# Holistic Representation for 3D Objects using Quantum Teleportation

CASIS 2018 meeting
Lawrence Livermore National Laboratory

G. Chapline
Lawrence Livermore National Laboratory

May 23, 2018

LLNL-PRES-XXXXXX
This work was performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under contract DE-AC52-07NA27344. Lawrence Livermore National Security, LLC

Lawrence Livermore National Laboratory

# Outline

- Construction of 3-dimensional shapes using quantum teleportation within an array of quantum oscillators of entangled states to build the #D shape from tetrahedral building blocks

- The “inverse” problem of generating the control instructions for emulating the shape of a given object
  - The inverse problem also provides a framework for using Bayesian methods to search for objects of interest

# Angular momentum a la Schwinger

## J. Schwinger 1951

$$J \equiv \hbar \sum_{\varsigma,\varsigma'=1,2} a_{\varsigma}^{+} \left\langle \varsigma \left| \frac{\sigma}{2} \right| \varsigma' \right\rangle a_{\varsigma'}$$

$$\left[a_{\varsigma}, a_{\varsigma'}\right] = \left[a_{\varsigma}^{+}, a_{\varsigma'}^{+}\right] = 0 \qquad \left[a_{\varsigma'}, a_{\xi}^{+}\right] = \delta_{\varsigma\varsigma'}$$

where $J_{+} = \hbar a_1^{+} a_2 \qquad J_{-} = \hbar a_2^{+} a_1 \qquad J_3 = \frac{\hbar}{2}\left(a_1^{+} a_1 - a_2^{+} a_2\right)$

$$J^2 = \hbar^2 \left(\frac{n^2}{4} + \frac{n}{2}\right) = \hbar^2 \frac{n}{2}\left(\frac{n}{2} + 1\right)$$

# 6j symbols

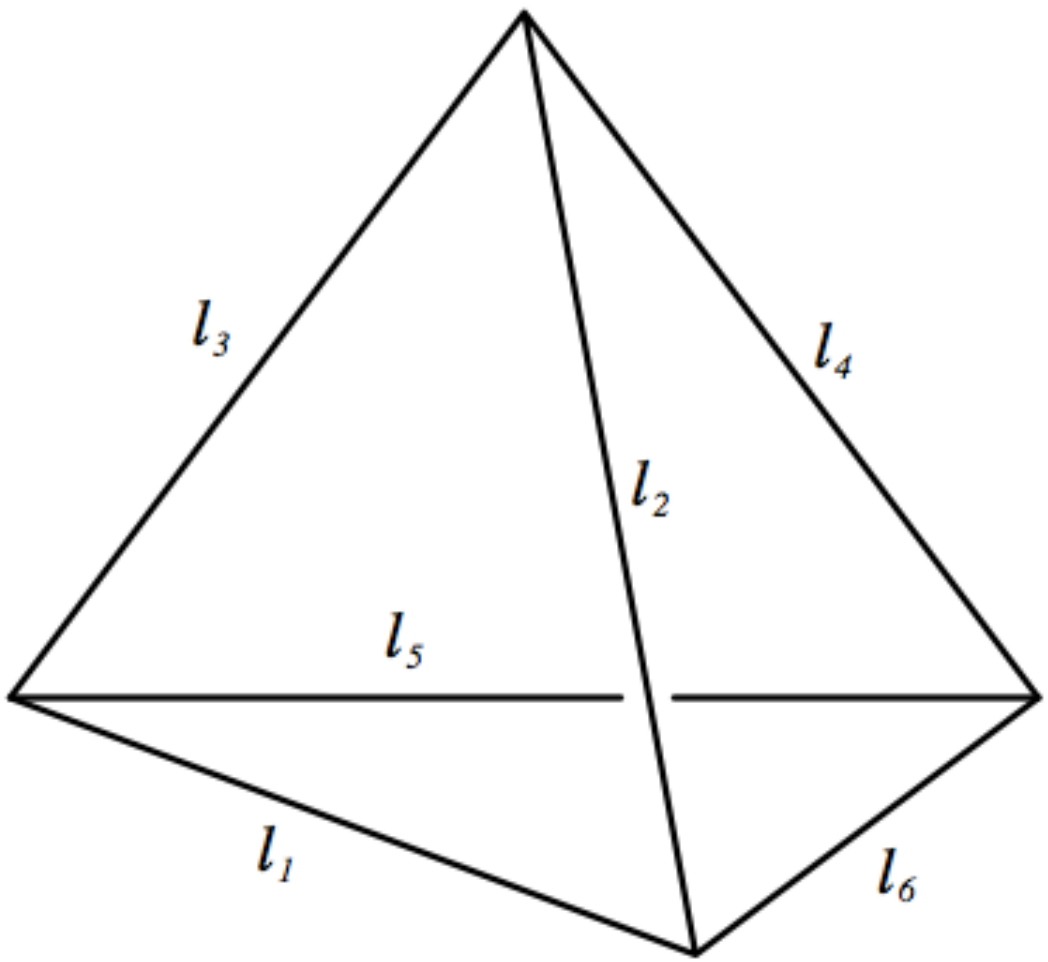
The sum of 3 angular momenta $j_1$, $j_2$ , $j_3$, can be carried out by by adding $j_1 + j_2$ to $j_3$ or $j_2 + j_3$ to $j_1$. The quantum states for the sum of $j_1$, $j_2$ , $j_3$, for these two coupling schemes are related by a unitary transformation:

