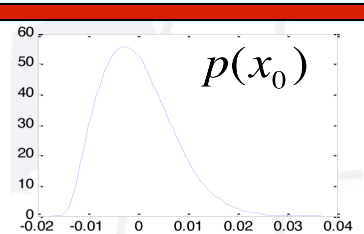
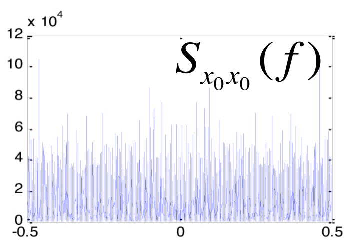
CASIS 2016:
20TH Annual Signal & Image Sciences Workshop
Center for Advanced Signal and Imaging Sciences (CASIS)

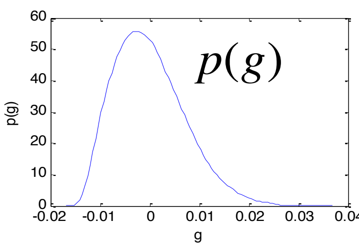
Stochastic Simulation of Non-Gaussian Correlated Processes for Bayesian Radar Target Classification

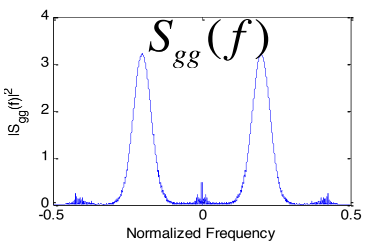




May 18, 2016

Grace Clark, Ph.D.,
IEEE Fellow





Contact Information for Grace Clark

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Grace Clark, Ph.D., IEEE Fellow, currently serves as a statistical signal processing consultant via her business, *Grace Clark Signal Sciences*, in Livermore, CA. She retired from the Lawrence Livermore National Laboratory in October 2013, after a career as a research engineer. She worked earlier at the Caltech Jet Propulsion Laboratory. Dr. Clark has served as thesis advisor for 10 graduate students: 5 MSECE students at the Naval Postgraduate School, and 3 MSECE plus 2 Ph.D. ECE students at U. of California Davis. She earned BSEE and MSEE degrees from the Purdue U. Electrical Engineering Honors Program, W. Lafayette, IN, and the Ph.D. in Electrical and Computer Engineering (ECE) from the U. of California Santa Barbara. Her technical expertise is in statistical signal/image processing, estimation, detection, pattern recognition/machine learning, sensor fusion, communication and control. She has contributed more than 230 publications on signal processing in acoustics, electromagnetics and particle physics. She is a Fellow of the IEEE and a member of the Acoustical Society of America (ASA) Technical Council on Signal Processing in Acoustics, the Society of Exploration Geophysicists (SEG), Eta Kappa Nu and Sigma Xi.

The World of Acoustics Before Signal Processing

My Current Research is Focused on *Nonlinear, Non-Gaussian* Signal Processing Problems

- **Mobility Modeling and Estimation for Ad Hoc Networks of Unmanned Ground Vehicles**
  - Estimate position, velocity and acceleration, given only measurements of Received Signal Strength Indicator (RSSI) signals from fixed or mobile base stations
  - with Prof. Preetha Thulasiraman, NPS

- **Illumination Waveform Design for Non-Gaussian Multi-Hypothesis Target Classification in Cognitive Radar**
  - with a student at NPS

- **Statistical Feature Selection for Non-Gaussian Target Classes**
  - with a student at NPS

- **Clock Synchronization Through Time-Variant Underwater Acoustic Channels**
  - with Prof. Joe Rice, NPS

Cognitive Radar Thesis Research Team

- **Grace Clark, Advisor,** *Grace Clark Signal Sciences*, Livermore, CA
  Former Visiting Research Professor, ECE, NPS

- **Ric Romero, Co-Advisor, Assistant Professor,** ECE, Naval Postgraduate School Monterey, CA

- **Ke Nan Wang, ENS, USN, Former MSEE Student,**
  Naval Postgraduate School, Monterey, CA
  Received Eta Kappa Nu (HKN) Best Thesis Award

Ke Nan & Grace

Ric Romero

Agenda

- **Problem Definition:**
  - In our previous non- Gaussian Cognitive Radar work, we ***simulated*** Non- Gaussian target signals with ***closed form pdfs***
  - In this work, we extend our capability so we can use ***measured*** Non-Gaussian target response signal exemplars to generate the desired samples:

> Given only *measured target responses*, we can draw correlated samples with *BOTH specified Non-Gaussian pdf and specified PSD for Cognitive Radar*

- **Brief Summary** of our work in *Cognitive Radar for Non-Gaussian distributed targets*

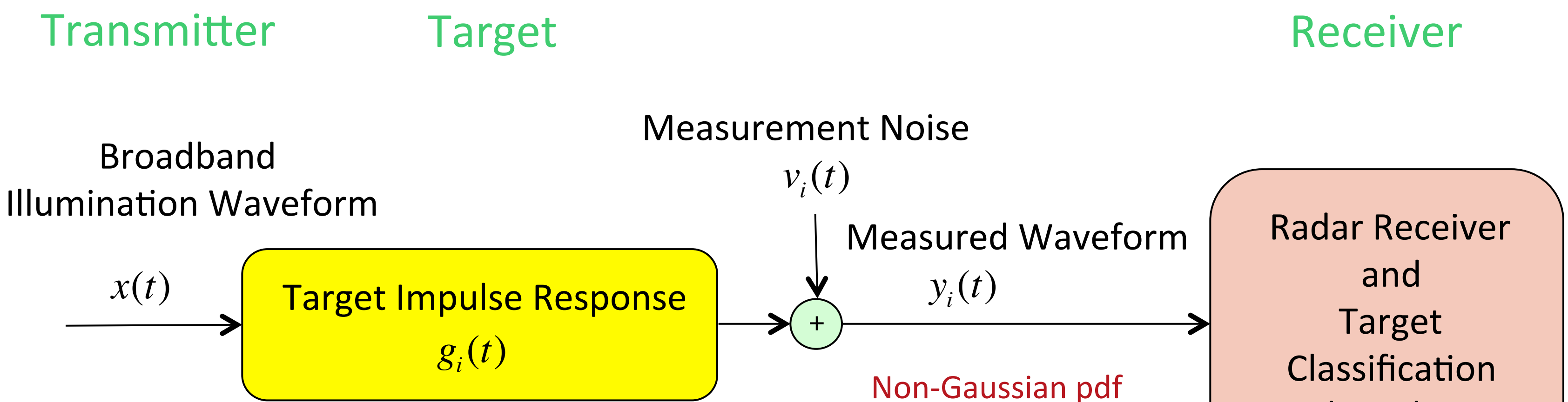
- **A complex stochastic simulation algorithm** that is simple, fast and provides high quality samples with specified pdf and PSD
  - *Example*

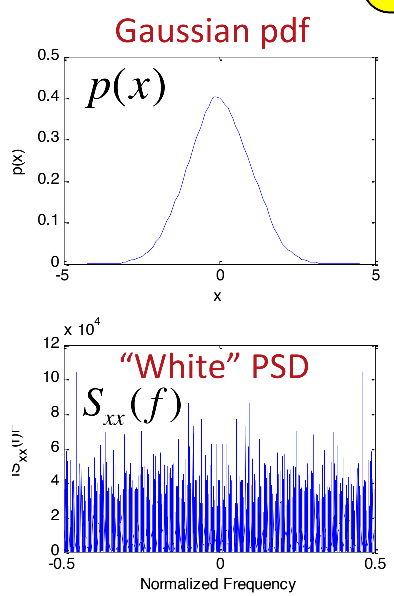
- **Conclusions**

Brief Summary of Our Work in Cognitive Radar for Non-Gaussian Target Distributions

Grace Clark

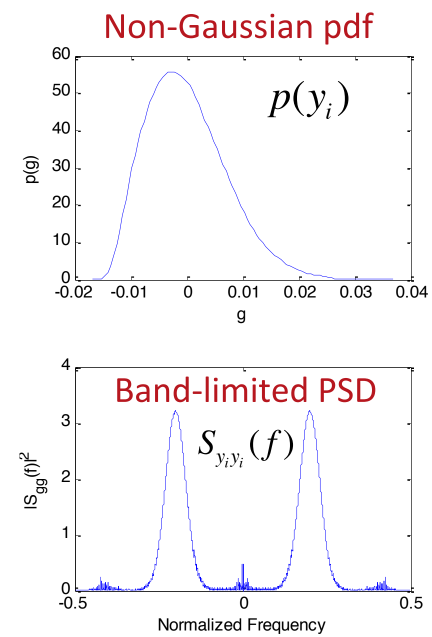
A Conventional Radar System *Illuminates the Target with a Broadband Waveform – Excites all Possible Target Modes*





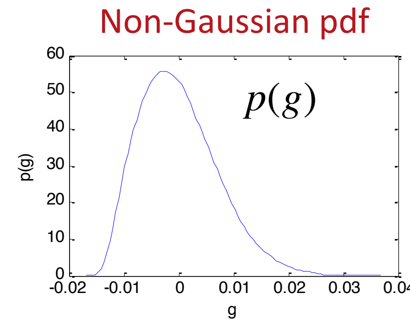
<u>Targets Often Have:</u>

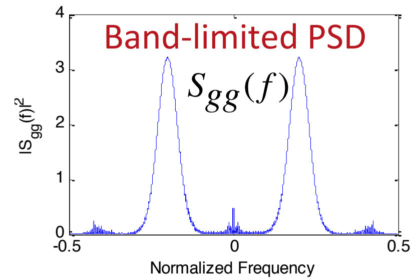
- Non-Gaussian pdf
- Band-limited (Correlated) Power Spectral Density (PSD)



Several Problems Motivate Us to Improve on Conventional Radars ➔ Cognitive Radar

- **Illumination waveform power** E_s limitations vs. receiver signal-to-noise ratio (**SNR**)
- Real-world targets have **band-limited radar responses** – sparse spectra, but we use a broadband pulse to illuminate the target
  - – wasted energy
- **Inadequate detection/classification performance** due to low SNR, etc.
  - *P(CC) = Probability of Correct Classification*
- Current classification theory assumes complex **Gaussian-distributed targets** - but real-world targets are often non-Gaussian, or arbitrarily-distributed
- **New shared-spectrum applications:**
  Sponsors would like to have communications and radar systems that can share the EM spectrum
  - – Not all frequencies are available at a given time





Cognitive Signal Processing Systems Learn from the Environment and Adapt their Inputs

A Cognitive Signal Processing system is one that observes and learns from the environment; then uses a dynamic closed-loop feedback mechanism to adapt the illumination waveform so as to provide system performance improvements over traditional systems

Early Reference:
Simon Haykin, McMaster University, Hamilton, Ontario, Canada
"Cognitive Radar, A Way of the Future,"
IEEE Signal Processing Magazine, February 2006

PWE(t) is a Weighted Sum of Individual Optimal Matched Target Illumination Waveforms

(((GCSS)))

- A single matched illumination waveform is estimated by Maximizing the SNR in the receiver:
- The PSD's of the individual targets are assumed known a priori from calibration experiments
- The optimal illumination waveform $x_i^{opt}(t)$ for a single target is an eigen-solution that has the form of a complex exponential function:

$$\lambda_{\max}\hat{x}(t) = \int_{-T/2}^{T/2} \hat{x}(\tau) R_g(t-\tau) d\tau$$

where $R_g(\tau)$ is the covariance obtained from the PSD of the target signal $g(t)$.

