# Turbulence Modeling Based on Multiscale Dynamical Analysis

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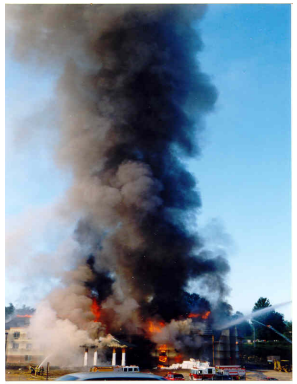
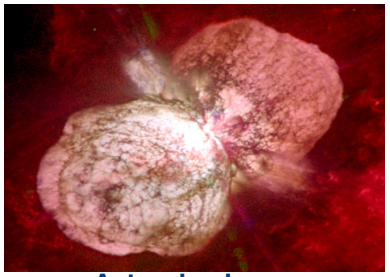
# Turbulent Flow is Embedded in Many Engineering Problems of Interest



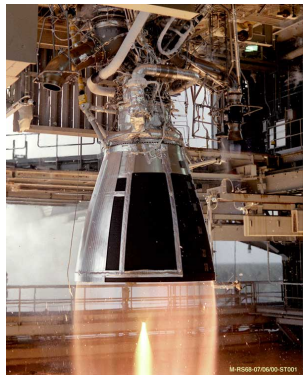
Industrial Pool Fires

Wind Energy



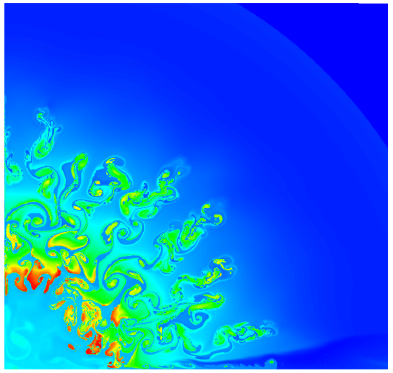
Astrophysics

Environmental Sciences



Rocket Propulsion


Scramjet Propulsion



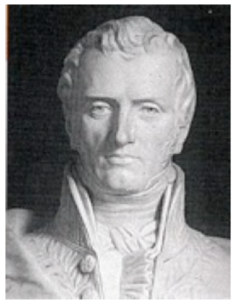
HE Blast Waves

Internal Combustion Engine Design

**Turbulence mixes scalars (fuel & oxidizer) and dissipates energy efficiently**

# Navier-Stokes: Non-linear Chaotic Dynamics (I)

Navier



(1787-1831)

## Navier-Stokes equation

$$\frac{\partial u_i}{\partial t} = -\frac{\partial}{\partial x_j} u_i u_j + \nu \frac{\partial^2 u_i}{\partial x_j \partial x_j} - \frac{1}{\rho}\frac{\partial p}{\partial x_i}$$

velocity change w/ time | convective forces | viscous forces | pressure forces

Stokes

(1808-1876)

$$F = m \cdot a \longrightarrow a = \frac{F}{m}$$

# Navier-Stokes: Non-linear Chaotic Dynamics (II)

Navier

(1787-1831)

## Navier-Stokes equation

$$\frac{\partial u_i}{\partial t} = -\frac{\partial}{\partial x_j} u_i u_j + \nu \frac{\partial^2 u_i}{\partial x_j \partial x_j} - \frac{1}{\rho}\frac{\partial p}{\partial x_i}$$

velocity change w/ time | convective forces | viscous forces | pressure forces

Stokes

(1808-1876)

$$F = m \cdot a \longrightarrow a = \frac{F}{m}$$

**The BIG problem: convective forces are nonlinear**

$$u_i u_j \equiv u_i(\underline{x}, t)\, u_j(\underline{x}, t)$$

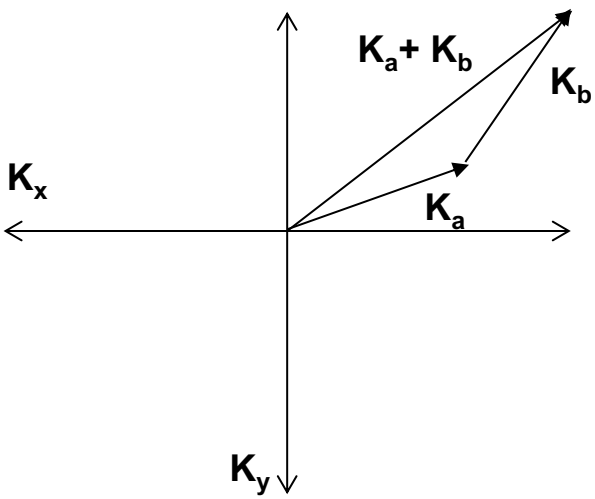
**Evolution of each scale depends on all the other scales**

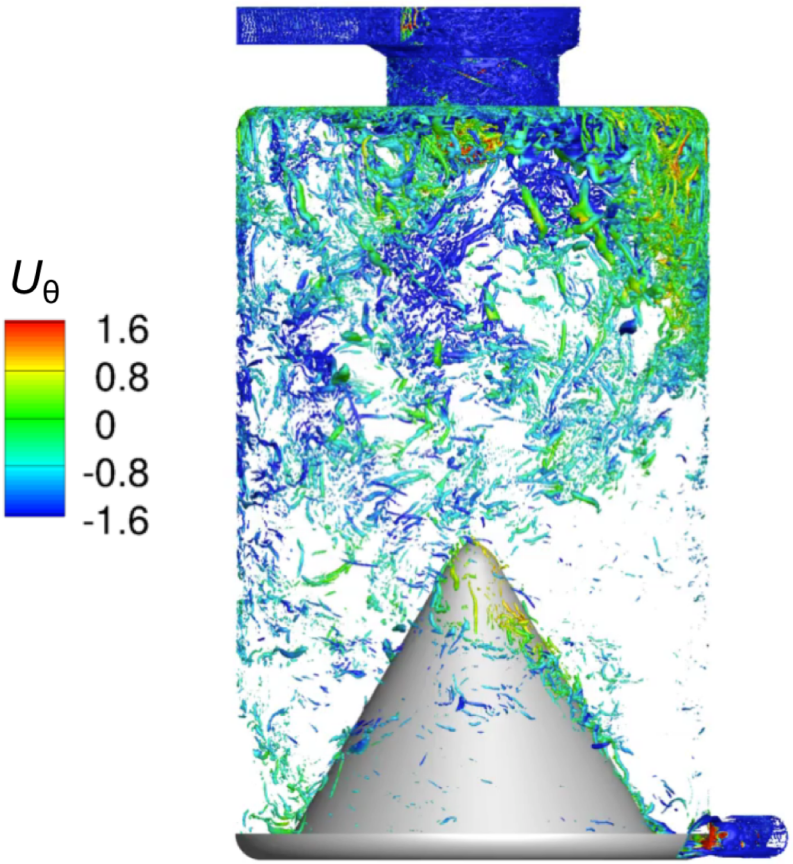
# Navier-Stokes: Non-linear Chaotic Dynamics (III)

