Morphological Diversity Extraction Method (MODEM) and Annulets for Surface Roughness and Radiation Asymmetry Characterization (ARAC)

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CASISConference
LLNL
5-18-2016
Livermore, CA

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Outline & Introduction

- Very brief introduction to inertial confinement fusion challenges tackled here.
- What is MRA? Wavelets, Curvelets, Ridgelets, Annulets, etc.
- Target surface imperfections characterization.
- MODEM: Y_{lm}'s, WLT's & DCT: Doing it on the Sphere. AFM Data.
- Filtering overlapping circular patches of a great circle around a spherical target surface: a MODEM technique for phase shifting spherical diffractive interferometric data.
- Should isolated defect specs incorporate coherence or clustering effects?
- ICE Surface Roughness: X Ray Phase Contrast Imaging Analysis
- Radiation asymmetry Characterization

The Scope of the Work on ICF Target Characterization

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- Apply advanced multiresolution analysis techniques to the characterization of target surfaces, be they internal or external, via optical interferometric, AFM or X-ray transmission data and X ray Phase Contrast Imaging for ICE.
- The first three methods of data collection accentuate global scale spherical features, local defect or bump sequence information and very fine scale imperfections on the microscopic level. The local defects have been isolated (decontaminated from artifacts), classified and sorted (statistics extracted)
- Our techniques are part of our Morphological Diversity Extraction Method (MODEM) here adapted to ICF capsule surface data.
- We have successfully separated the Global features which are best captured by Y_{lm}'s, local defects best captured by isotropic undecimated wavelet transforms, and artifacts best captured by overlapping local DCT's.
- PSSDI data has also been separated into global scale variations, localized defects and diffractive artifacts. The local defects have then been detected, estimated, classified and sorted (ie their statistics extracted)
- We have also characterized the precision radiography data using fractal signatures of the wavelet decompositions of these noisy data sets.
- X Ray PCI data has been denoised using Annulets and the resulting ICE surface nonuniformity quantified using one or more images (out of 200) to see how many Annulets require: 1.

What Are Wavelets? Start @ (www.wavelets.org) & Surf (Mathsoft, amara, ...)

Mallat, Meyer, Daubechies, Coifman, Vetterli, Jaffard, Donoho, Starck, Candes...

- Wavelets are localized kernels or atoms in PHASE SPACE.
- You may think of them as basis functions with prescribed dilation and translation properties.
- They may or may not be orthonormal or have compact support or be differentiable everywhere, or be fractal, or have many zero moments.
- Wavelets are like breathing wave packet envelopes which can home in on structures in phase space better than FT or WFT ever could.

$$\psi_{j,k}(x) = 2^{j/2}\,\Psi\left[2^j\left(x-\frac{k}{2^j}\right)\right];\ j,k\in Z$$

$$\Psi_n(x) = (-1)^n \frac{d^n}{dx^n}\left[\exp\left(-\kappa(x-x_c)^2/2\right)\right]$$

When the scale is decreased translation steps between wavelets should likewise be decreased

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What is MRD or Multi-resolution Decomposition?

- Multiresolution: Zoom in and out on a number of successively finer scales in a sequence of nested approximation subspaces $\{V_j\}_{j \text{ in } Z}$.
- In general, get an overcomplete basis set in $L_2(R)$.
Approximate (or truncate) by bounding the scales of interest.

Scaling functions and the scaling equation:

Low pass filter

$$\varphi(x) = 2\sum_{k=0}^{2N-1} h_k \varphi(2x-k)$$

$$\sum_k h_k = 1 \qquad \int_{-\infty}^{\infty} \varphi(x)\,\mathrm{d}x = 1$$

The Wavelets:

High pass filter

$$\psi(x) = 2\sum_{k=0}^{2N-1} g_k \varphi(2x-k)$$

$$g_k = (-1)^k h_{2N-1-k}$$

These filters decompose a sampled signal into 2 sub-sampled channels: the coarse approximation of the signal and the missing details at finer scales. The original signal can be reconstructed from these channels by interpolation.