- The overall illumination waveform $PWE^k(t)$ is the weighted sum of the individual optimal target waveforms. The weights P_i^k are prior probabilities:

$$PWE^k(t) = \sqrt{E_s}\sum_{i=1}^{M}\sqrt{P_i^k}\, x_i^{opt}(t) = \text{Probability Weighted Energy}$$

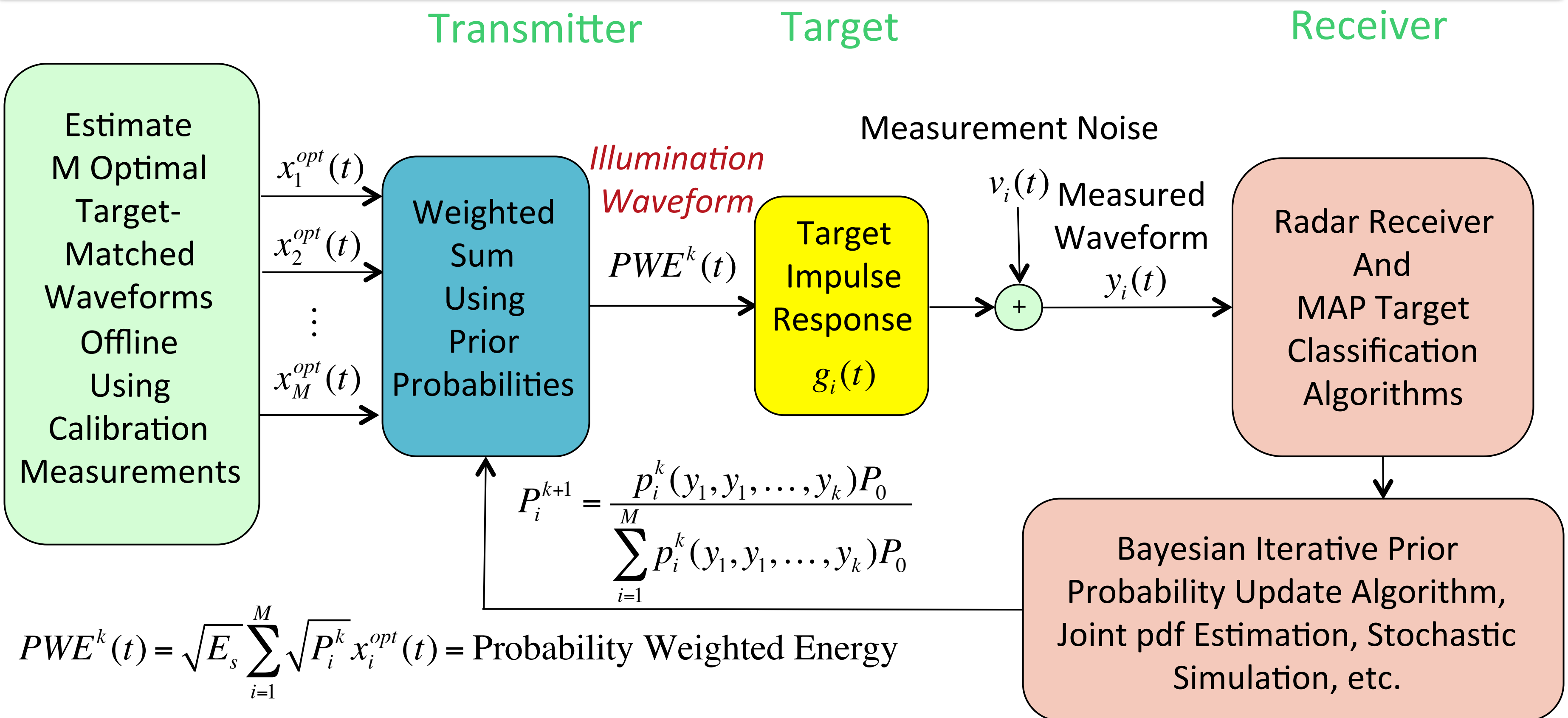
$k =$ Illumination Iteration Index $= 0,1,2,\ldots$

$i =$ Target Index $= 1,2,\ldots M$

$E_s =$ Energy in the Illumination Waveform

$P_i^k =$ Prior probability for target i at illumination iteration k

A Cognitive Radar System *Can Illuminate the Target with a Waveform Matched to the Target Classes Known "A Priori"*



$$PWE^k(t) = \sqrt{E_s}\sum_{i=1}^{M}\sqrt{P_i^k}\,x_i^{opt}(t) = \text{Probability Weighted Energy}$$

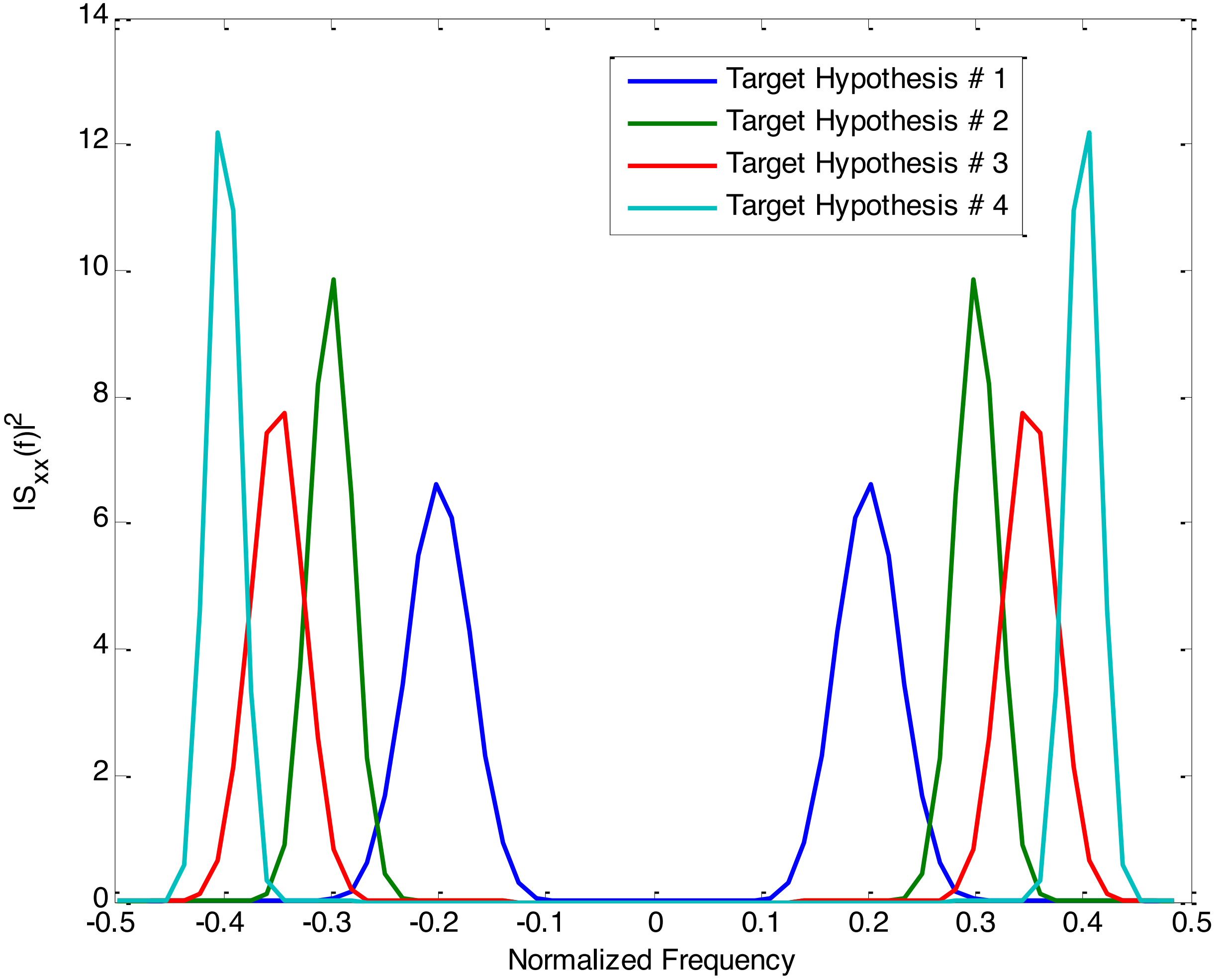
$k =$ Illumination Index $= 0, 1, 2, \ldots$

$i =$ Target Index $= 1, 2, \ldots M$

$E_s =$ Energy in the Illumination Waveform

$P_i^k =$ Bayesian prior used to weight the optimal matched waveform $x_i^{opt}(t)$

Experiment: Specified PSDs Corresponding to the Four Target Classes (Hypotheses)



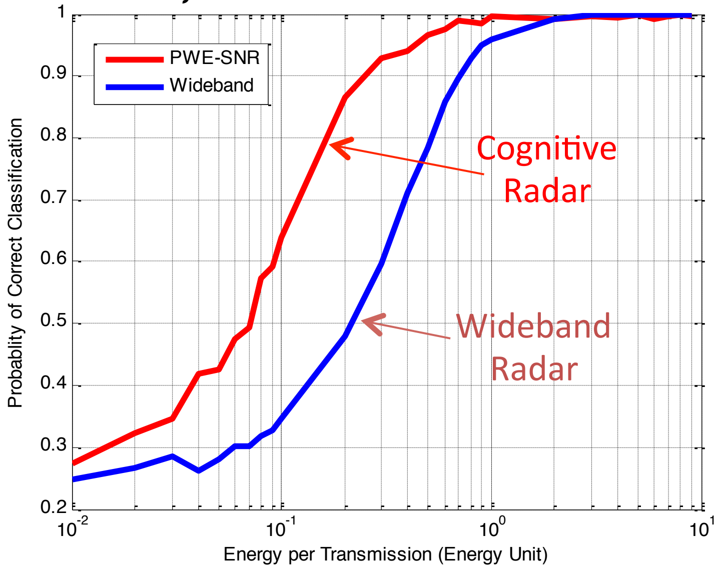
Cognitive Radar Promises Solutions to Several Key Problems in Radar Target Classification

(((GCSS)))

- *Recent research with my NPS student created a new Cognitive Radar Algorithm for Non-Gaussian distributed targets.*
  - *Using 4 Non-Gaussian targets, we showed:*

> *For a given Illumination Waveform Energy, the Cognitive Radar (red) achieves an approximately 100% gain in Probability of Correct Classification over the Conventional Wideband Radar (Blue).*