- Fourier Analysis and Triadic Interactions

$$u(x) = \sum_{k} \hat{u}_k e^{ik\cdot x}$$

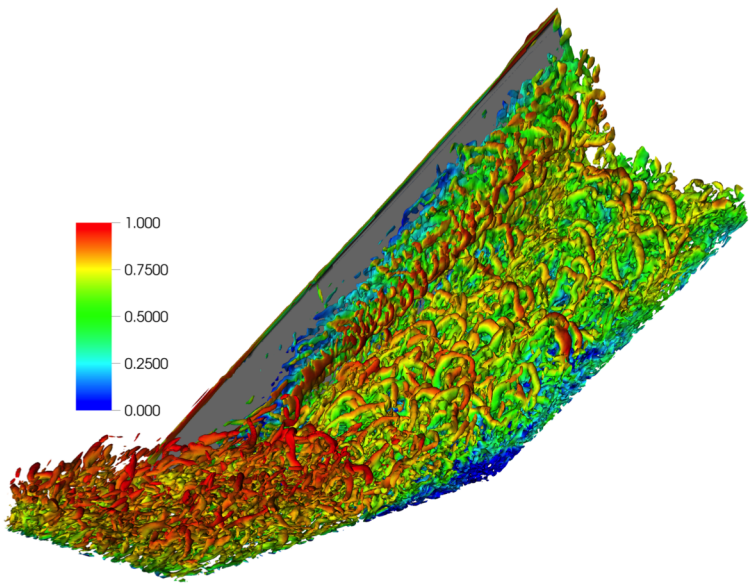
$$u_i u_j \equiv u_i(\underline{x},t)\, u_j(\underline{x},t)$$



