tolerance to:

- Misalignment
 - Atmospheric turbulence
- Free-space communications is most compelling over long links, where:
 - Alignment and capturing the entire beam can be difficult.
 - Atmospheric turbulence can be significant.

These conditions are more favorable for WDM than SDM.

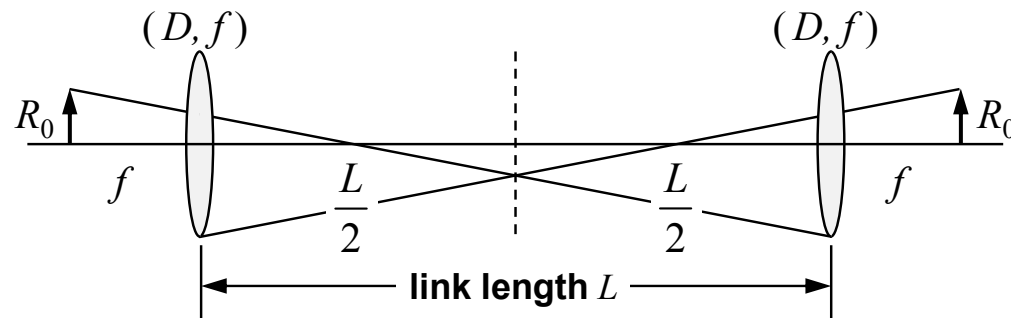
Do free-space links require more capacity than WDM offers?



Backup Slides



Symmetric Two-Lens Link



- Assume $L \gg f$. Then

$$M = \frac{\pi R_0 NA}{\lambda}, \text{ where } NA = \frac{D}{2f},$$

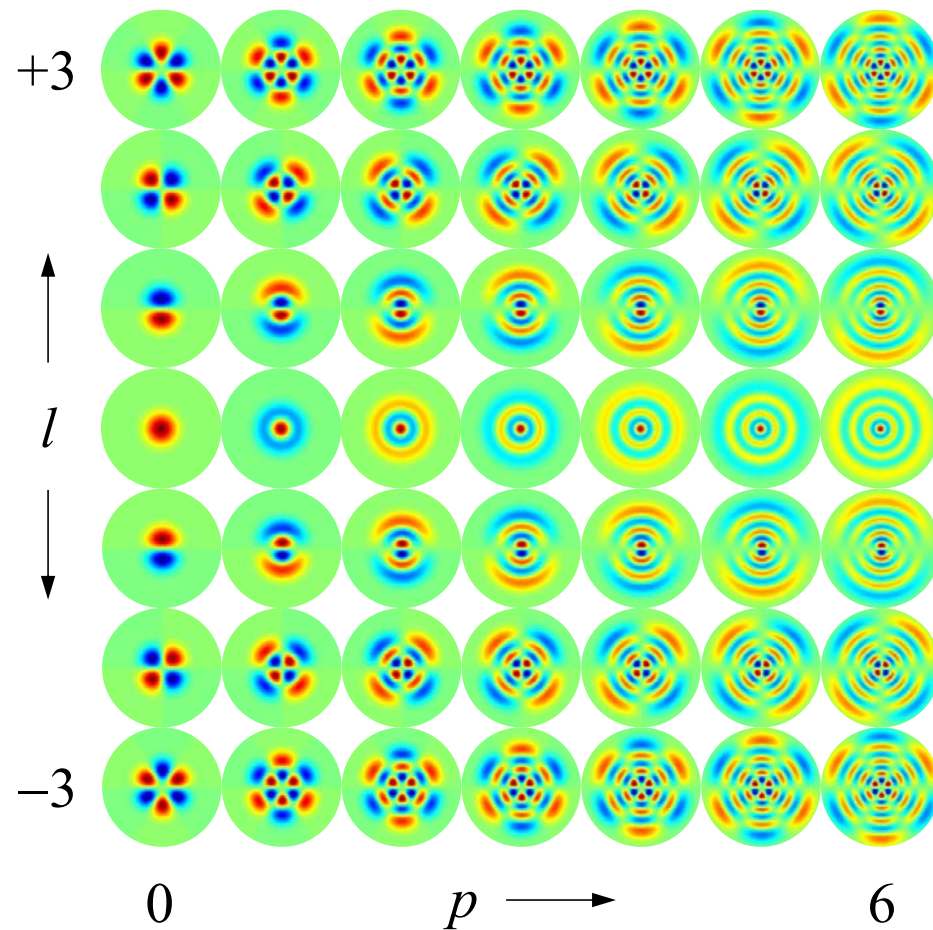
provided $D \geq 2 R_0 L / f$ to avoid clipping.