$$\left|Jmj_{23}\right\rangle = \sum_{m_1,m_2} \left\langle j_{12}\middle|j_{23}\right\rangle \left|Jmj_{12}\right\rangle$$

Using the relations between $\langle m_1 m_2 | jm \rangle$ for j = $j_{12}$, $j_{23}$, J and the 3j symbols one obtains:

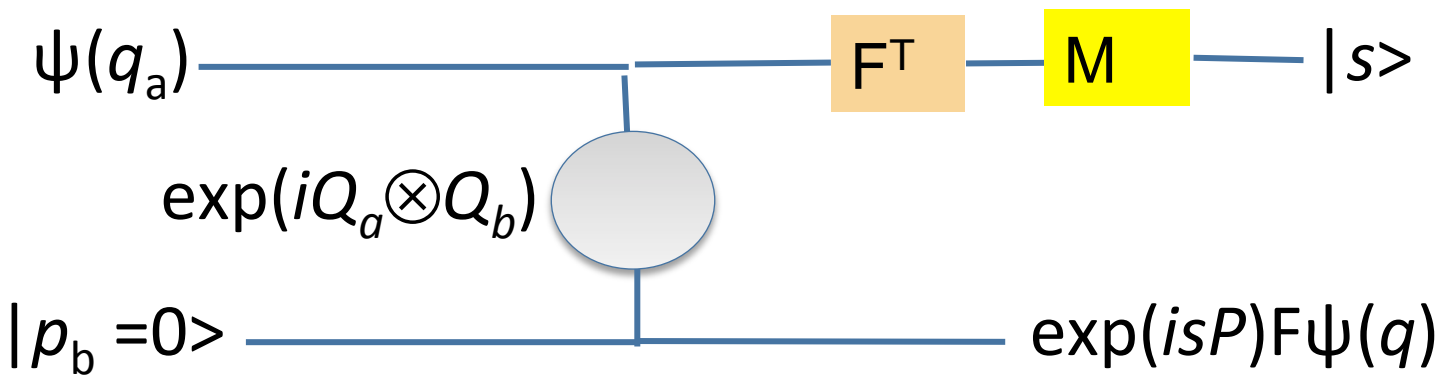
$$\left\langle j_{12}\middle|j_{23}\right\rangle = (-1)^{j_1+j_2+j_3+J}\sqrt{(2j_{12}+1)(2j_{23}+1)}\sum_{m_1,m_2}\begin{pmatrix} j_1 & j_2 & j_{12} \\ j_3 & J & j_{23} \end{pmatrix}$$

# Wigner noticed that the 6j symbol can be associated with a tertrahedron whose sides have lengths equal to the quantized angular momenta that appear in the 6j symbol