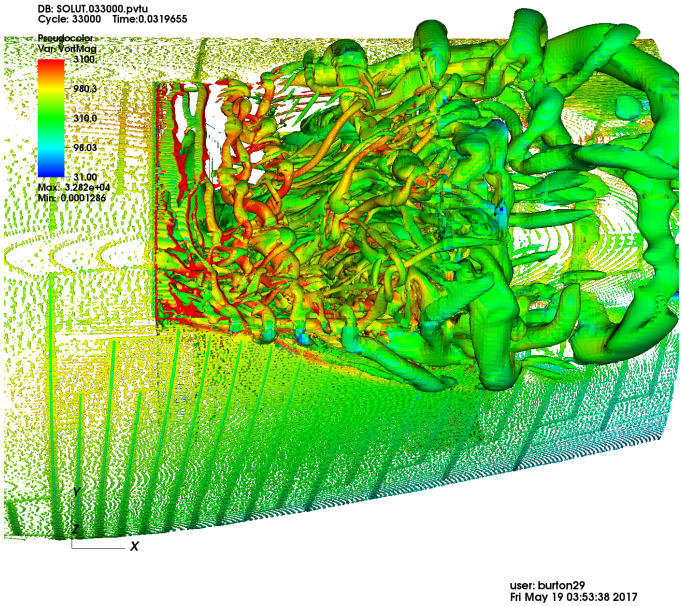



# Navier-Stokes: Non-linear Chaotic Dynamics (III)

- Coherent Vorticity




**Wall-bounded Hairpin Vortices**

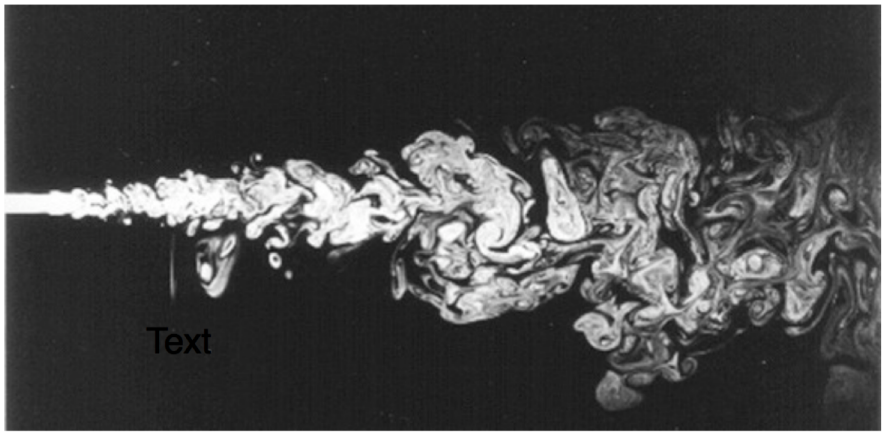



**Driven Cavity-BL Vortices**

# Rapidly Increasing Scale Range with Reynolds Number: $N \sim Re^{9/4}$

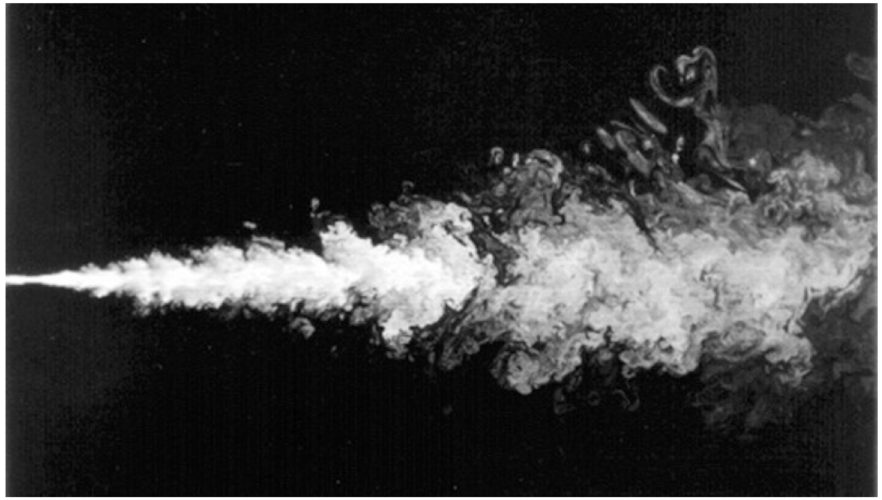
**Low Speed Jet**

**Coarse structures: Less Detail**




**High Speed Jet**

**Finer structures: More Detail**

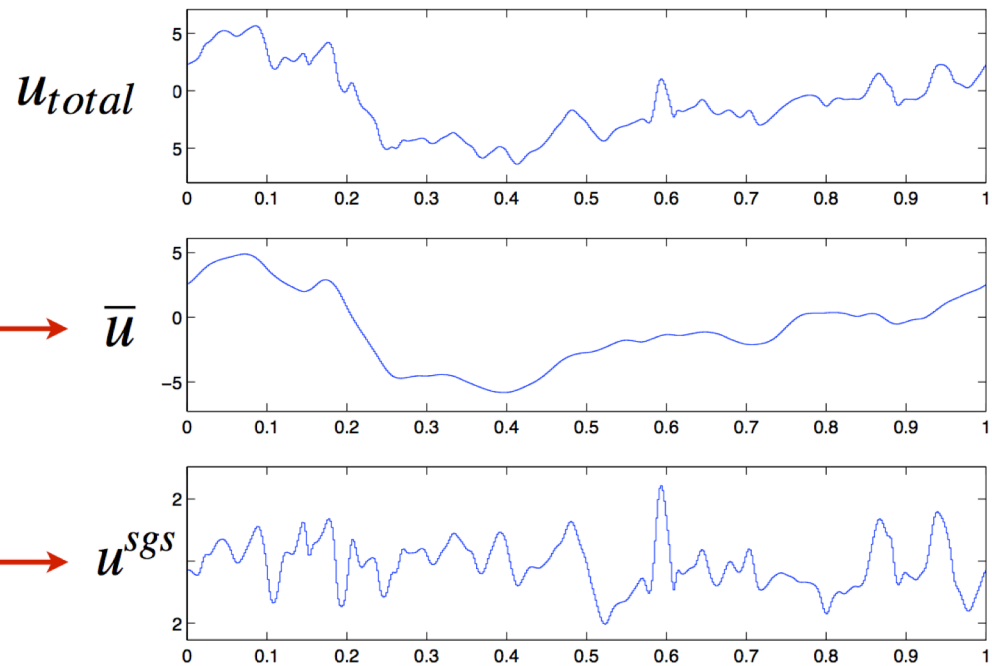


# Large-Eddy Simulation (LES): Resolve the large scales, but model the small

**Filtered Navier-Stokes Equation:**

$$\frac{\partial \overline{u}_i}{\partial t} + \frac{\partial}{\partial x_j}\overline{u_i u_j} - \nu \frac{\partial^2 \overline{u}_i}{\partial x_j \partial x_j} + \frac{1}{\rho}\frac{\partial \overline{p}}{\partial x_i} = 0$$

**Decompose velocity field:** $u_{total} \equiv \overline{u} + u^{sgs}$




**Total velocity**

**Resolved velocity**

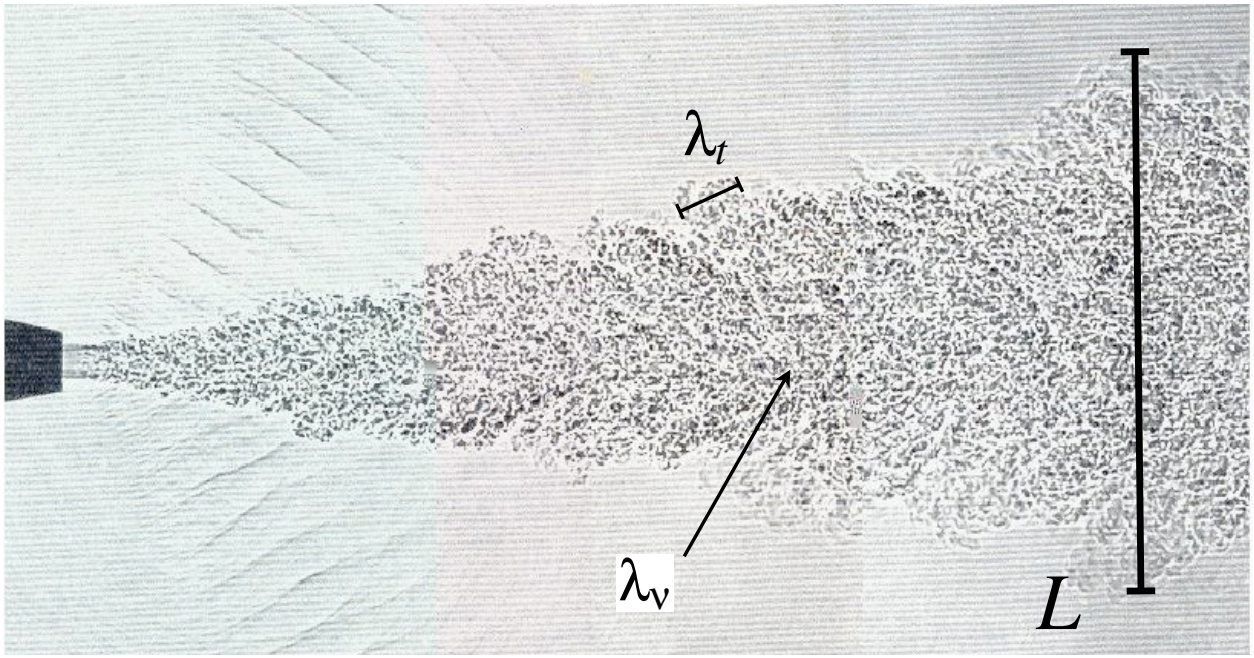
**Subgrid velocity**

**Explicitly calculate this** → $\overline{u}$

**Model this** → $u^{sgs}$

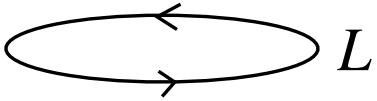
**Unclosed nonlinear term:** $\overline{u_i u_j} \equiv \overline{\overline{u}_i \overline{u}_j} + \overline{\overline{u}_i u_j^{sgs}} + \overline{u_i^{sgs} \overline{u}_j} + \overline{u_i^{sgs} u_j^{sgs}}$

# Traditional Energy-Transfer Paradigm (1897, 1921)




Energy input here →

Outer Scale




Taylor Scale

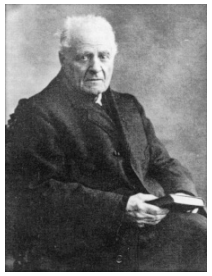

One-Way Transfer

Inner, Viscous Scale


Energy dissipated to heat here →

# Eddy viscosity: 19th century paradigm

**Boussinesq (1897)**



Channel Flow

**"Turbulent friction"**
eddies remove energy, descriptive not physical

**Von Neumann (1950)**

Shock Calculations

**Artificial Viscosity:**
damps numerical oscillations

**Smagorinsky (1963)**

Geophysical Turbulence

**Eddy Viscosity:**
removes turbulent energy in large eddy simulations

**Filtered Navier-Stokes equation:**

$$\frac{\partial \overline{u}_i}{\partial t} + \frac{\partial}{\partial x_j}\overline{u_i u_j} - \nu \frac{\partial^2 \overline{u}_i}{\partial x_j \partial x_j} + \frac{1}{\rho}\frac{\partial \overline{p}}{\partial x_i} = 0$$