- Matched designs ($D = 2 R_0 L / f$) for Gaussian beams, 30- μm pitch in Tx/Rx planes:

R_0 (μm)	f (cm)	D (cm)	NA	L (m)	M
65	12.5	1	0.04	10	5
65	125	10	0.04	1000	5
250	25	2	0.04	10	20
250	250	20	0.04	1000	20

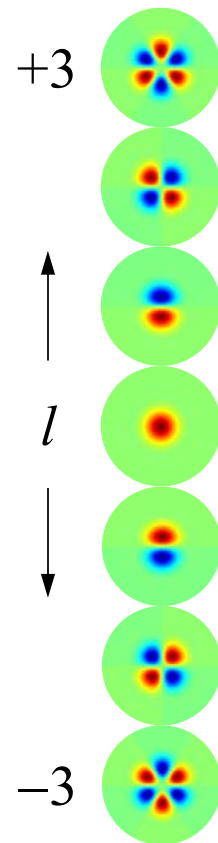
L-G and OAM Modes

$$LG_{pl}(r, \phi) = \sqrt{\frac{2p!}{\pi(p+|l|)!}} \frac{1}{\omega_0} \exp\left(\frac{-r^2}{\omega_0^2}\right) L_p^{|l|}\left(\frac{2r^2}{\omega_0^2}\right) \left(\frac{r\sqrt{2}}{\omega_0}\right)^{|l|} e^{il\phi}$$



L-G and OAM Modes

$$OAM_l(r, \phi) = LG_{0l}(r, \phi)$$



Counting Laguerre-Gaussian Modes

- Laguerre-Gaussian mode of order (p, l) :

$$LG_{pl}(r, \phi) = \sqrt{\frac{2p!}{\pi(p+|l|)!}} \frac{1}{\omega_0} \exp\left(\frac{-r^2}{\omega_0^2}\right) L_p^{|l|}\left(\frac{2r^2}{\omega_0^2}\right) \left(\frac{r\sqrt{2}}{\omega_0}\right)^{|l|} e^{il\phi}$$

- Define mode order:

$$m = 2p + |l| + 1$$

$$\begin{aligned} \omega_m &= \sqrt{m} \omega_0 && \text{r.m.s. waist size} \\ \theta_m &= \sqrt{m} \theta_0 && \text{r.m.s. divergence} \\ \frac{\pi}{\lambda} \omega_m \theta_m &= m && \text{space-bandwidth product} \end{aligned}$$

- Modes up to order $m = M$ can propagate through the system.
Counting (p, l) such that $2p + |l| + 1 \leq M$:

$$Q_{\text{LG/HG}} \geq \frac{M(M+1)}{2}$$

- For tighter bounds, see N. Zhao et al, *Nature Photonics* **9**, 822 (2015).