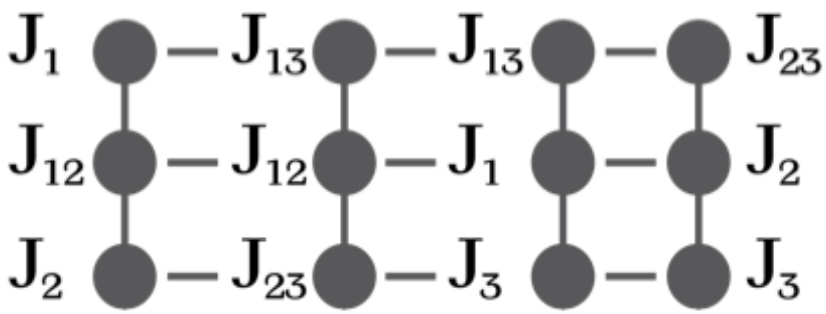
# The centerpiece of quantum computing is teleportation (Nielson & Chuang 2000)

- Teleportation of a continuous variable state $\psi(q_a)$ defined on a set of nodes $\{i_a\}$ at one end of a quantum wire begins by using a controlled phase gate $\exp(iQ_a \otimes Q_b)$ to entangle the input and initial target state $|P_b = 0\rangle$. A measurement of $P_a$ after entanglement results in a distortion of the initial state that depends on the result of the measurement.



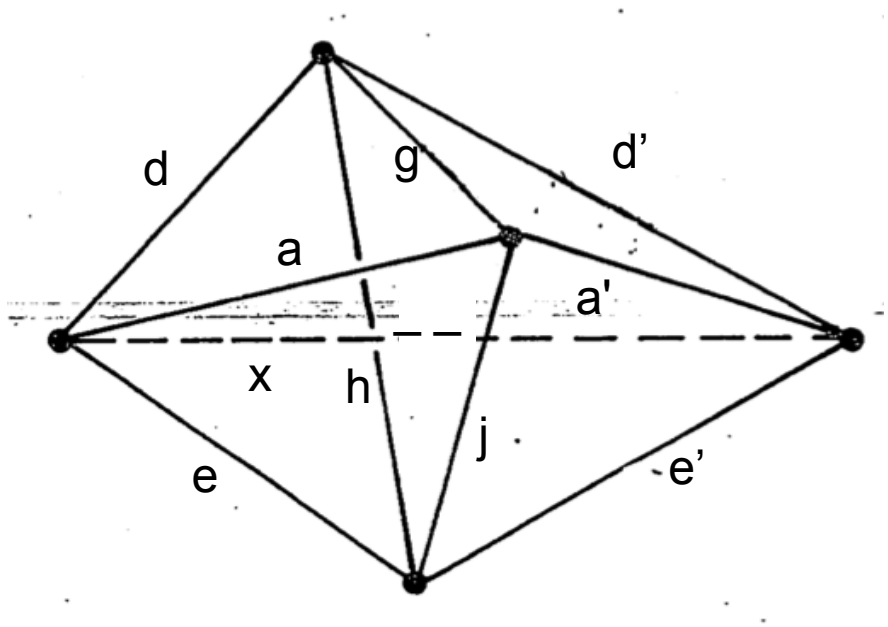
# Fundamental invention for constructing 3-dimensional objects

- An input state $|J_1 + J_{12} + J_2 = 0\rangle$ representing a triangle localized on column 1 can be teleported to any other location in 3-dimensions using a quantum wire consisting of triples of 2D oscillators



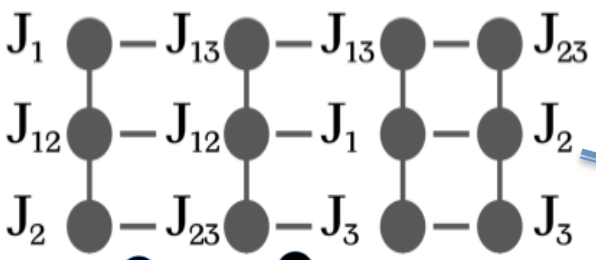

$$\prod_{a,b,c} \left|\phi_{i+1} = 0\right\rangle \rightarrow R(\phi_i) \sum_{m_a, m_b, m_c} X(m_i) \begin{pmatrix} j_{ia} & j_{ib} & j_{ic} \\ m_{ia} & m_{ib} & m_{ic} \end{pmatrix} \left|j_{ia}, m_{ia}\right\rangle \left|j_{ib}, m_{ib}\right\rangle \left|j_{ic}, m_{ic}\right\rangle$$