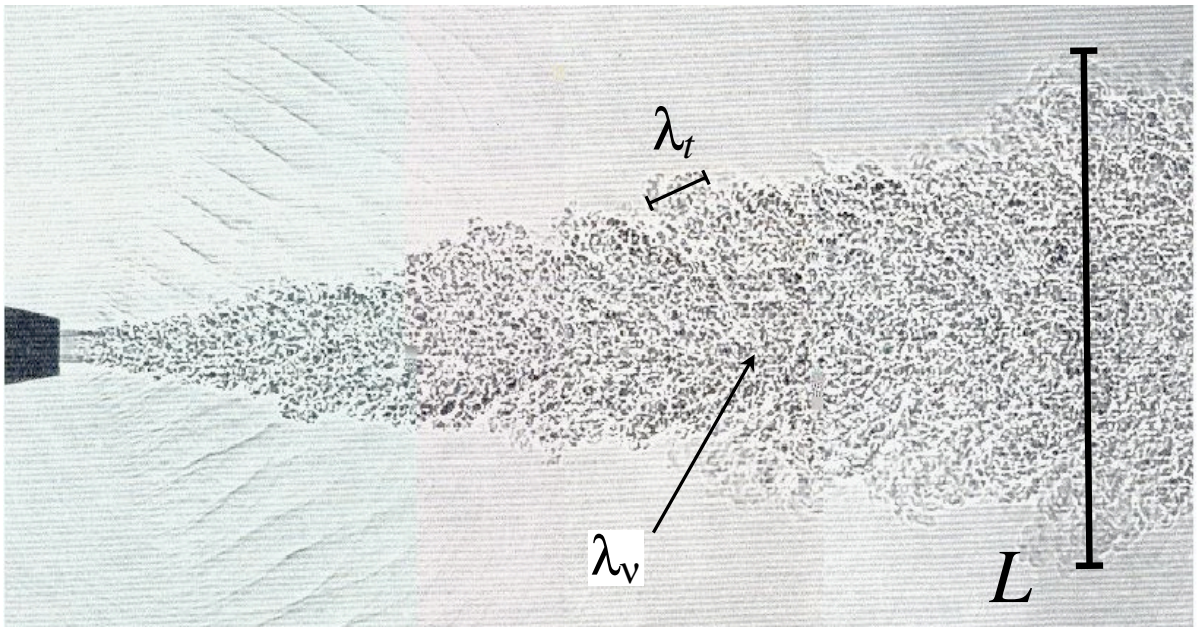
**Smagorinsky's modified equation:**

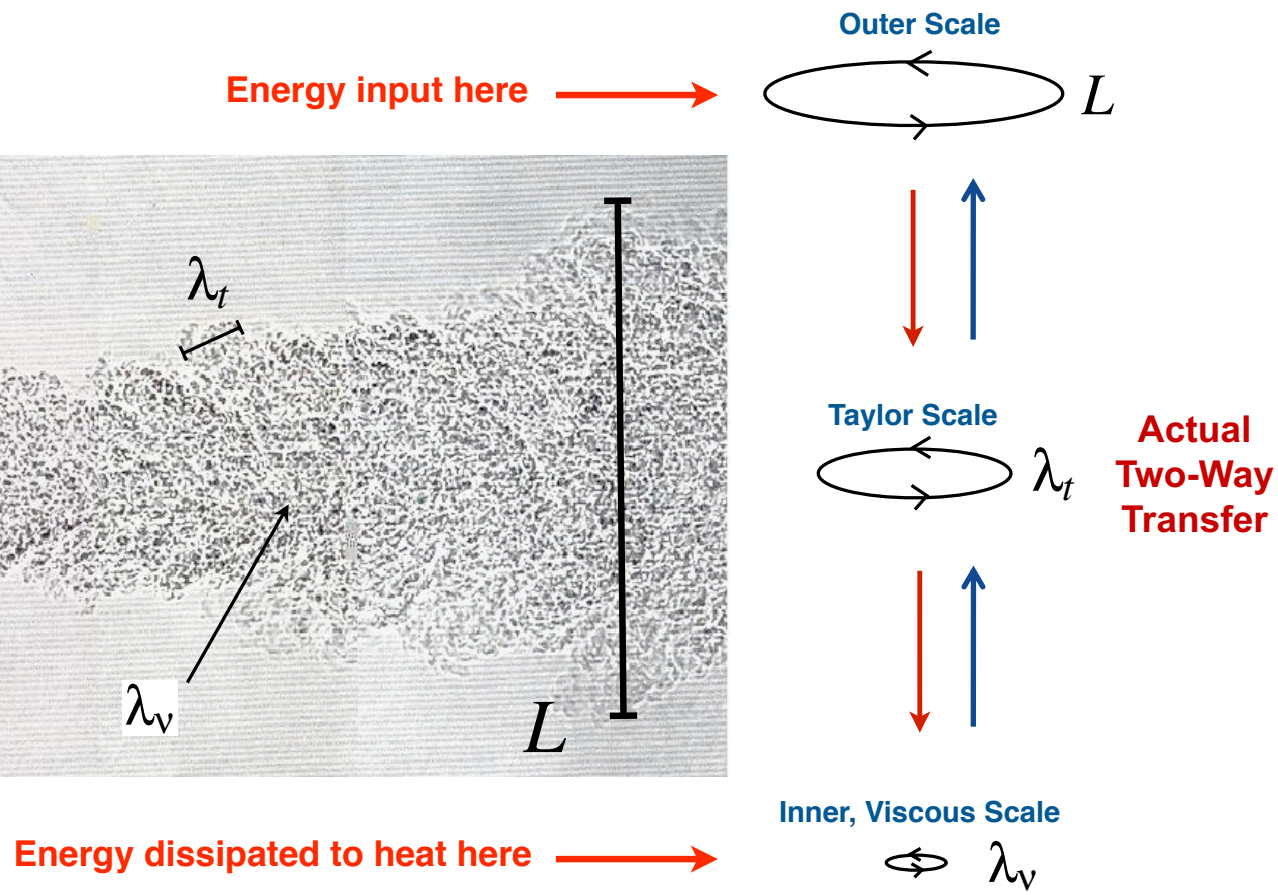
$$\frac{\partial \overline{u}_i}{\partial t} + \frac{\partial}{\partial x_j}\overline{u}_i \overline{u}_j - \nu \frac{\partial^2 \overline{u}_i}{\partial x_j \partial x_j} + \frac{1}{\rho}\frac{\partial \overline{p}}{\partial x_i} = -\frac{\partial}{\partial x_j}\tau_{ij}$$