Discrete Wavelet Transforms & Perfect Reconstruction Subband Coding (Quadrature Mirror) Filters

DWTs can be orthonrmal decompositions:

$$f(t) = \sum_k c_k \, \phi_k(t) + \sum_{j=0}^{J} \sum_k d_{jk} \, \psi_{jk}(t)$$

$$c_m = \int f(t) \, \phi_m(t) \, dt, \quad d_{lm} = \int f(t) \, \psi_{lm}(t) \, dt$$

The number of operations required to perform DWTs with a filter of length L (with L taps) is of order L x N (Even FFTs require N log N operations)

$$LN\left(1 + \frac{1}{2} + \frac{1}{2^2} + \ldots\right) < 2LN$$

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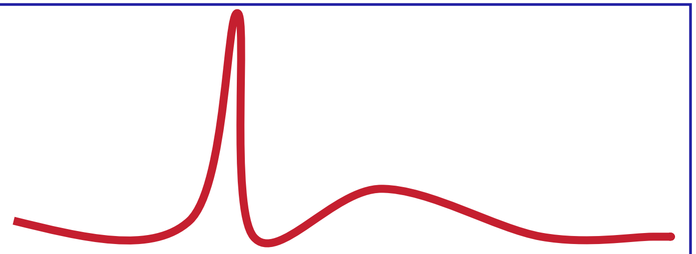
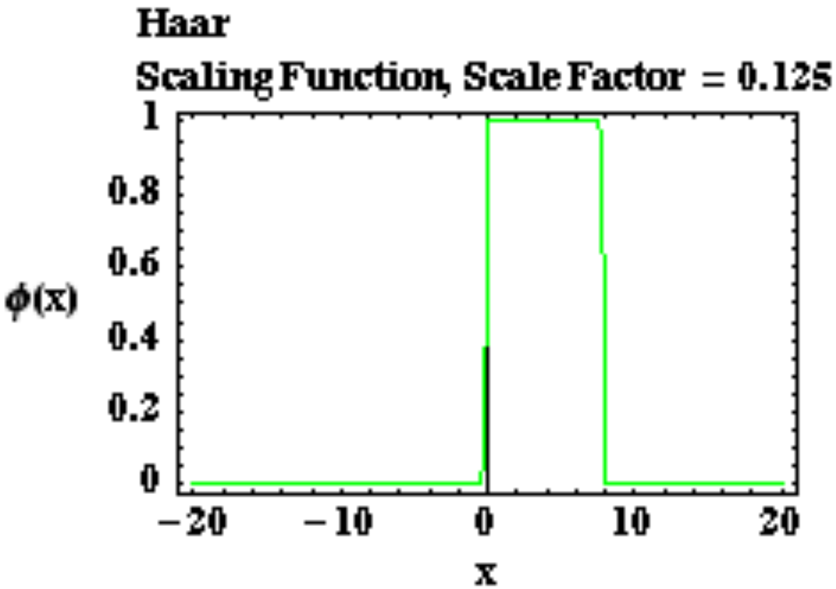
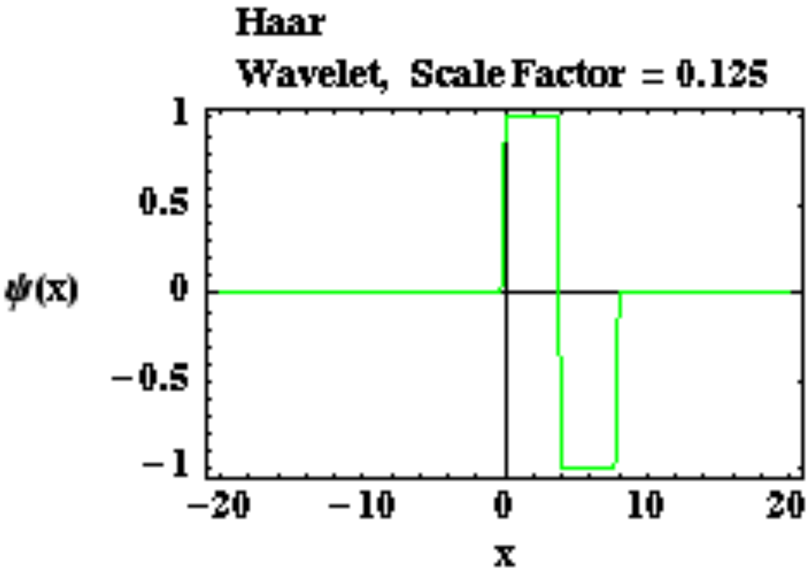
The Key to Multi-Rresolution Analysis Using Wavelets Is:



- THRESHOLDING
- Two Ways to do it:
- Linear or Scale Thresholding
- Nonlinear or Largest Coefficient Thresholding
- Linear is Fourier like: Keep down to some scale and chop off the rest
- Nonlinear Thresholding is the true breakthrough: Keep those wavelets which have the largest coefficients no matter where they are and on whatever scale they are. No need to keep intermediate scales or intermediate locations. Just keep the BIG stuff. Automatically denoise, automatically compress and automatically bring out significant patterns.