PCC, for 10 Transmissions



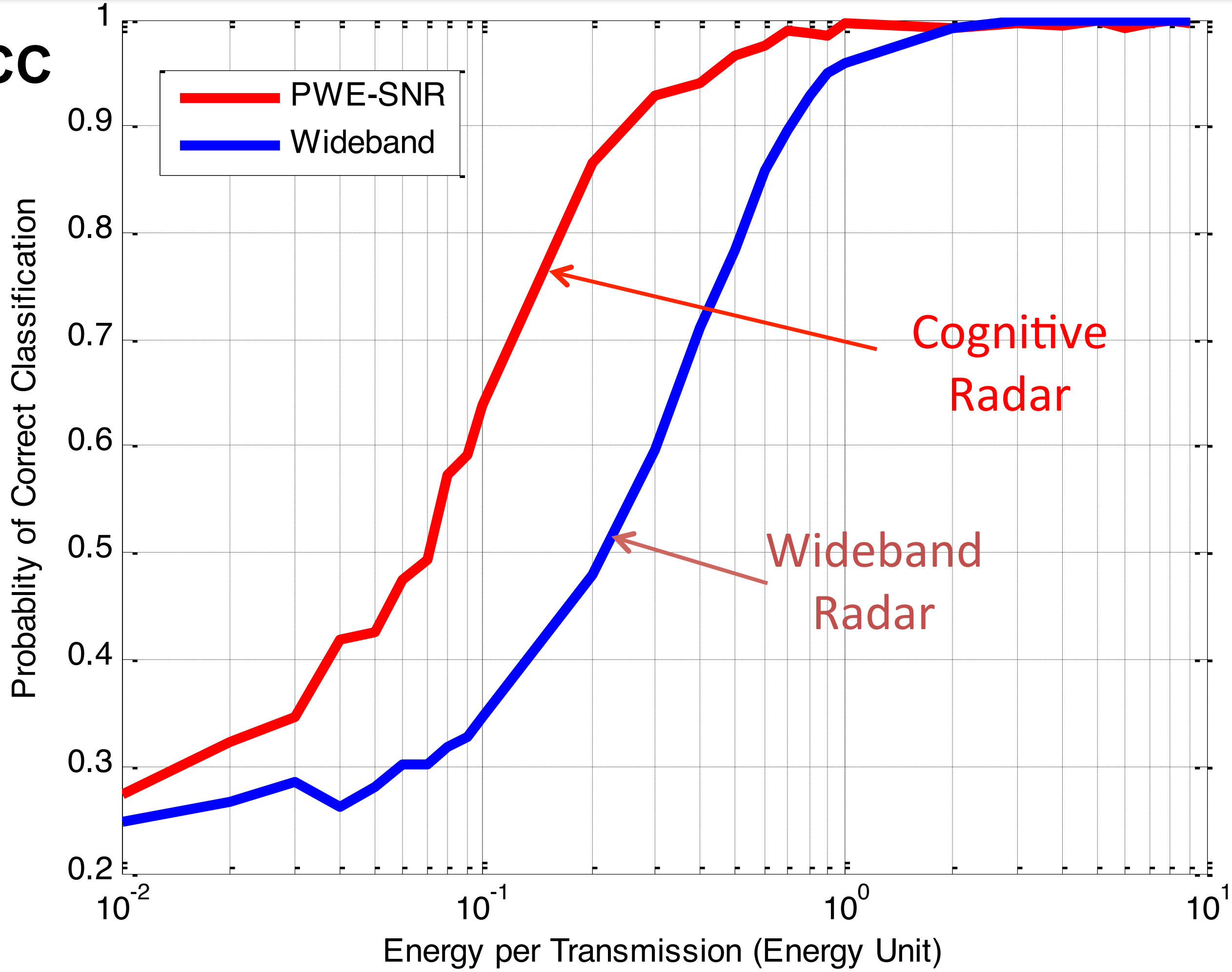
Illumination energy

- *We exploit the spectral sparsity of the target responses and create matched waveforms with band-limited spectra:*
  - *Saves spectral energy*
  - *Good for low-power, low SNR applications*
  - *Good for shared-spectrum applications*

- *We can deal with Non-Gaussian distributed targets*

Classification Performance of the NGCCR Algorithm for 10 Transmissions

PCC



Illumination Energy

<u>Monte Carlo Setup:</u>

- 50 Target Realizations
- 10 Noise Realizations

<u>NGCCR Algorithm Setup:</u>

- 40 Target Realizations for the ensemble averaging

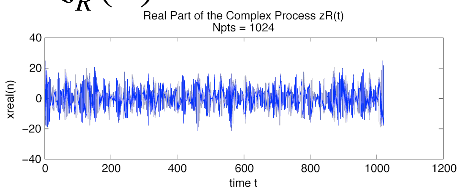
Stochastic Simulation for Drawing Samples with BOTH Specified pdf and Specified PSD

Grace Clark

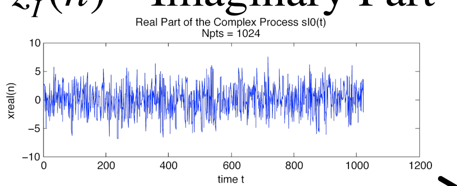
Given Only Measurements, We Need to Simulate Large Ensembles of Target Response Signals for Use With Monte Carlo Algorithms

Given:
Measured Complex Radar Target Response Signals

$z_R(n) = \text{Real Part}$



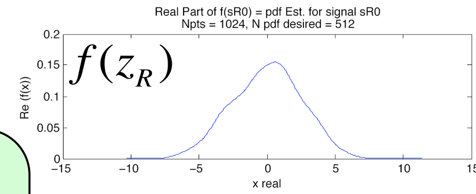
$z_I(n) = \text{Imaginary Part}$



Estimate pdfs

and

PSDs

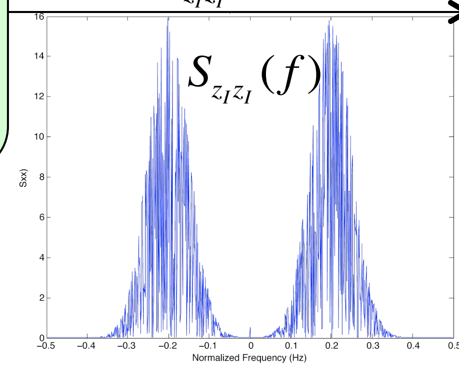


$f(z_R)$

$f(z_I)$

$S_{z_R z_R}(f)$

$S_{z_I z_I}(f)$



Simulate an Ensemble of Signals With the Same pdfs and PSDs

Ensemble of M Simulated Complex Target Response Signals

$\left\{ \hat{x}_{R_i}(n) \right\}_{i=1}^{M}$

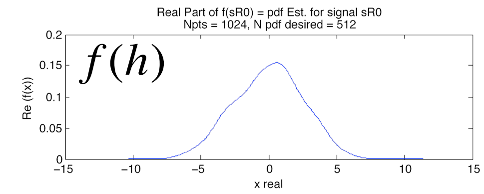
$\left\{ \hat{x}_{I_i}(n) \right\}_{i=1}^{M}$

We Are Accustomed to Drawing *i.i.d. Samples from a Specified Distribution with a Given pdf*

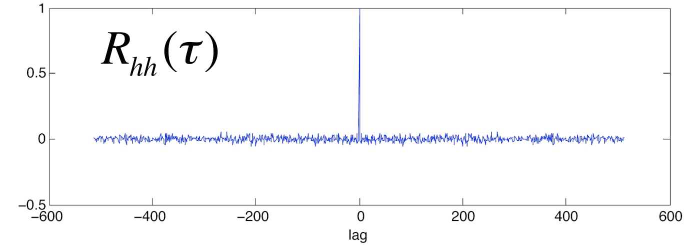
- **Markov Chain Monte Carlo (MCMC) Methods**
    - Metropolis-Hastings Sampling
    - Gibbs Sampling
    - Rejection Sampling
    - Slice Sampling
    - Importance Sampling

    etc.
- **Sequential Monte Carlo (Particle Filter) Methods**

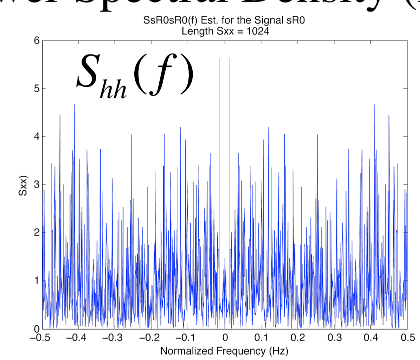
Probability density function (pdf)



Autocorrelation



Power Spectral Density (PSD)



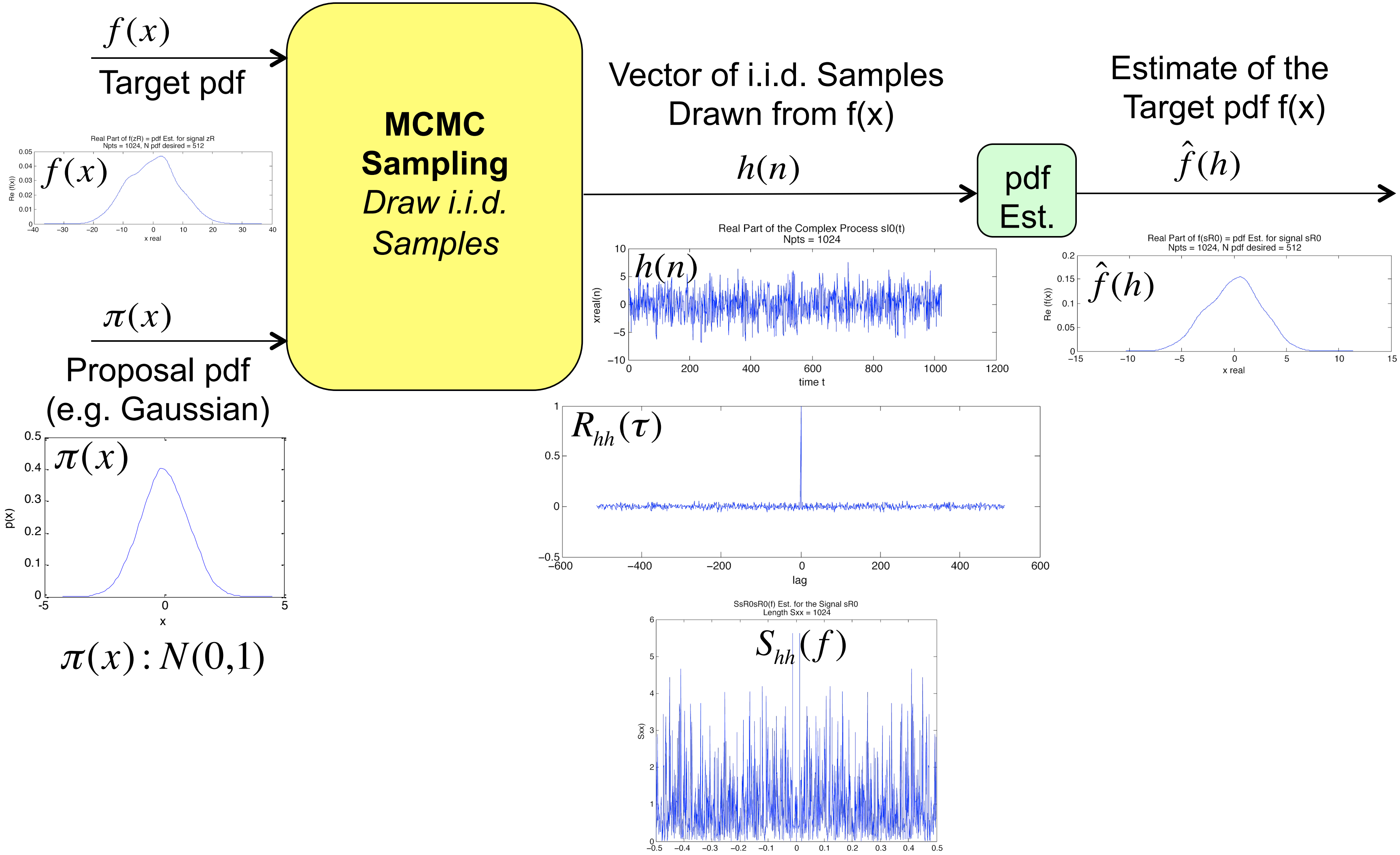
Autocorrelation of $x(t)$:

$$R_{xx}(\tau) = E\left\{x(t)x^{*}(t+\tau)\right\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{xx}(f)e^{j2\pi f\tau}d\omega$$

Power Spectral Density (PSD) of $x(t)$:

$$S_{xx}(f) = \int_{-\infty}^{\infty} R_{xx}(\tau)e^{-j2\pi f\tau}d\tau$$

Example: MCMC Sampling Algorithms Draw i.i.d. Samples from the Target Distribution You Provide



For Non-Gaussian Cognitive Radar, We Need to Draw *Non-Gaussian Correlated Samples*

For the Cognitive Radar Problem, we need to draw correlated samples from a specified pdf (Probability Density Function) and specified Power Spectral Density (PSD)

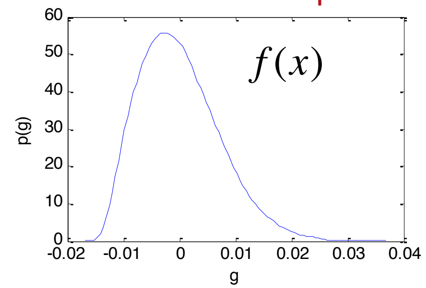
Autocorrelation of $x(t)$:

$$R_{xx}(\tau) = E\left\{x(t)x^*(t+\tau)\right\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{xx}(f)e^{j2\pi f\tau}d\omega$$