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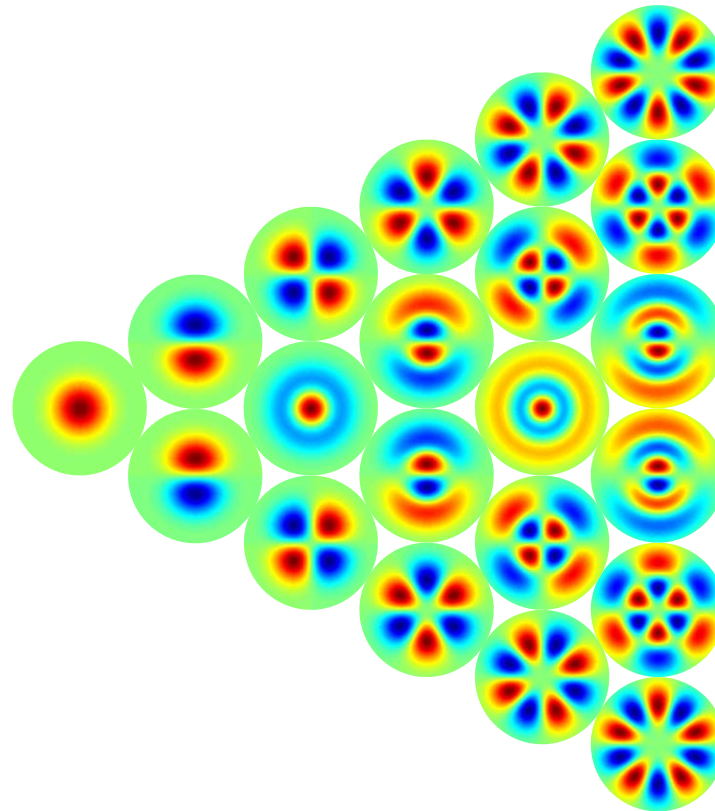
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Counting Laguerre-Gaussian Modes



$$1 \quad m = 2p + |l| + 1 \longrightarrow 6$$

$$\underbrace{\hspace{15em}}_{Q \approx 21 \text{ for } M = 6}$$



Counting Orbital Angular Momentum Modes

- For a given l , the mode with $p=0$ has the smallest space-bandwidth product. OAM multiplexing typically uses modes with $p=0$:

$$OAM_l(r, \phi) = LG_{0l}(r, \phi)$$

- Modes up to order $m = M$ can propagate through the system. Counting l such that $|l| + 1 \leq M$:

$$Q_{\text{OAM}} \approx 2M - 1$$



Counting Orbital Angular Momentum Modes

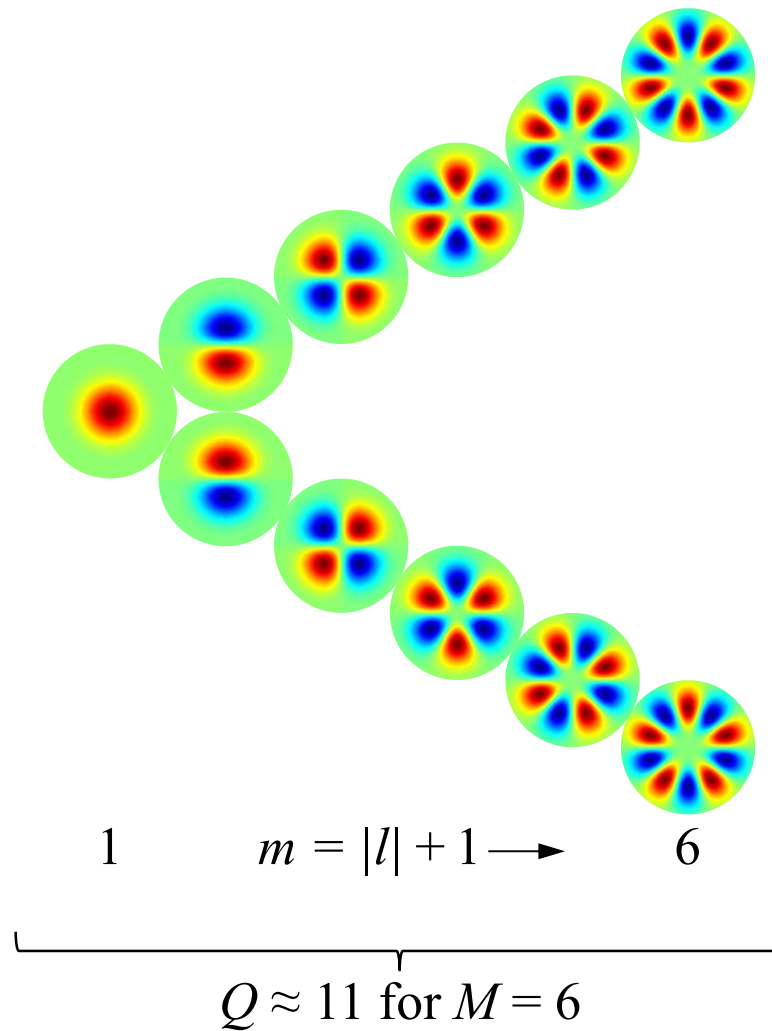
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Counting Orbital Angular Momentum Modes