**New "subgrid stress" term:**

$$\tau_{ij} \equiv \overline{u_i u_j} - \overline{u}_i \overline{u}_j \quad \Longrightarrow \quad \tau_{ij} \approx 2\nu_t \overline{S}_{ij}$$

# True Energy Transfer in Turbulent Flows (1990)



# Multifractals & Turbulence

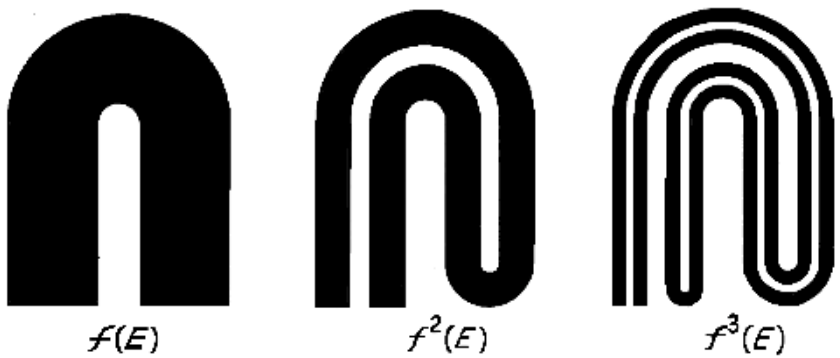
- Gradient-squared quantities exhibit scale-invariant multifractal structure at inertial range scales
- Created by combined effects of strain and vorticity which repeatedly stretch and fold flow back on itself
- Multifractal cascade: a concentration process

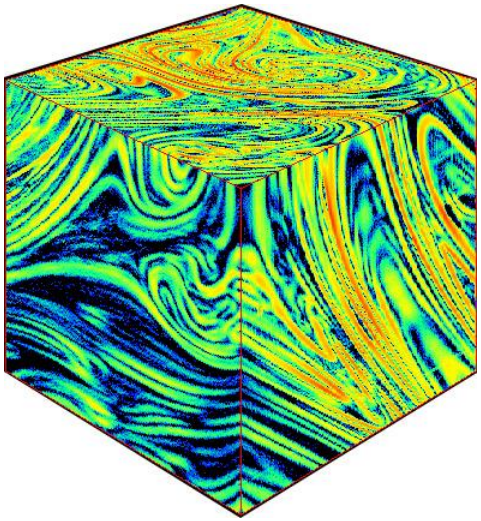
$$2Q \equiv \omega_i \omega_i$$

$$\chi \equiv D \nabla \phi_i \nabla \phi_i$$

$$\varepsilon \equiv 2\nu S_{ij} S_{ij}$$

Horseshoe Mapping



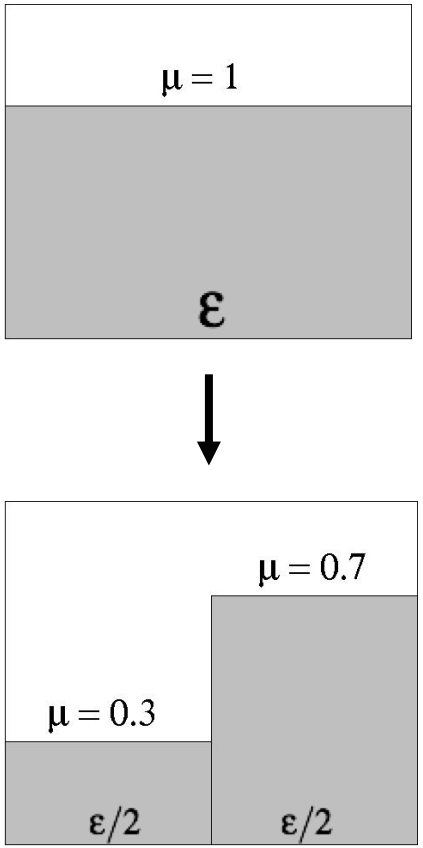
Passive-Scalar Dissipation

# Testing for Multifractal Structure

**Kernel of Multifractal Analysis:**
**1-D Multiplier Ratio**

$$\mathcal{M}_\varepsilon \equiv \frac{\int_{\varepsilon/\alpha} \mu(x)\, dx}{\int_\varepsilon \mu(x)\, dx}$$