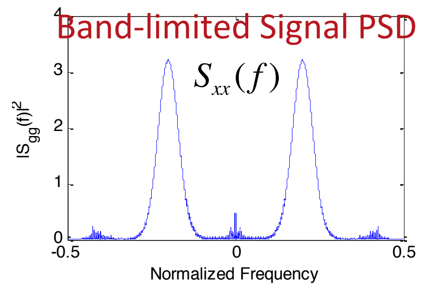
Power Spectral Density (PSD) of $x(t)$:

$$S_{xx}(f) = \int_{-\infty}^{\infty} R_{xx}(\tau)e^{-j2\pi f\tau}d\tau$$

Non-Gaussian pdf

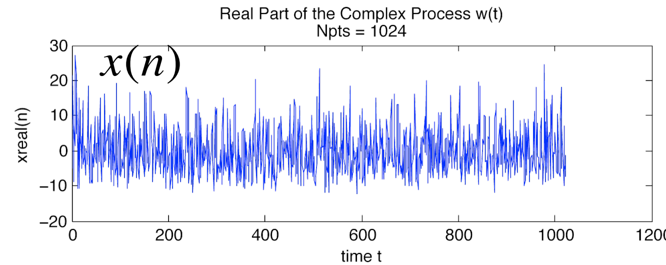


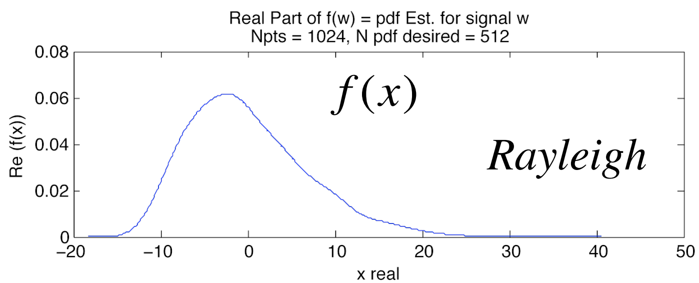
Band-limited Signal PSD

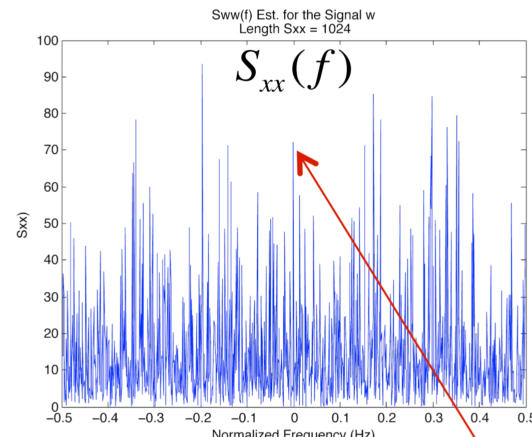


The pdf and PSD of a Stochastic Process Cannot be Specified Independently

The pdf $f(x)$ and the PSD $S_{xx}(k)$ are linked through the signal mean $\overline{x}$ and signal variance σ_x^2







$$E\left\{x\right\} = \overline{x} = \int_{-\infty}^{\infty} f(x)\,dx$$

$$\sigma_x^2 = E\left\{\left(x-\overline{x}\right)^2\right\} = E\left\{x^2\right\} - \overline{x}^2$$

$$\Rightarrow \quad \overline{x}^2 = \sigma_x^2 - E\left\{x^2\right\}$$

$$S_{xx}(0) = N\left(\overline{x}\right)^2 T$$

$$= N\left[\sigma_x^2 - E\left\{x^2\right\}\right]$$

Proof that the pdf and PSD of a Stochastic Process Cannot be Specified Independently

(((GCSS)))

We can show that the pdf $f(x)$ and the PSD $S_{xx}(k)$ are linked through the signal mean $\bar{x}$ and signal variance σ_x^2

$$R_{xx}(m) = T\sum_{n=0}^{N-1} x(nT)x\left[(n+m)T\right]$$

$$S_{xx}(k) = \sum_{m=0}^{N-1} R_{xx}(mT)e^{-\frac{j2\pi}{N}km}$$

$$= \sum_{m=0}^{N-1}\left\{T\sum_{n=0}^{N-1} x(nT)x\left[(n+m)T\right]\right\}e^{-\frac{j2\pi}{N}km}$$

$$S_{xx}(0) = \left\{\sum_{m=0}^{N-1} x(nT)\right\}\left\{T\sum_{n=0}^{N-1}\left[(n+m)T\right]\right\}$$

$$= \quad N\bar{x} \quad \bullet \quad T\bar{x}$$

$$\boxed{S_{xx}(0) = N\left(\bar{x}\right)^2 T}$$

We know that the variance can be written:

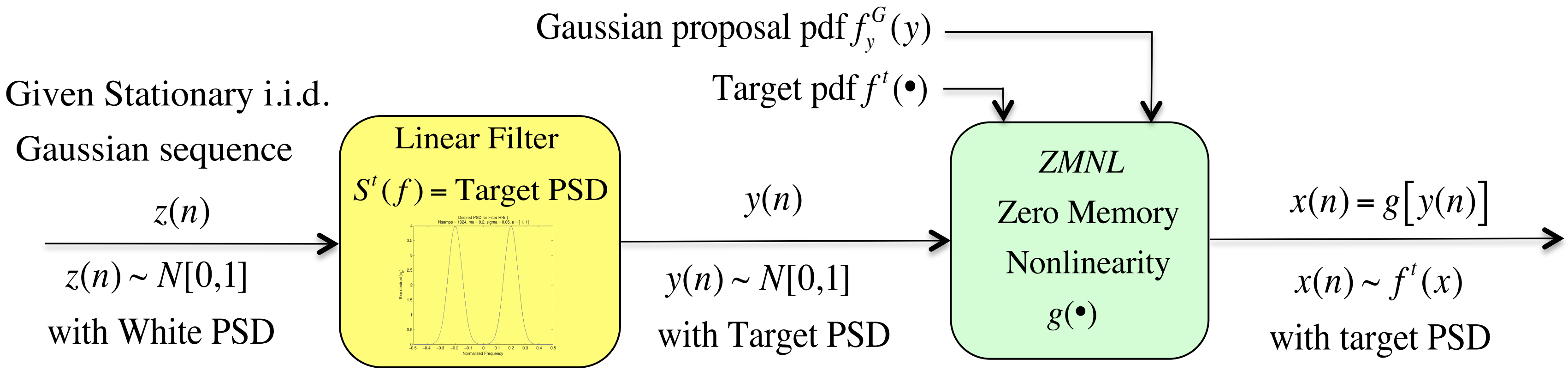
$$\sigma_x^2 = E\left\{\left(x-\bar{x}\right)^2\right\} = E\left\{x^2\right\} - \bar{x}^2$$

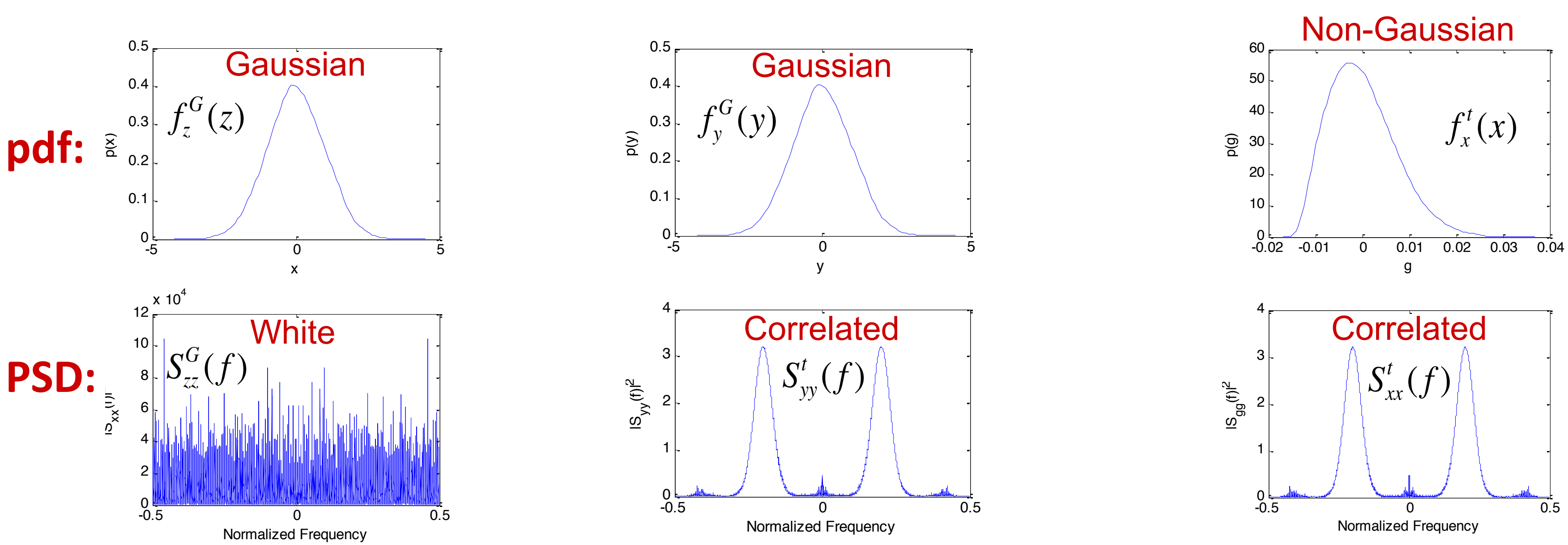
$$\Rightarrow \boxed{\bar{x}^2 = \sigma_x^2 - E\left\{x^2\right\}}$$

We see that:

$$\boxed{S_{xx}(0) = N\left[\sigma_x^2 - E\left\{x^2\right\}\right]}$$

Literature Survey: *The General Approach Uses a Zero Memory Nonlinearity (ZMNL)*





Brief Literature Survey: Generating Correlated Samples with Desired pdf and Desired PSD

- **The Problem:**
  Given i.i.d. Gaussian sequence sequence *z(n)*, desired *target pdf/CDF*, and *desired PSD*
  Generate sequence x(n) with desired pdf and PSD

- **Inverse CDF Methods can provide a *Zero Memory Nonlinearity (ZMNL)***
  - Use a linear filter to obtain y(n) and to assign the desired spectral properties
  - The ZMNL function g(.) is given by:

$$x(n) = g[y(n)] = F_t^{-1}\left\{F_y^G[y(n)]\right\}$$

$$F_t(\cdot) = \text{Desired Target CDF}$$

$$F_y^G[y(n)] = \text{Gaussian Proposal CDF}$$

  - $g(\cdot)$ is expanded in terms of Hermite polynomials, so the autocorrelation of the ZMNL output can be written as a power series of the autocorrelation of y(n).

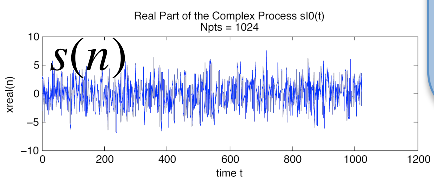
  - Solve for the autocorrelation associated with y(n) which makes the ZMNL output best approximate the autocorrelation associated with y(n)

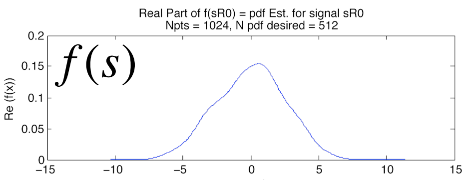
  - *The main problem is that $F(\cdot)$ is often not invertible analytically, and Finding $F^{-1}(\cdot)$ numerically is detrimental to the simplicity and accuracy of the method*

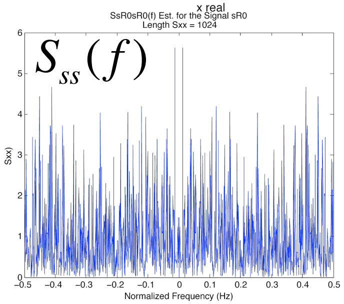
New Iterative Algorithm by Nichols et Al.