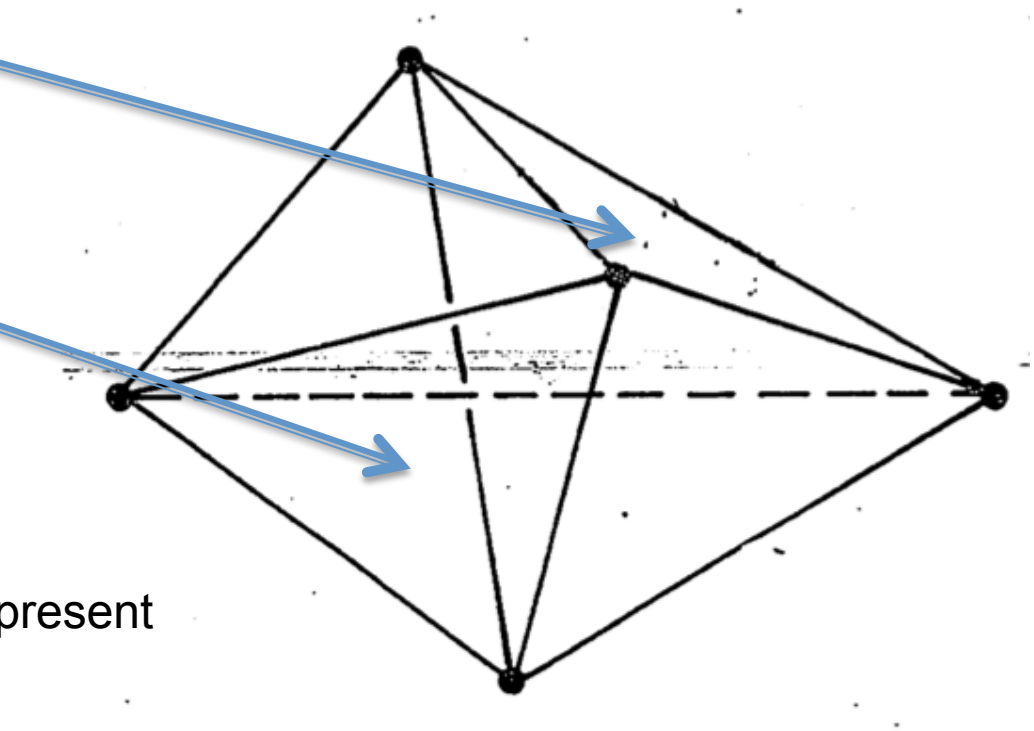
# Example: A double pyramid can be constructed by teleportation along a “figure 8” knot




| a | a | d | e | j | g | h | h | d | d |
|---|---|---|---|---|---|---|---|---|---|
| j | j | d’ | e’ | e’ | d’ | d’ | g | g | h |
| e | e | x | x | a’ | a’ | e’ | j | a | e |