**Ratio of measure at adjoining scales**

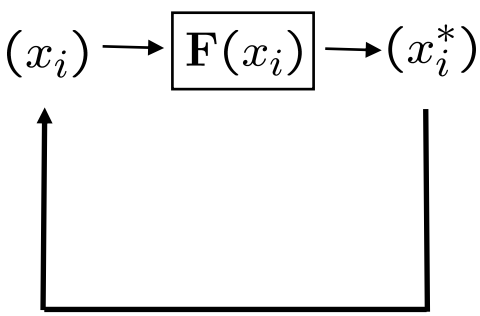


# Multifractals & Iterative Functions

**Multifractal Cascades:**

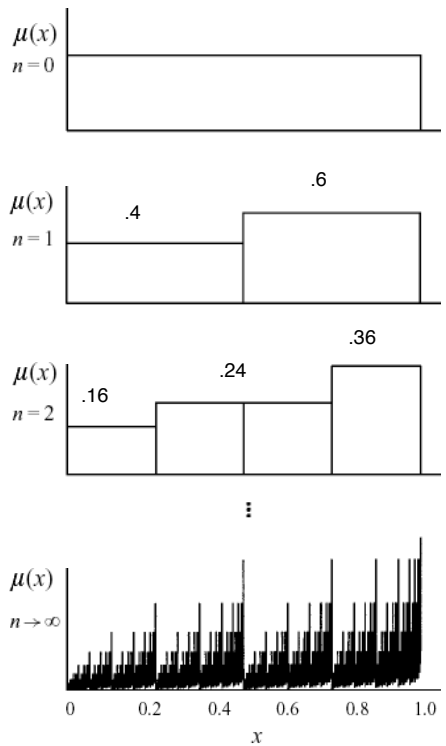
**Deterministic I.F.S.**

**Stochastic I.F.S.**

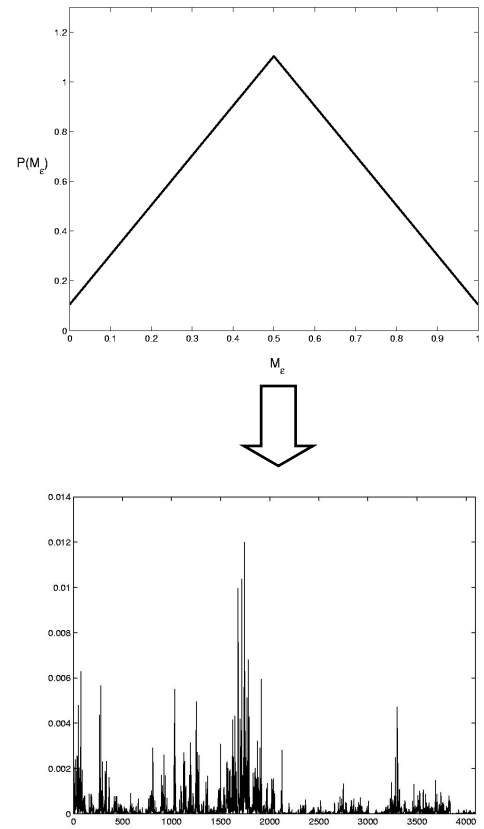



**Repeated application of function produces multifractal field**

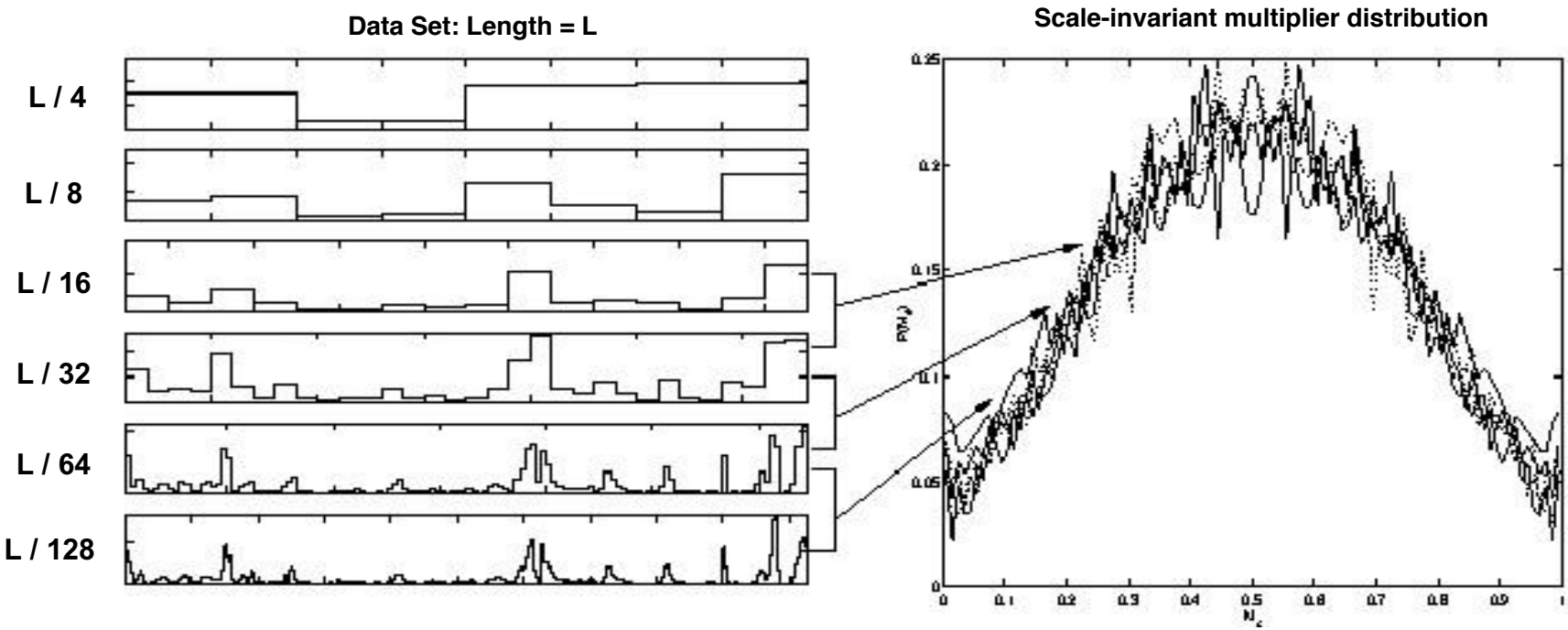
## Deterministic I.F.S.




## Stochastic I.F.S.



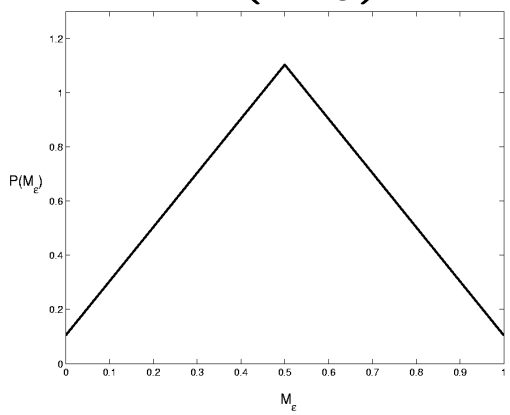
# Testing for Multifractal Structure: $\mathcal{M}_\varepsilon$




**Multifractal structure confirmed over scales where multiplier distribution exhibits invariance**

# Constructing a Multifractal Field

$$P(\mathcal{M}_\varepsilon)$$



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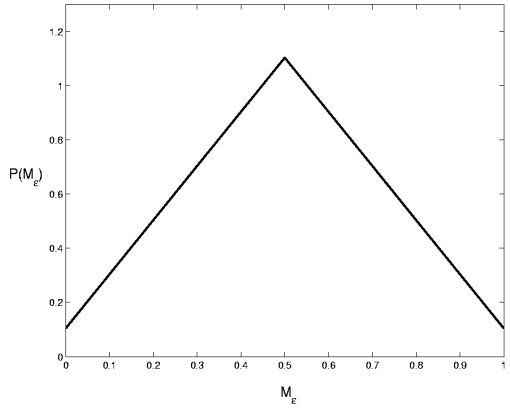
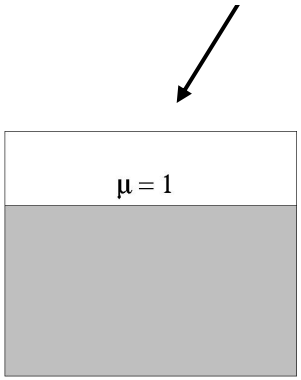
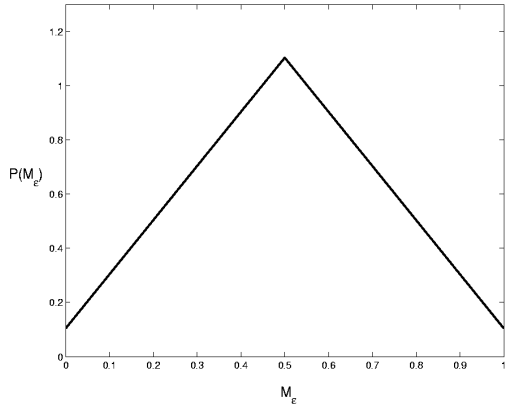
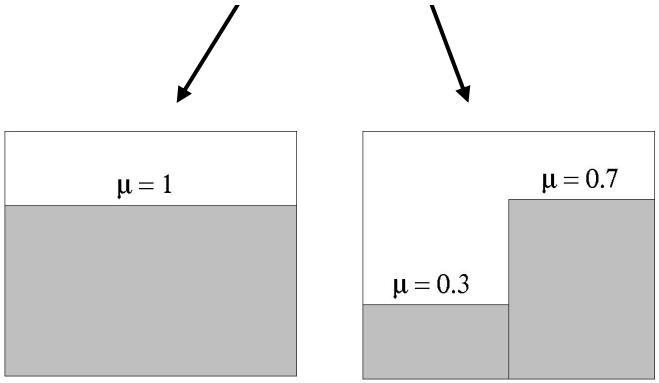