Good for Generating *Real* Correlated Samples

i.i.d. Samples $s(n)$







Scaling so the signal has variance consistent with Parseval's Theorem and the Desired PSD. Remove the mean.

$x_0(n)$

Sort from smallest to largest and store result

$x_0(I_n)\,,\;\; n = 0,1,\,\ldots\,, N-1$

$x_0(I_1) < x_0(I_2) < \cdots < x_0(I_{N-1})$

Give signal same phase, but *Target* Fourier magnitudes

$x(n)$

Bring pdf in line with target pdf: Shuffle $x_0(n)$ so it has same rank as $x(n)$. Smallest value of $x_0(n)$ is given same position in signal as smallest value of $x(n)$, etc.

$x(J_n)$

$x(J_n) = x_0(I_n)\,,\;\; n = 0,1,\,\ldots\,, N-1$

$x(J_1) < x(J_2) < \cdots < x(J_{N-1})$

$x(J_n)$

$Rank(k) = Rank(k-1)?$

No

$k = k+1$

Yes

$x(J_n) = x(J_n) + \bar{x}$

Stop

$x(J_n)$ has the same pdf as $x_0(n)$ and similar PSD

For a Real-World Application with a Non-Gaussian pdf, The Overall Sampling Process Involves Several Steps

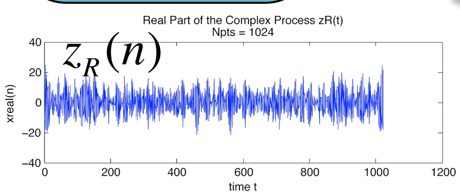
Simulate Correlated Complex Measurements *with Non-Gaussian pdf & Correlated PSD*

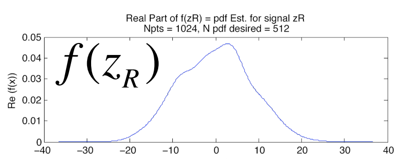
$z_R(n)$ → pdf Est. → $f(z_R)$

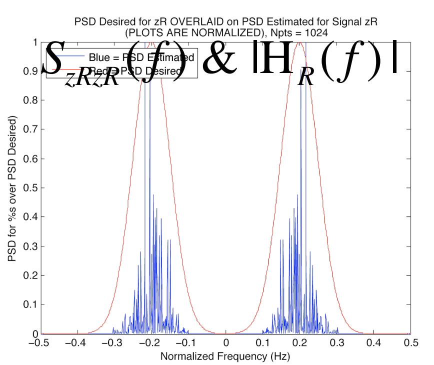
$z_I(n)$ → pdf Est. → $f(z_I)$

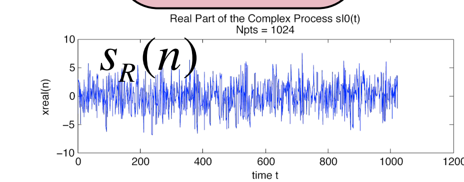
$f(z_R)$, $f(z_I)$ → **Markov Chain Monte Carlo Sampling** *Draw i.i.d. Samples* → $s_R(n)$, $s_I(n)$

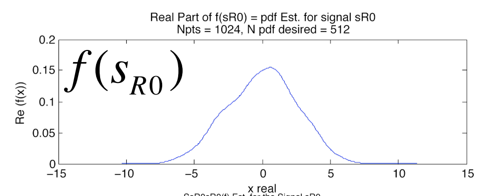
$s_R(n)$, $s_I(n)$ → **Correlated Sampling Algorithm:** Iterative Spectral Constraints and Magnitude Sorting *Draw Correlated Samples* → $\hat{x}_R(n)$, $\hat{x}_I(n)$

