Holistic Representation for 3D Objects using Quantum Teleportation

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Outline

- Construction of 3-dimensional shapes using quantum teleportation within an array of quantum oscillators of entangled states to build the #D shape from tetrahedral building blocks

- The “inverse” problem of generating the control instructions for emulating the shape of a given object
  - The inverse problem also provides a framework for using Bayesian methods to search for objects of interest
Angular momentum a la Schwinger

J. Schwinger 1951

\[ J \equiv \hbar \sum_{\xi, \xi' = 1, 2} a_{\xi}^+ \braket{\xi | \frac{\sigma}{2} | \xi'} a_{\xi'} \]

\[
\left[ a_{\xi}, a_{\xi'} \right] = \left[ a_{\xi}^+, a_{\xi'}^+ \right] = 0 \quad \left[ a_{\xi'}, a_{\xi}^+ \right] = \delta_{\xi \xi'}
\]

where

\[
J^+ = \hbar a_1^+ a_2 \quad J^- = \hbar a_2^+ a_1 \quad J_3 = \frac{\hbar}{2} \left( a_1^+ a_1 - a_2^+ a_2 \right)
\]

\[
J^2 = \hbar^2 \left( \frac{n^2}{4} + \frac{n}{2} \right) = \hbar^2 \frac{n}{2} \left( \frac{n}{2} + 1 \right)
\]
6j symbols

The sum of 3 angular momenta $j_1$, $j_2$, $j_3$, can be carried out by by adding $j_1 + j_2$ to $j_3$ or $j_2 + j_3$ to $j_1$. The quantum states for the sum of $j_1$, $j_2$, $j_3$, for these two coupling schemes are related by a unitary transformation:

$$|Jm j_{23}\rangle = \sum_{m_1,m_2} \langle j_{12} | j_{23}\rangle |Jm j_{12}\rangle$$

Using the relations between $<m_1m_2|jm>$ for $j = j_{12}, j_{23}, J$ and the 3j symbols one obtains:

$$\langle j_{12} | j_{23}\rangle = (-1)^{j_1+j_2+j_3+J} \sqrt{(2j_{12}+1)(2j_{23}+1)} \sum_{m_1,m_2} \left( \begin{array}{ccc} j_1 & j_2 & j_{12} \\ j_3 & J & j_{23} \end{array} \right)$$
Wigner noticed that the 6j symbol can be associated with a tetrahedron whose sides have lengths equal to the quantized angular momenta that appear in the 6j symbol.
The centerpiece of quantum computing is teleportation (Nielsen & Chuang 2000)

- Teleportation of a continuous variable state \( \psi(q_a) \) defined on a set of nodes \( \{i_a\} \) at one end of a quantum wire begins by using a controlled phase gate \( \exp(iQ_a \otimes Q_b) \) to entangle the input and initial target state \( |P_b = 0\rangle \). A measurement of \( P_a \) after entanglement results in a distortion of the initial state that depends on the result of the measurement.
Fundamental invention for constructing 3-dimensional objects

- An input state $|J_1 + J_{12} + J_2 = 0\rangle$ representing a triangle localized on column 1 can be teleported to any other location in 3-dimensions using a quantum wire consisting of triples of 2D oscillators.

\[
\prod_{a,b,c} |\phi_{i+1} = 0\rangle \rightarrow R(\phi_i) \sum_{m_a,m_b,m_c} X(m_i) \begin{pmatrix}
\dot{j}_{ia} & \dot{j}_{ib} & \dot{j}_{ic} \\
\dot{m}_{ia} & \dot{m}_{ib} & \dot{m}_{ic}
\end{pmatrix} |j_{ia},m_{ia}\rangle |j_{ib},m_{ib}\rangle |j_{ic},m_{ic}\rangle
\]
Example: A double pyramid can be constructed by teleportation along a “figure 8” knot.
Extending a quantum wire for triangles to a tetragonal lattice allows one to construct arrays of tetrahedrons in 3-dimensions.

Adding columns in 3\textsuperscript{rd} dimension allows to represent teleportation in a 3D tetrahedral array.
Arrays of 4-dimensional quantum oscillators are the stage for representing 3D geometries.
A feed-back loop for controlling the shape of a 3D object is defined by a matrix $A(\sigma,x,t')$ which translates a signal $I(\sigma,t)$ measuring the shape of the surface into a corrective displacement $\delta(\sigma,t)$ of the surface and a matrix $B(\sigma,x)$ which relates the difference $e(\sigma,t) \equiv \delta(\sigma,t) + n(\sigma,t)$ between the actual and desired shapes of the surface to the change in the measurement signal $I(\sigma,t) - I_0(\sigma)$ resulting from the feedback and noise $n(\sigma,t)$. These matrices satisfy the Dyson equations:

$$A = KB^T I_0^{-1},$$

where $K(\sigma_1,\sigma_2,t_1,t_2)$ is a causal matrix:

$$K + K^T + KB^T I_0^{-1}BK^T + N = 0$$

where $N$ is the time averaged covariance function $< n(\sigma)n(\sigma') >$ for the quantum fluctuations in the shape of the surface.
Uncertainties in the shape of a surface arising from quantum fluctuations in the orientations of the triangles used to represent a 3D object opens the door to using a Gaussian process neural network with a representing kernel generated by $C(I^n(\sigma),I^{n'}(\sigma)) + \sigma_n^2$, where $C(I^n(\sigma),I^{n'}(\sigma))$ is the covariance matrix for shape measurements for a training set of “quantum surfaces”. Numerical inversion of the kernel matrix allows one to rapidly generate modulo Gaussian noise the oscillator Fock space amplitudes corresponding to an observed shape.

Specialized AI chips could be used to rapidly carry out the required matrix operations; however recognition of complex shapes would be exponentially faster using a quantum GPNN to generate the wave function of the oscillator array!
Summary

- Entangled Fock states of 4-dimensional quantum oscillators can be used to provide a holistic representation of any 3-dimensional object.

- The “inverse” problem of generating the control instructions for arrays of quantum oscillators is a work in progress (A. Peterson, J.Dubois, L.Poyneer).

Longer term it appears that large arrays of quantum oscillators could be used to not only generate 3D shapes, but also use measurements of shapes to generate oscillator array wave functions that can be directly compared with the wave function for a particular shape being sought.