The Scaling Function and Wavelet for Haar or Daubechies 1 in X-Space





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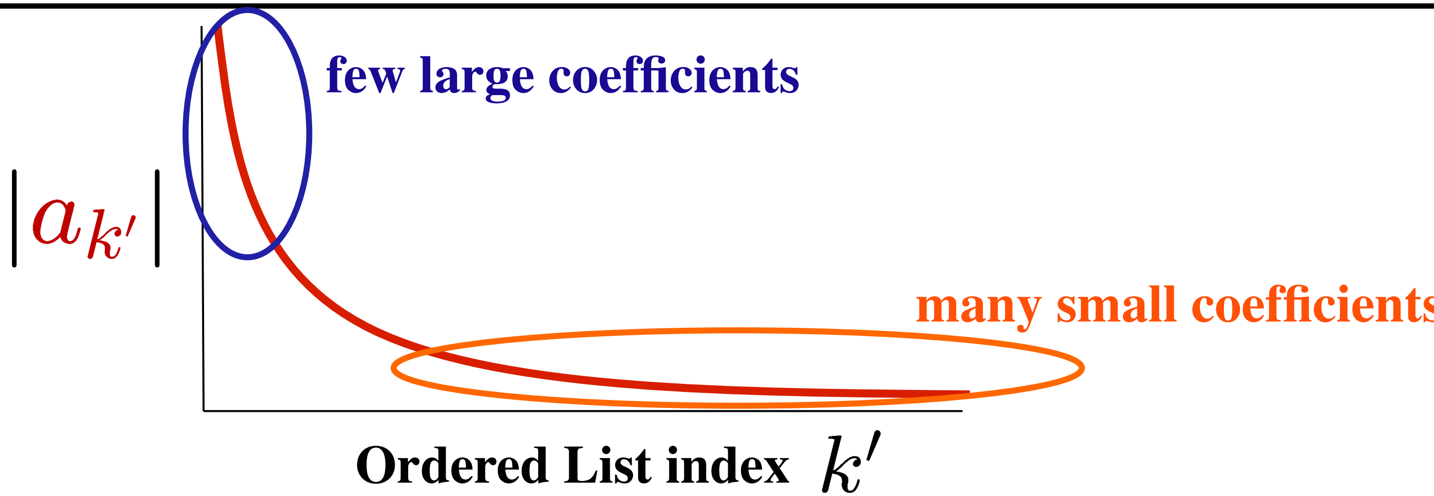
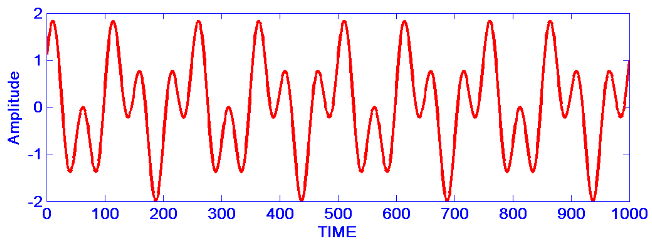
The Secret is to Exploit Sparsity in the Appropriate (Dual) Space



$$s = \sum_{\gamma} \alpha_{\gamma} \phi_{\gamma}$$

In as much as you can decompose a given signal in some redundant library of functions where the large ones will be far fewer in number than the small ones, a hard thresholding technique will give rise to a sparse representation with controllable error.