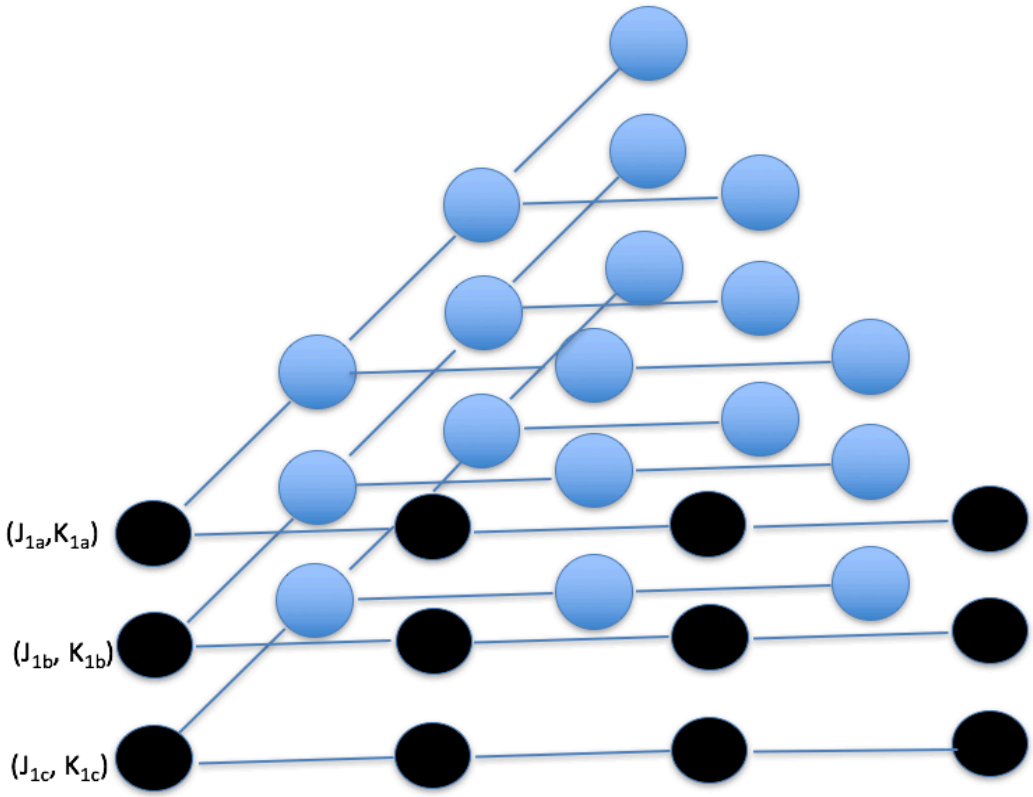
# Extending a quantum wire for triangles to a tetragonal lattice allows one to construct arrays of tetrahedrons in 3-dimensions



Adding columns in 3rd dimension allows to represent teleportation in a 3D tetrahedral array

# Arrays of 4-dimensional quantum oscillators are the stage for representating 3D geometries



# Inverse problem: determining the control parameters, viz $R(\theta_{ij})$, for constructing a given shape

A feed-back loop for controlling the shape of a a 3D object is defined by a matrix **A**(σ,x,t') which translates a signal I(σ,t) measuring the shape of the surface into a corrective displacement δ(σ,t) of the surface and a matrix **B**(σ,x) which relates the difference e(σ,t) ≡ δ(σ,t) + n(σ,t) between the actual and desired shapes of the surface to the change in the measurement signal I(σ,t) - $I_0$(σ) resulting from the feedback and noise n(σ,t). These matices satisfy the Dyson equations:

$A = KB^T I_0^{-1}$, where **K**($\sigma_1$,$\sigma_2$ ,$t_1$ ,$t_2$) is a causal matrix:

$$K+K^T+KB^T I_0\text{-}1BK^T+N=0$$

where N is the time averaged covariance function < n(σ)n(σ') > for the quantum fluctuations in the shape of the surface.

# Application to Bayesian object recognition

**Uncertainties in the the shape of a surface arising from quantum fluctuations in the orientations of the triangles used to represent a 3D object opens the door to using a Gaussian process neural network with a representing kernel generated by $C(I^n(\sigma),I^{n'}(\sigma)) + \sigma_n^2$ , where $C(I^n(\sigma),I^{n'}(\sigma))$ is the covariance matrix for shape measurements for a training set of "quantum surfaces". Numerical inversion of the kernel matrix allows one to rapidly generate modulo Gaussian noise the oscillator Fock space amplitudes corresponding to an observed shape**

**Specialized AI chips could be used to rapidly carry out the required matrix operations; however recognition of complex shapes would be exponentially faster using a quantum GPNN to generate the wave function of the oscillator array!**

# Summary

- Entangled Fock states of 4-dimensional quantum oscillators can be used to provide a holistic representation of any 3-dimensional object
- The “inverse” problem of generating the control instructions for arrays of quantum oscillators is a work in progress (A. Peterson, J.Dubois, L.Poyneer)

Longer term it appears that large arrays of quantum oscillators could be used to not only generate 3D shapes, but also use measurements of shapes to generate oscillator array wave functions that can be directly compared with the wave function for a particular shape being sought.