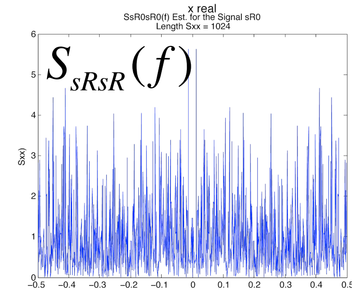


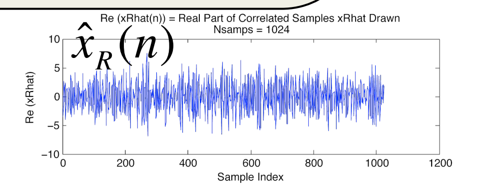


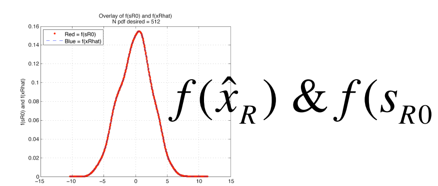


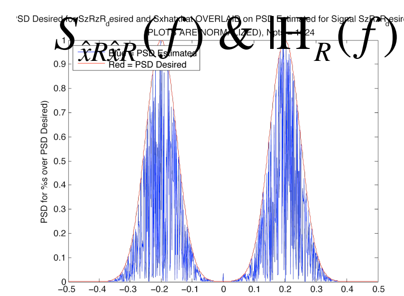




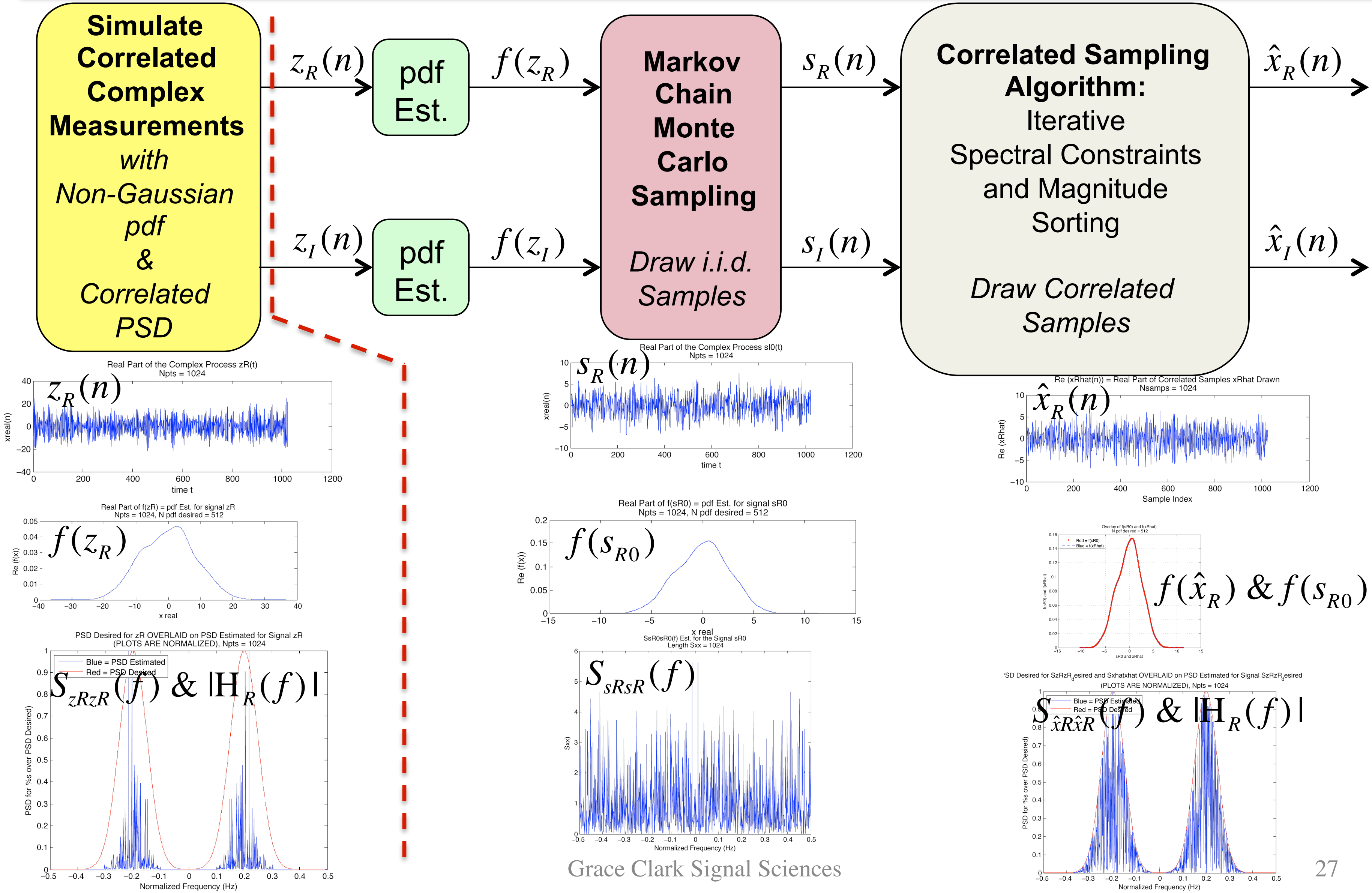




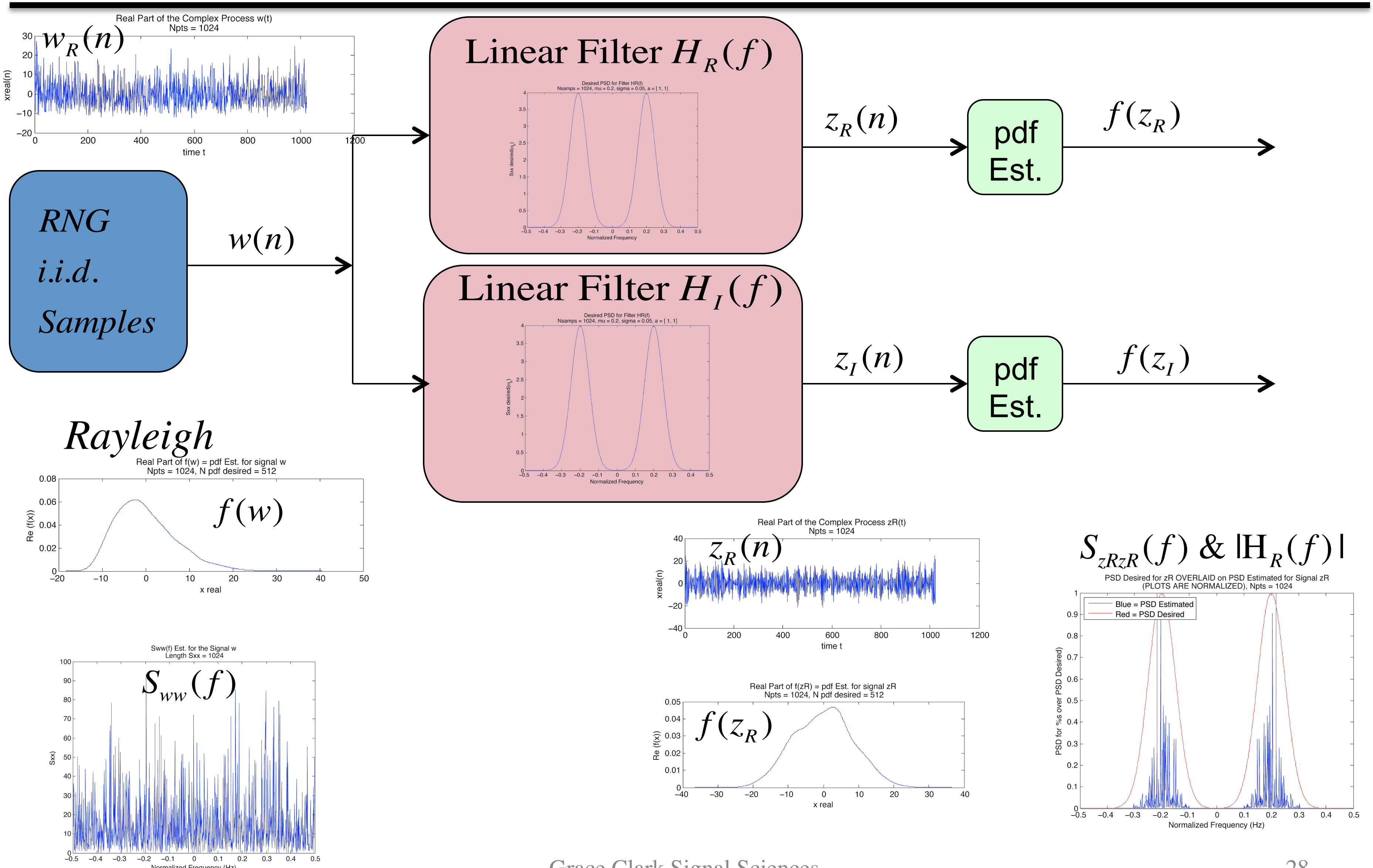




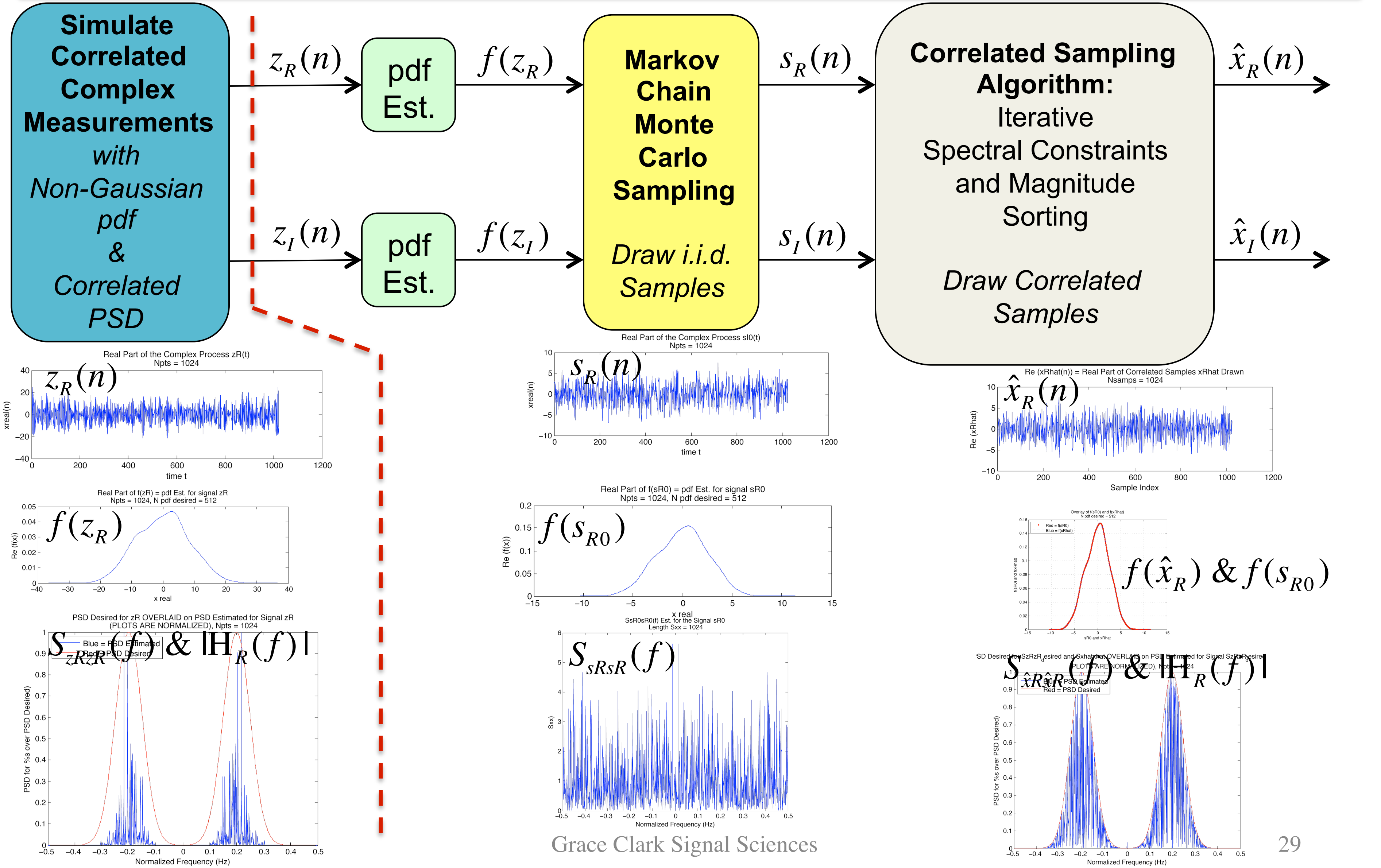
Close-Up Block Diagram (See Yellow):
Simulate Correlated Complex Measurements



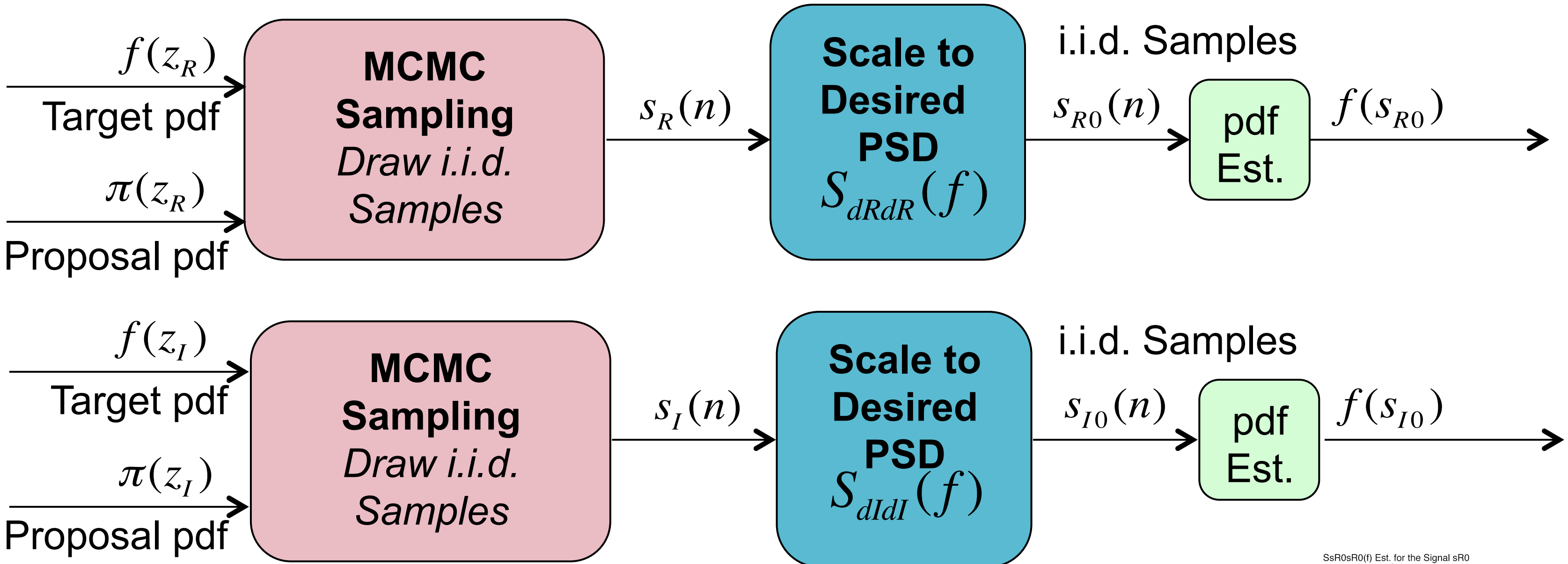
Close-Up Block Diagram for:
Simulating Correlated Complex Signal Measurements

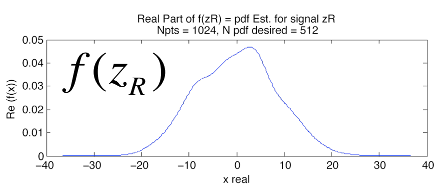


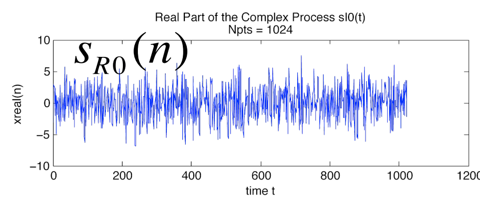
For a Real-World Application with a Non-Gaussian pdf,
The Overall Sampling Process Involves Several Steps

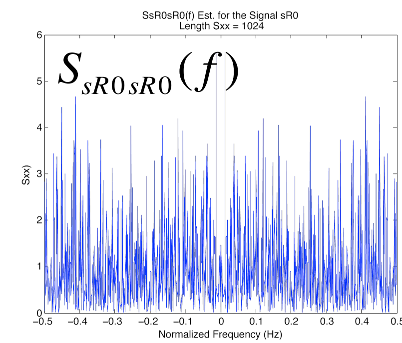


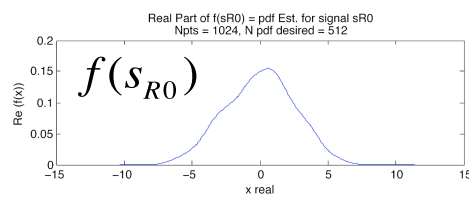
Close-Up Block Diagram for:
The MCMC Sampling Step to Generate i.i.d. Samples