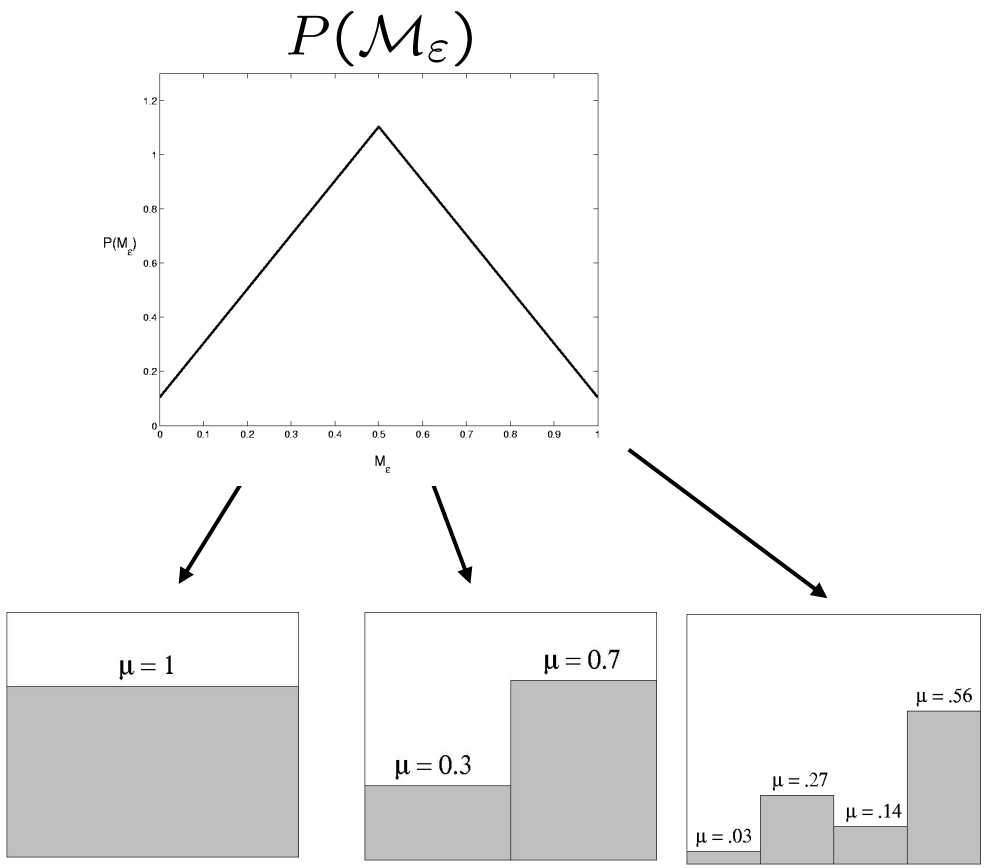
# Constructing a Multifractal Field

$$P(\mathcal{M}_\varepsilon)$$



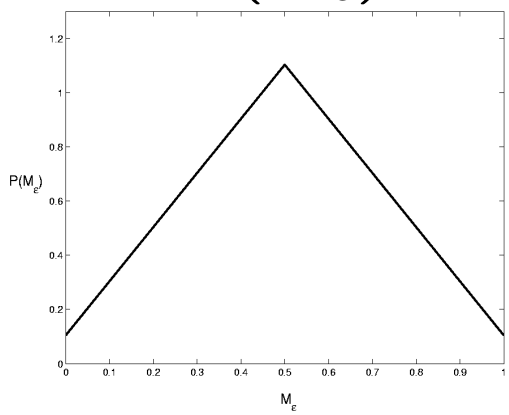



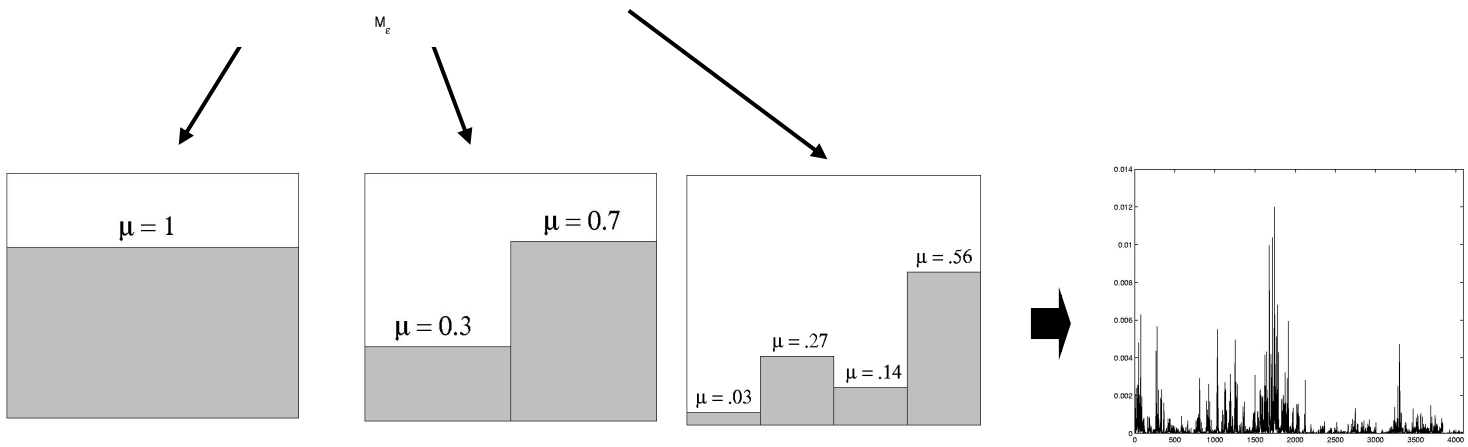
# Constructing a Multifractal Field



# Constructing a Multifractal Field

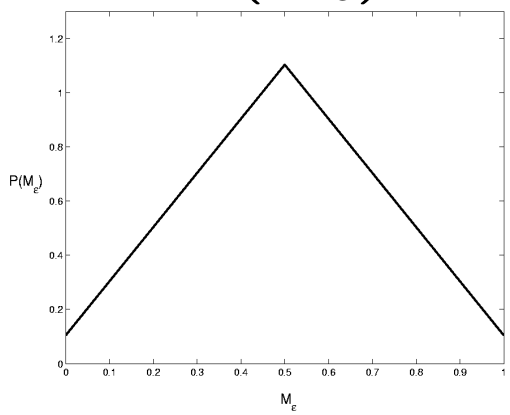
$$P(\mathcal{M}_\varepsilon)$$



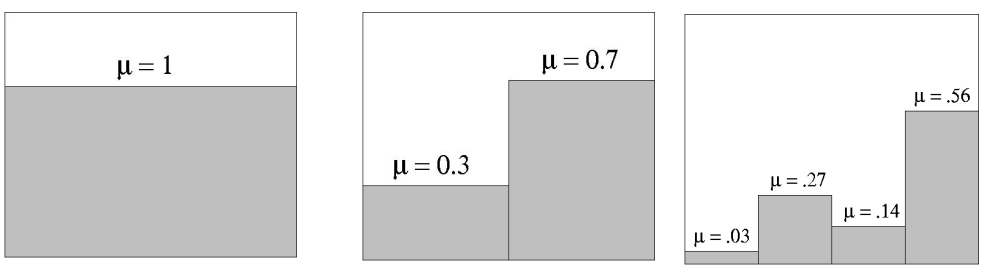



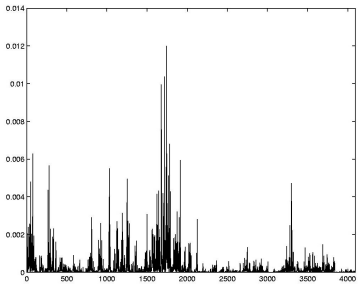
# Constructing a Multifractal Field

$$P(\mathcal{M}_\varepsilon)$$

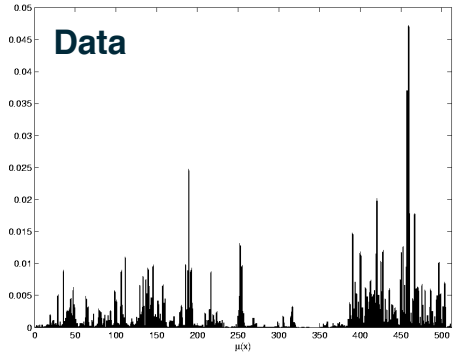



$$\mu(\mathbf{x}) = \mu_o \prod_{i=1}^{\mathcal{N}} \mathcal{M}(\mathbf{x})_i$$

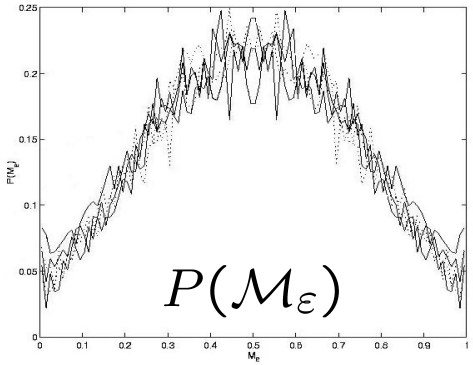


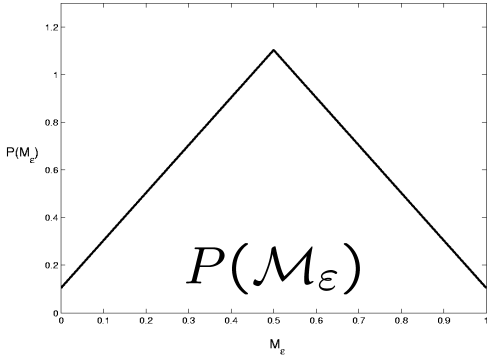



# Multifractal Cascades: Summary

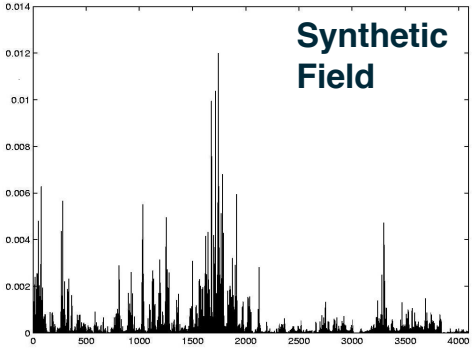



Recovery of Multiplier Ratio




Construction of Multifractal Field

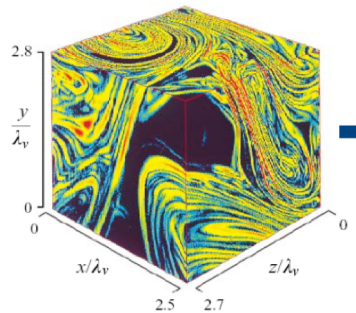