Counting Gaussian Beams

- Each Gaussian beam can have the same r.m.s. divergence angle as an M th-order L-G mode:

$$\theta_{0,G} = \theta_{M,LG} \quad \text{or} \quad \frac{\lambda}{\pi \omega_{0,G}} = \frac{M\lambda}{\pi \omega_{M,LG}}$$

- Hence:

$$\omega_{0,G} = \frac{\omega_{M,LG}}{M} = \frac{\omega_{0,LG}}{\sqrt{M}}$$

- The number of Gaussian beams that fit in the same area as an M th-order L-G mode is equivalent to the number of circles of radius $\omega_{0,LG} / \sqrt{M}$ that fit in a circle of radius $\sqrt{M} \omega_{0,LG}$, which is estimated to be:

$$Q_G \leq 0.9 M^2$$

This bound is loose for small M .

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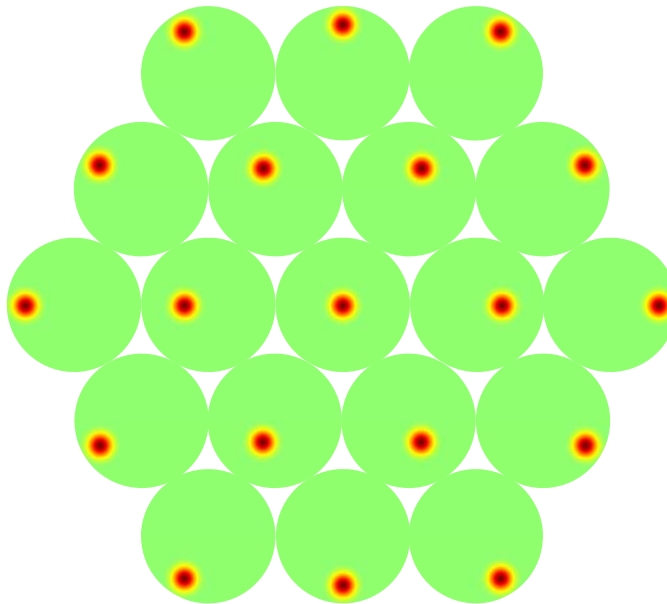
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Counting Gaussian Beams