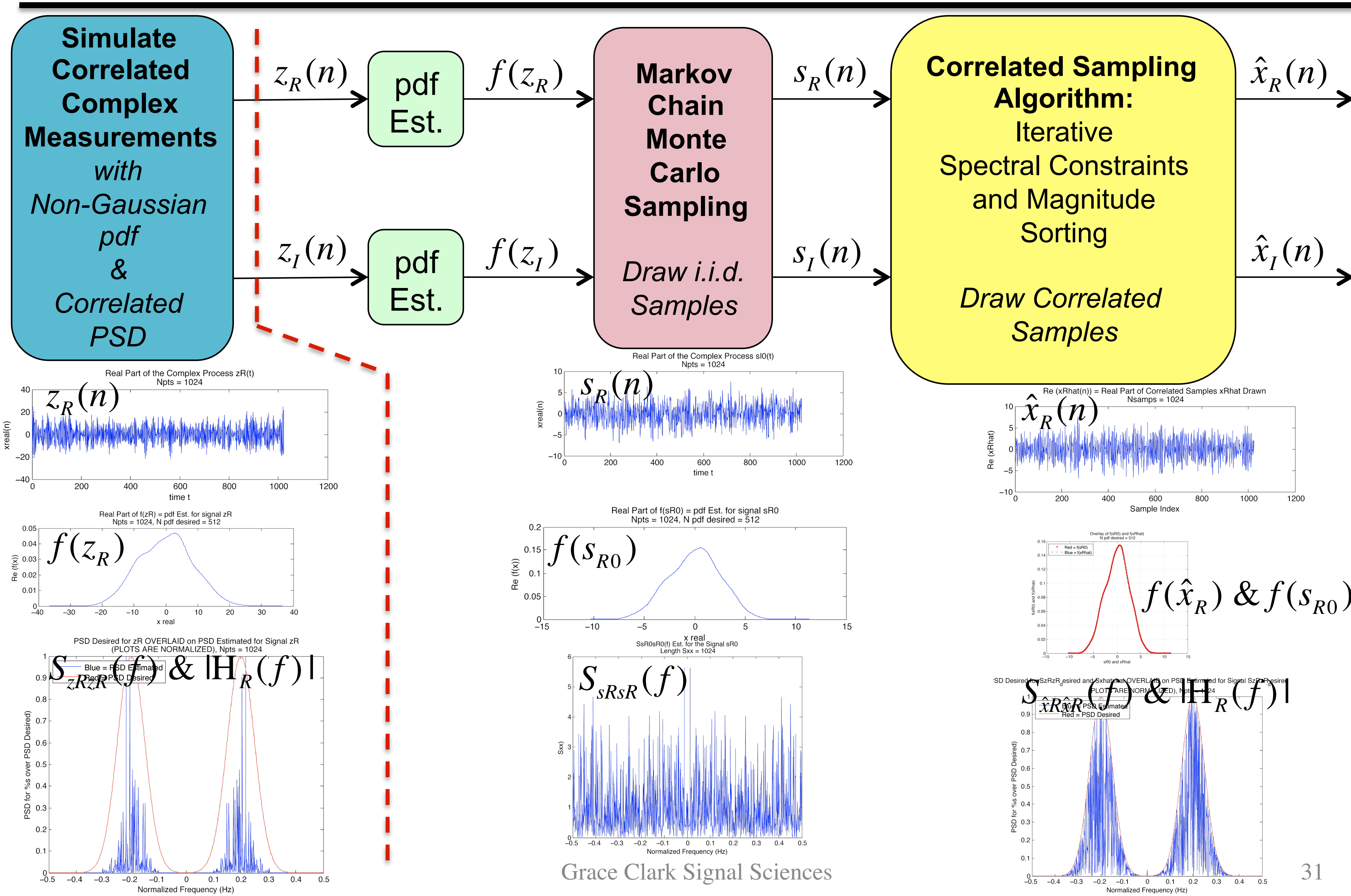




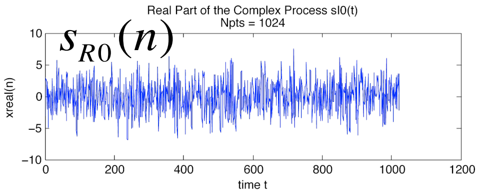


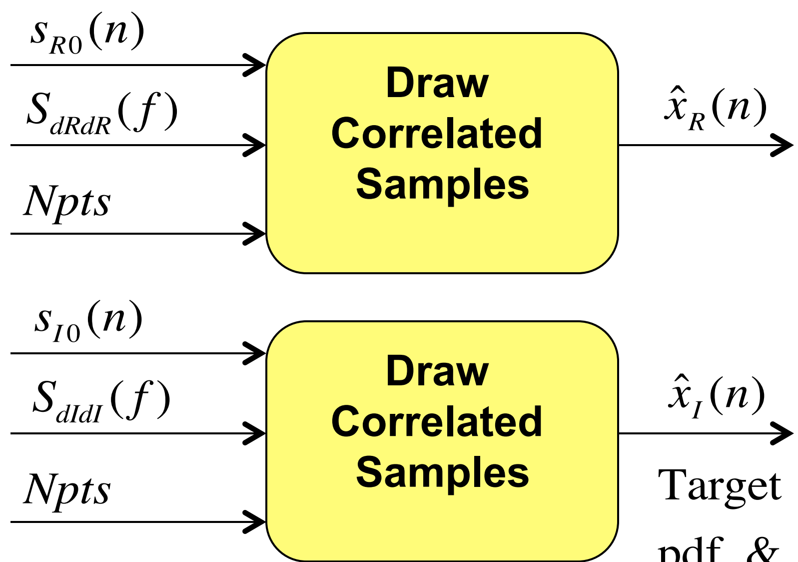


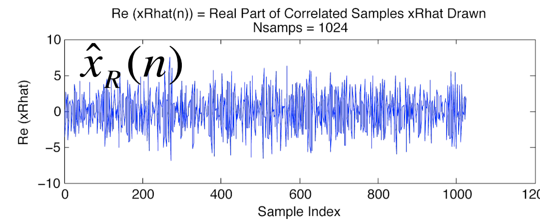
For a Real-World Application with a Non-Gaussian pdf, The Overall Sampling Process Involves Several Steps

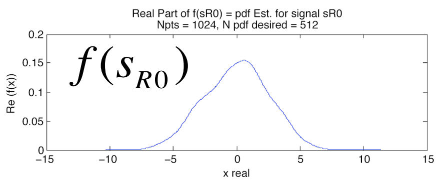


Close-Up Block Diagram for:
Correlated Sampling Algorithm

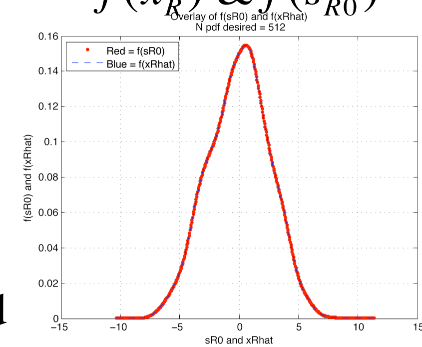




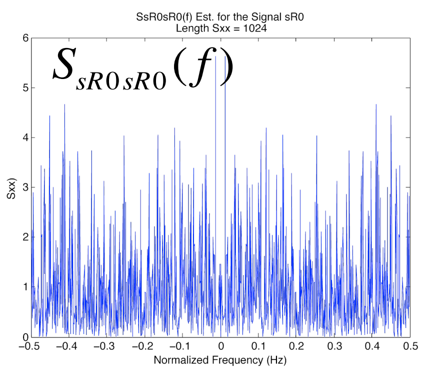




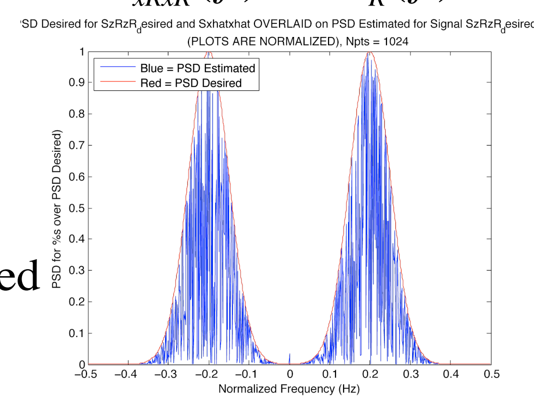
$f(\hat{x}_R)$ & $f(s_{R0})$

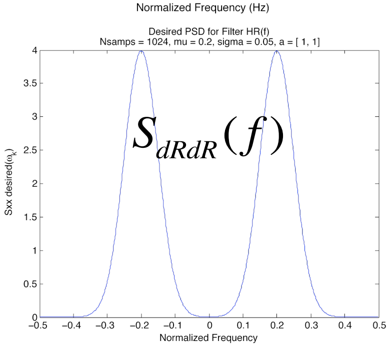


Target pdf & Estimated pdf



$S_{\hat{x}R\hat{x}R}(f)$ & $|H_R(f)|$





Desired Target PSD

Target PSD & Estimated PSD

Conclusions

- **Earlier**, we used target responses with *simulated closed-form pdfs* for proof of principle of our non-Gaussian Cognitive Radar algorithms

- The pdf and the PSD cannot be specified independently, because they are linked through the signal mean and variance

- ***New Capability for Using Real-World Signals in Cognitive Radar:*** *Given only* ***measured*** *complex non-Gaussian target responses,* we can now simulate large ensembles of these target responses that have specified pdfs and specified band-limited PSDs

  - Combined the simple and efficient Nichols algorithm with MCMC sampling
  - "Extended" the algorithm for use with complex signals

Future Work:

- Work with realistic simulated target impulse responses
- Work with real-world target impulse responses
- Strategies to reduce computational complexity

The World of Acoustics Before Signal Processing

Contingency VG's

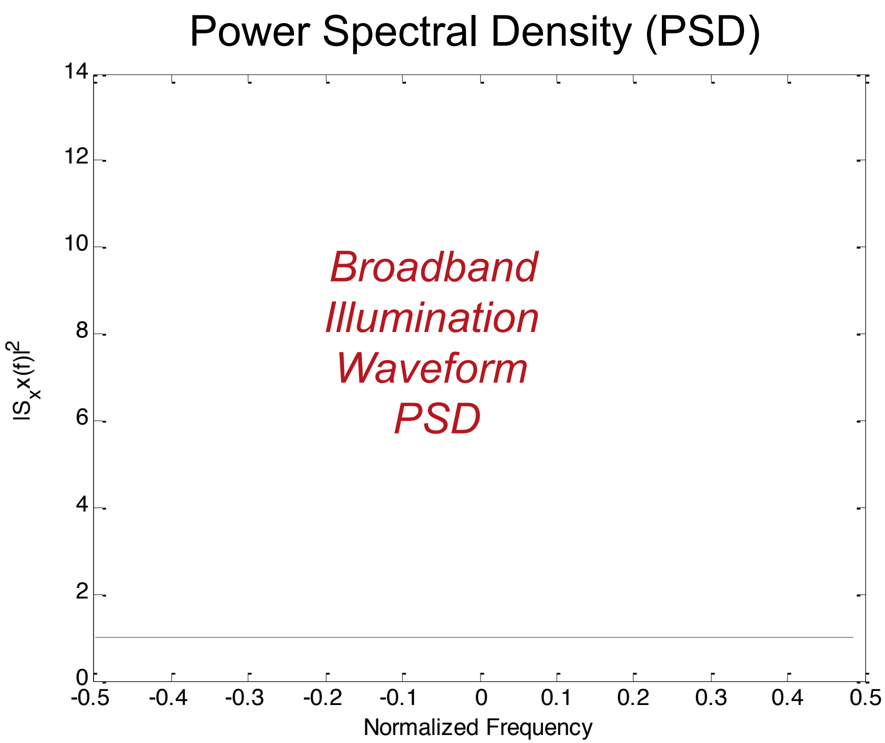
Grace Clark

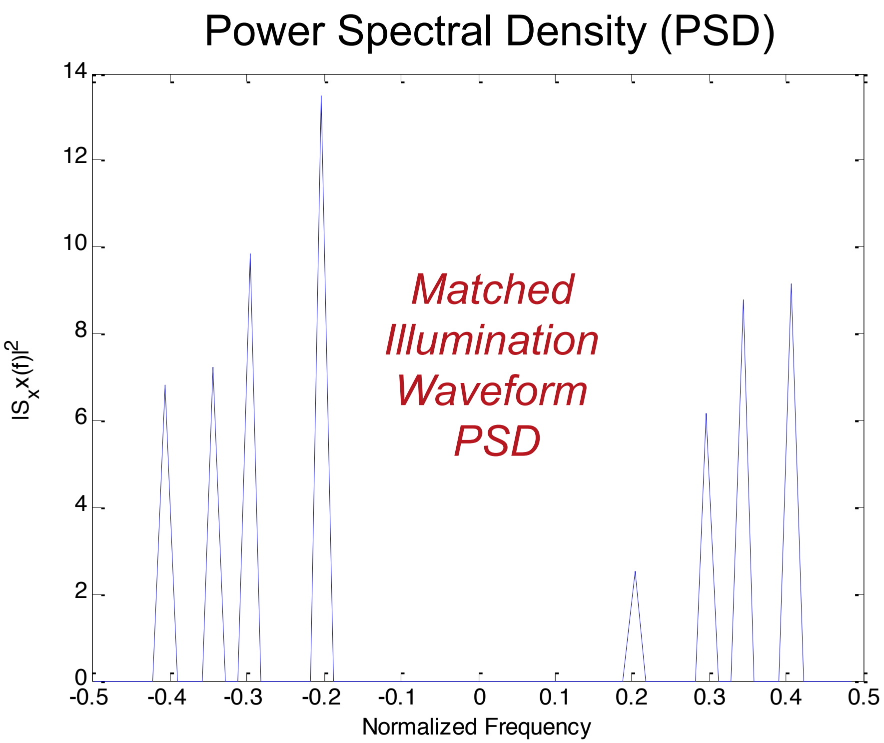
In Cognitive Radar, Illumination Waveform Design Exploits the Sparsity of the Bandlimited Target Spectra

Toy Example:

Four complex targets, each with a different PSD

The matched illumination waveform focuses the spectral energy where the target spectra reside





Both Waveforms Have Total Energy = 1 unit

Illumination Waveform Design Assumes that the Radar Can Transmit *"Arbitrary Waveforms"*

- Generally, radar systems are built to transmit broadband waveforms
- ***"Arbitrary Waveform Generators (AWG's)"*** are available commercially
    - Given a digital file containing the desired illumination waveform, the AWG, the radar system and antenna convert the digital file to an analog EM field used to illuminate the target(s)
    - For the approach defined here, the desired illumination waveform PWE(t) is computed as described in the figures and stored in a digital file:

$$PWE^k(t) = \sqrt{E_s}\sum_{i=1}^{M}\sqrt{P_i^k}\,x_i^{opt}(t) = \text{Probability Weighted Energy}$$

k = Illumination Iteration Index = $0,1,2,\ldots$

i = Target Index = $1,2,\ldots M$

E_s = Energy in the Illumination Waveform

P_i^k = Prior probability for target i at illumination iteration k

PWE(t) is a Weighted Sum of Individual Optimal Matched Target Illumination Waveforms

- A single matched illumination waveform is estimated by Maximizing the SNR in the receiver:
- The PSD's of the individual targets are assumed known a priori from calibration experiments
- The optimal illumination waveform $x_i^{opt}(t)$ for a single target is an eigen-solution that has the form of a complex exponential function:

$$\lambda_{\max}\hat{x}(t) = \int_{-T/2}^{T/2} \hat{x}(\tau) R_g(t-\tau)\, d\tau$$

where $R_g(\tau)$ is the covariance obtained from the PSD of the target signal $g(t)$.