**Synthetic field statistically identical to original**

# Direct Modeling of Unresolved Scales

- **Subgrid scalar model: based on multifractal cascade of scalar-dissipation**

SCALAR-ENERGY DISSIPATION




**Green's Function Inversion**

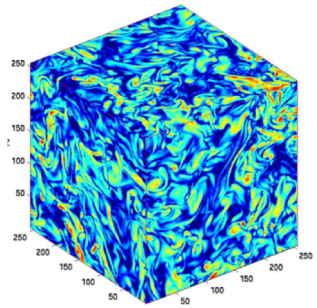
$$\langle \phi^{sgs} \rangle = \nabla \cdot \frac{1}{4\pi} \int_{\mathbf{x}'} \langle \nabla \phi^{sgs} \rangle \frac{1}{|\mathbf{x} - \mathbf{x}'|} d\mathbf{x}' .$$

**Scalar Model**

$$\langle \phi^{sgs} \rangle \approx \mathcal{D} \phi^{\Delta} + H.O.T.$$

- **Subgrid velocity model: based on multifractal cascade of enstrophy**

ENSTROPHY



**Biot-Savart Inversion**

$$\langle u^{sgs} \rangle \equiv \frac{1}{4\pi} \int_{x'} \langle \omega^{sgs} \rangle \times \frac{x - x'}{|x - x'|^3} d^3 x'$$

**Velocity Model**

$$\langle u_i^{sgs} \rangle \approx \mathcal{B} u_i^{\Delta} + H.O.T.$$

**Scaling transformation of smallest-resolved fields**

# THE END