$$Q \approx 19 \text{ for } M = 6$$

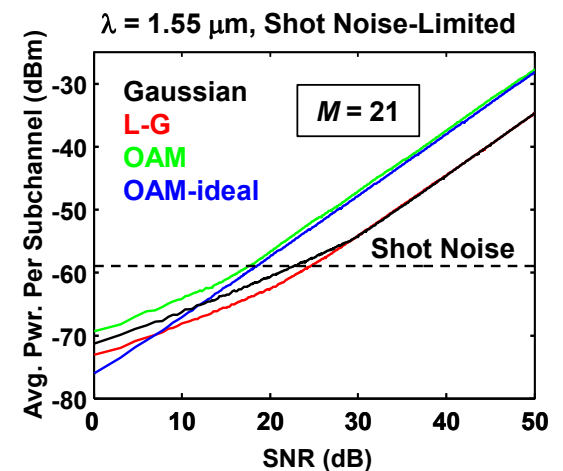
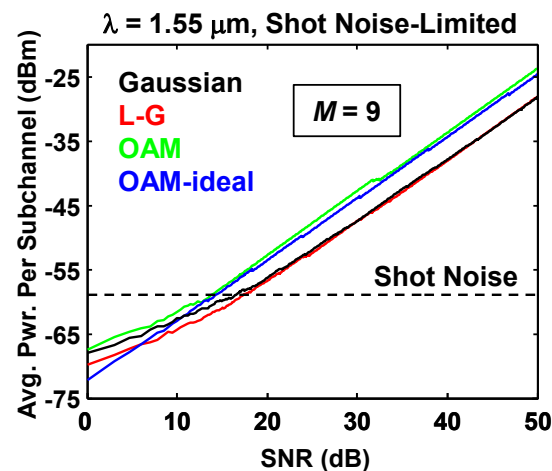
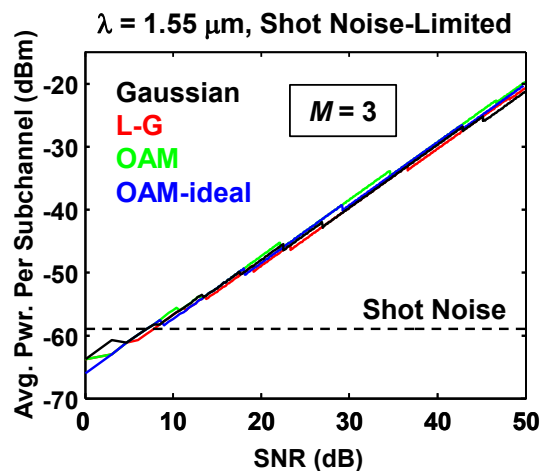
Transmitted Powers

Assume

- $R_s = 10$ Gbaud, $\lambda = 1550$ nm.
- Shot noise-limited coherent receiver with noise variance per receiver corresponding to one photon per symbol:

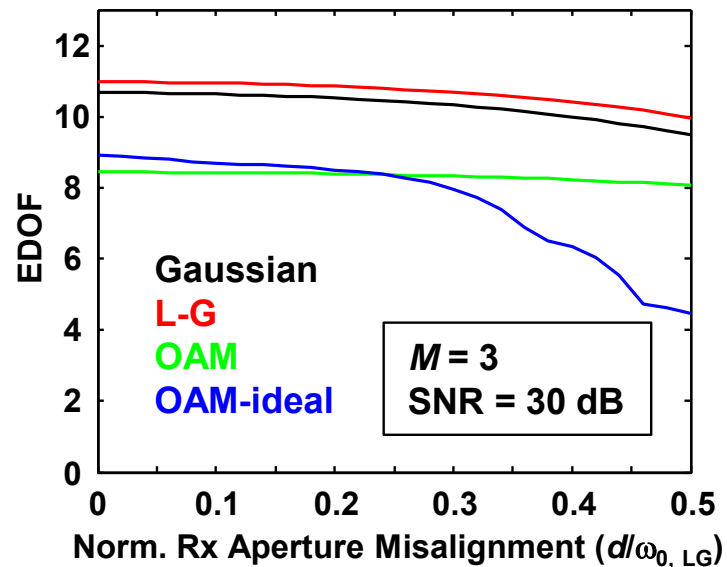
$$\sigma^2 = \frac{hcR_s}{\lambda} = -59 \text{ dBm}$$

Average Transmit Power per Subchannel



Impact of Receiver Misalignment

- Consider $M = 3$, where OAM is most competitive.



- OAM with ideal demultiplexer tolerates least misalignment because of phase mismatch.
- Gaussian, L-G or OAM with imaging demultiplexer tolerate greater misalignment.