- The overall illumination waveform $PWE^k(t)$ is the weighted sum of the individual optimal target waveforms. The weights P_i^k are prior probabilities:

$$PWE^k(t) = \sqrt{E_s}\sum_{i=1}^{M}\sqrt{P_i^k}\, x_i^{opt}(t) = \text{Probability Weighted Energy}$$

k = Illumination Iteration Index = $0, 1, 2, \ldots$

i = Target Index = $1, 2, \ldots M$

E_s = Energy in the Illumination Waveform

P_i^k = Prior probability for target i at illumination iteration k

Pseudo-Code Block Diagram of One Radar Classification Evolution

For k = 1: # of transmissions

Compute the $PWE(t)$ based on the SNR criterion with the proper probability update

Target realization of $\hat{\underline{g}}_i$ based on the i^{th} hypothesis

Simulate measurement data $\underline{y}_k = \sqrt{E_s} X_k \underline{g}_i + \underline{n}_k$

For i = 1: # of hypotheses

Compute the conditional PDF for each target hypothesis (for k^{th} transmission),
J is the number of online targets in the target ensemble ($J = 40$)

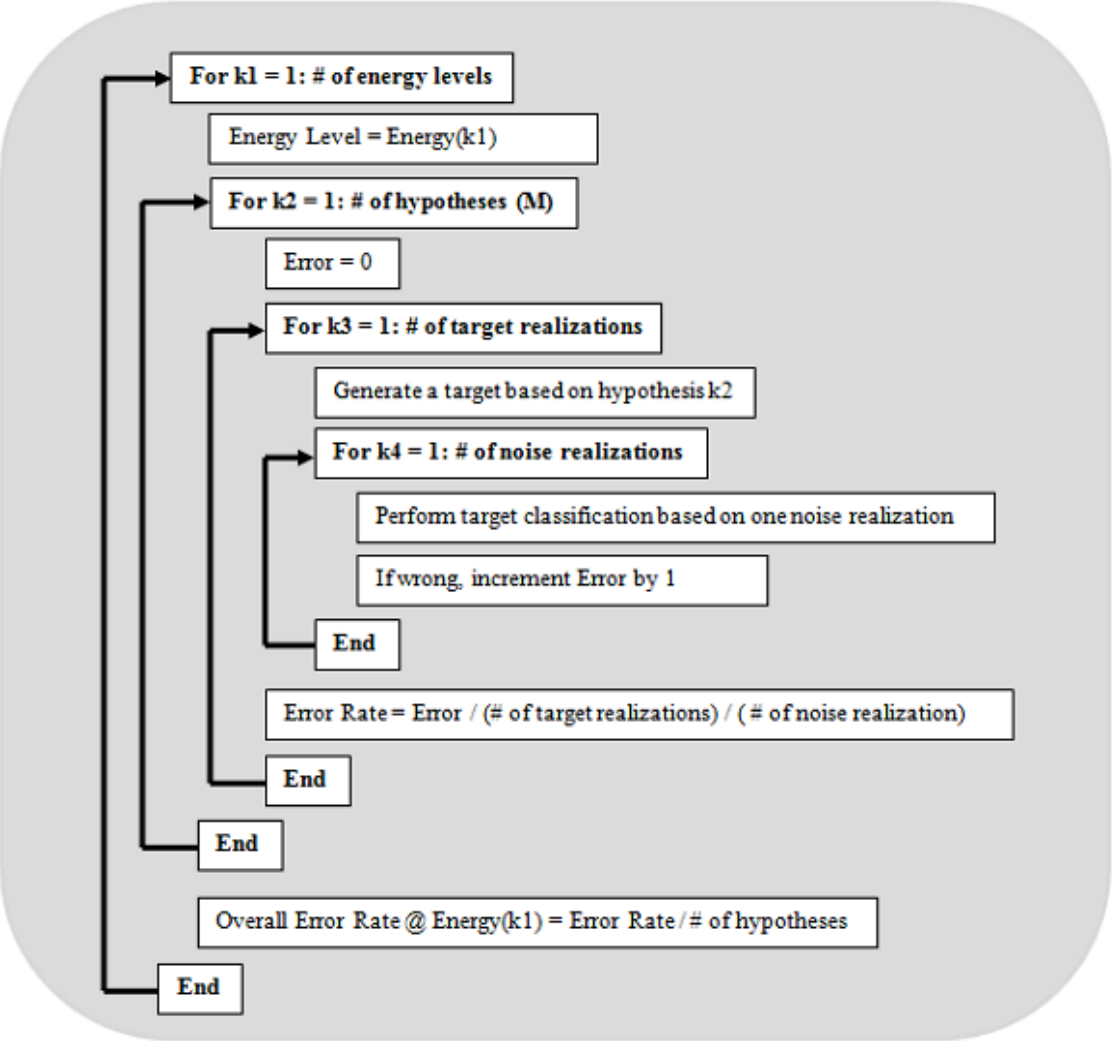
$$p_i^k(y_1,\dots,y_k) = \frac{1}{J}\sum_{j=1}^{J}\exp\left[-\sum_{k=1}^{K}(y_k-\hat{y}_i^j)^H K_N^{-1}(y_k-\hat{y}_i^j)\right]$$

End

Update the prior by $P_i^{k+1} = \dfrac{p_i^k(y_1,y_2,\dots,y_k)P_0}{\sum_{i=1}^{M} p_i^k(y_1,y_2,\dots,y_k)P_0}$

End

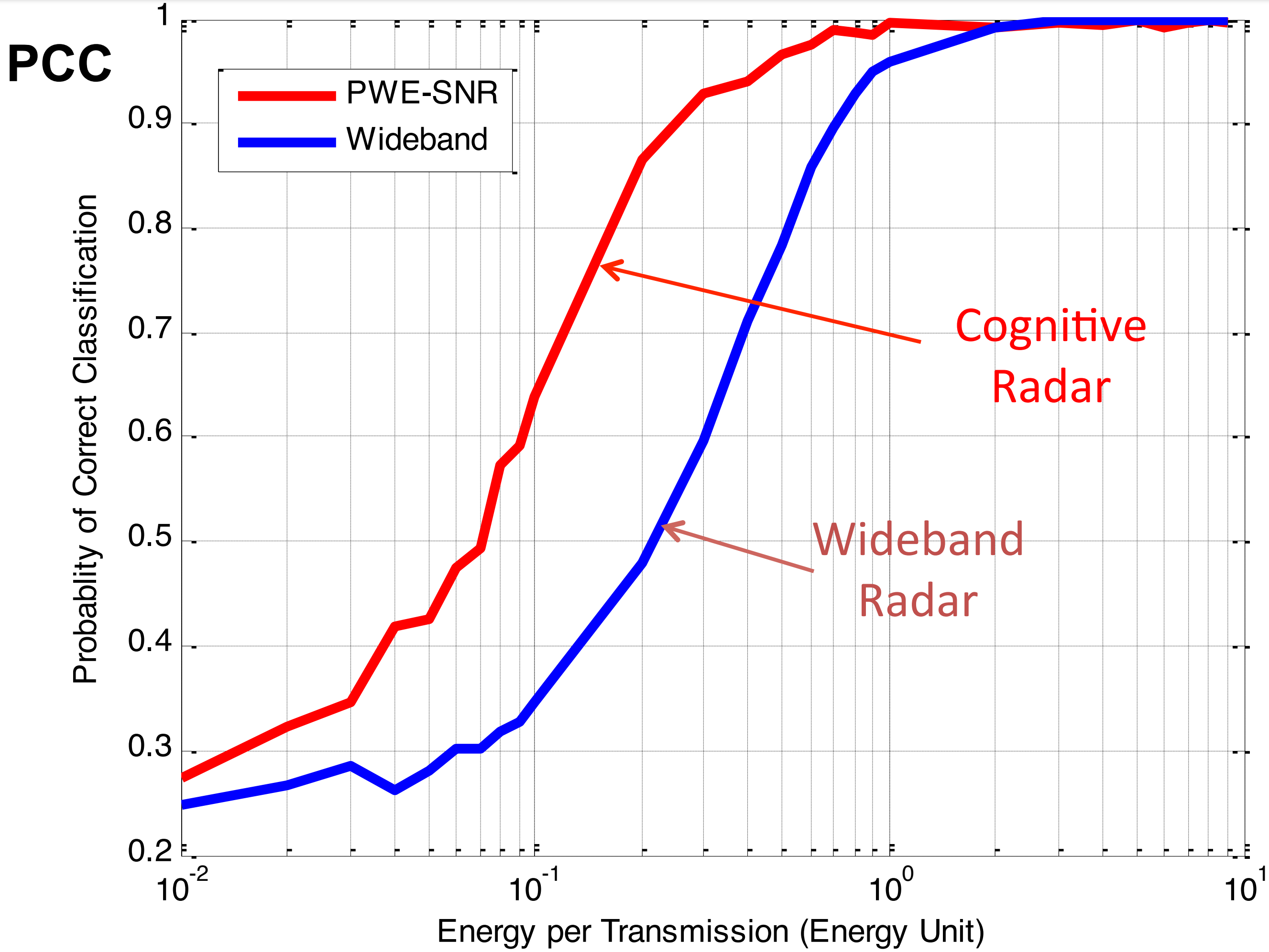
An Example of the Monte Carlo Simulation Experiments Used to Evaluate Classification Performance



Experiment: Probability Density Functions (pdf's) Specified for the Four Target Classes (Hypotheses)

Target Hypotheses	Target PDFs (PDF parameters)	μ	σ
Target # 1	Complex Rayleigh $(\sigma = 10)$	0.15	0.01
Target # 2	Complex Exponential $(\mu = 2)$	0.2	0.015
Target # 3	Complex Gamma $(k = 2, \theta = 2)$	0.25	0.02
Target # 4	Complex Log-Normal $(\mu = 0, \sigma = 1)$	0.3	0.025

Classification Performance of the NGCCR Algorithm for 10 Transmissions



Illumination Energy

<u>Monte Carlo Setup:</u>

- 50 Target Realizations
- 10 Noise Realizations

<u>NGCCR Algorithm Setup:</u>

- 40 Target Realizations for the ensemble averaging

The End

Grace Clark