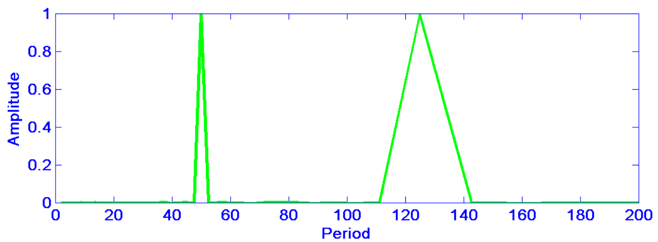
Discrete Wavelet Transform (MRA) vs a Continuous Fourier Transform



A time domain signal

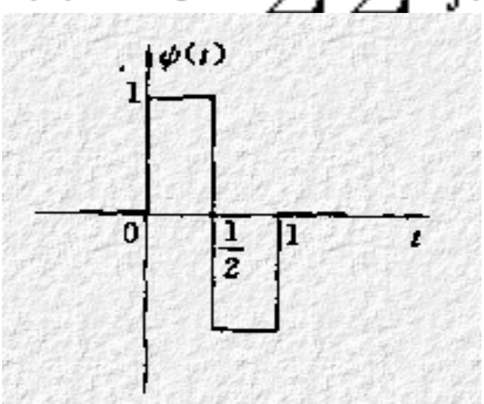
$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-2\pi i f t}\, dt$$

$$x(t) = \int_{-\infty}^{\infty} X(f) e^{2\pi i f t}\, df$$

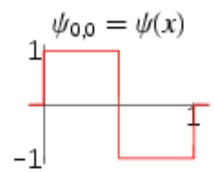


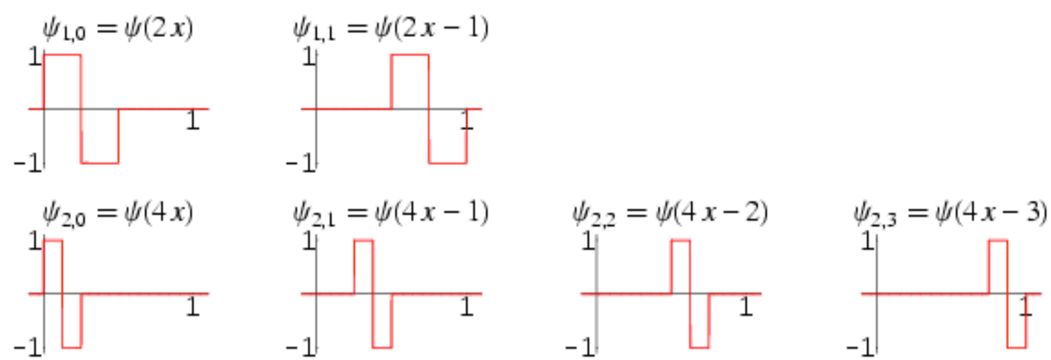
Its Fourier Transform Coefficients (circa 1800)

$$f(x) = c_0 + \sum^{\infty} \sum^{2^j - 1} c_{jk} \psi_{jk}(x).$$



Haar Wavelet decomposition (1909):



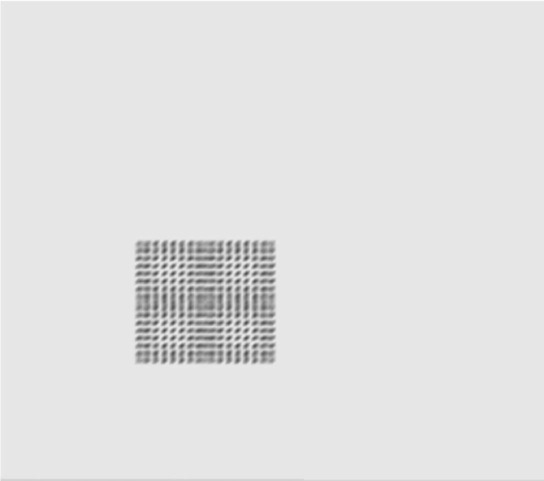


Three Handy MRA Tools for Generic Feature Detection:

Stationary textures

Locally oscillatory patterns

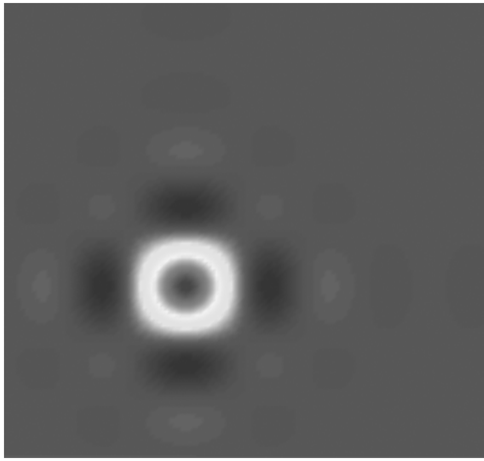
Use DCT



Piecewise smooth objects

Isotropic structures

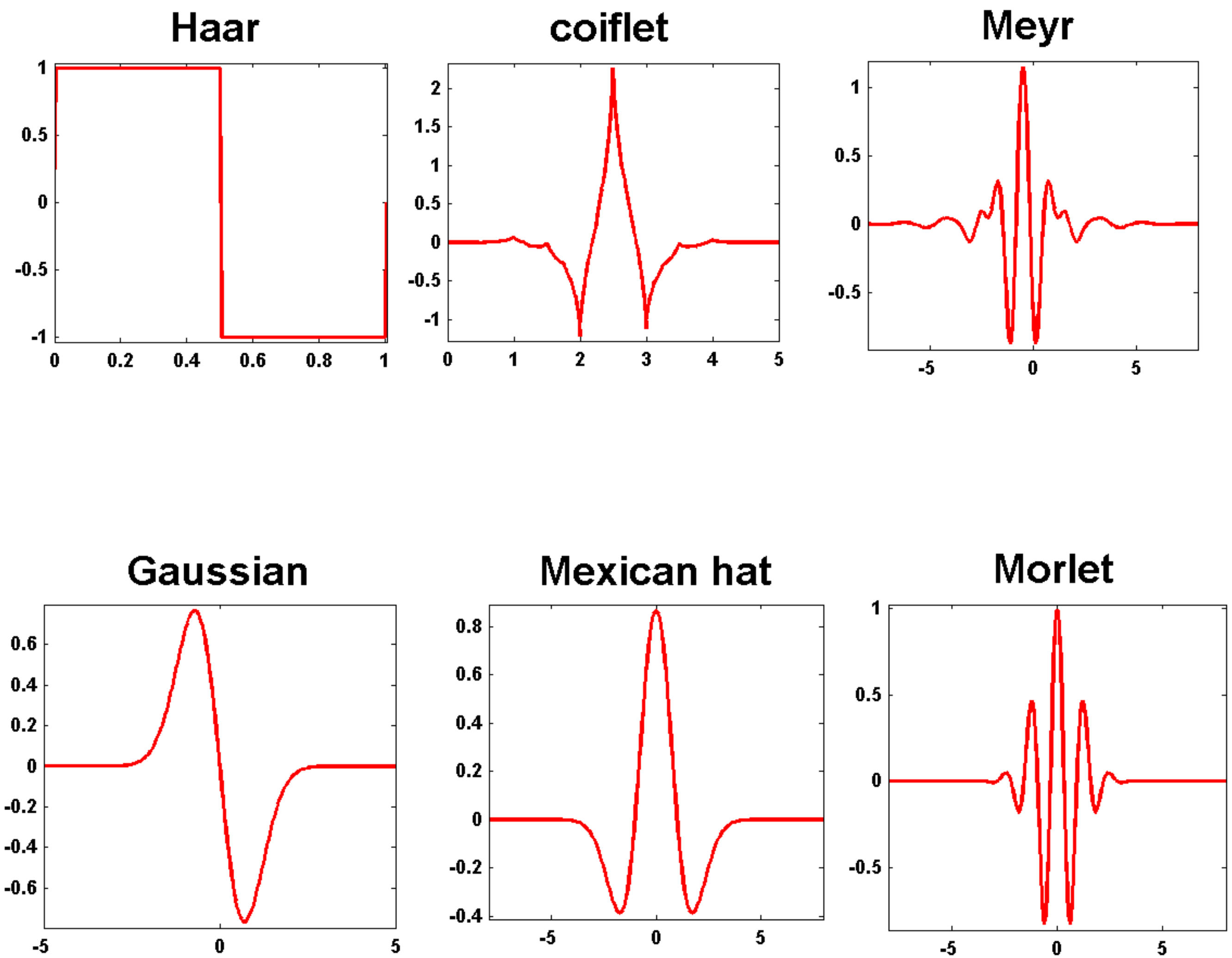
Use Isotropic Undecimated Wavelet Transforms



Piecewise smooth anisotropic objects, edges

Use Curvelets

Some Typical Wavelets (They Have Different phase Space Localization Features)



How Is Phase Space Tiled Differently in the Case of Diracs, DCT, STFT and WLT?

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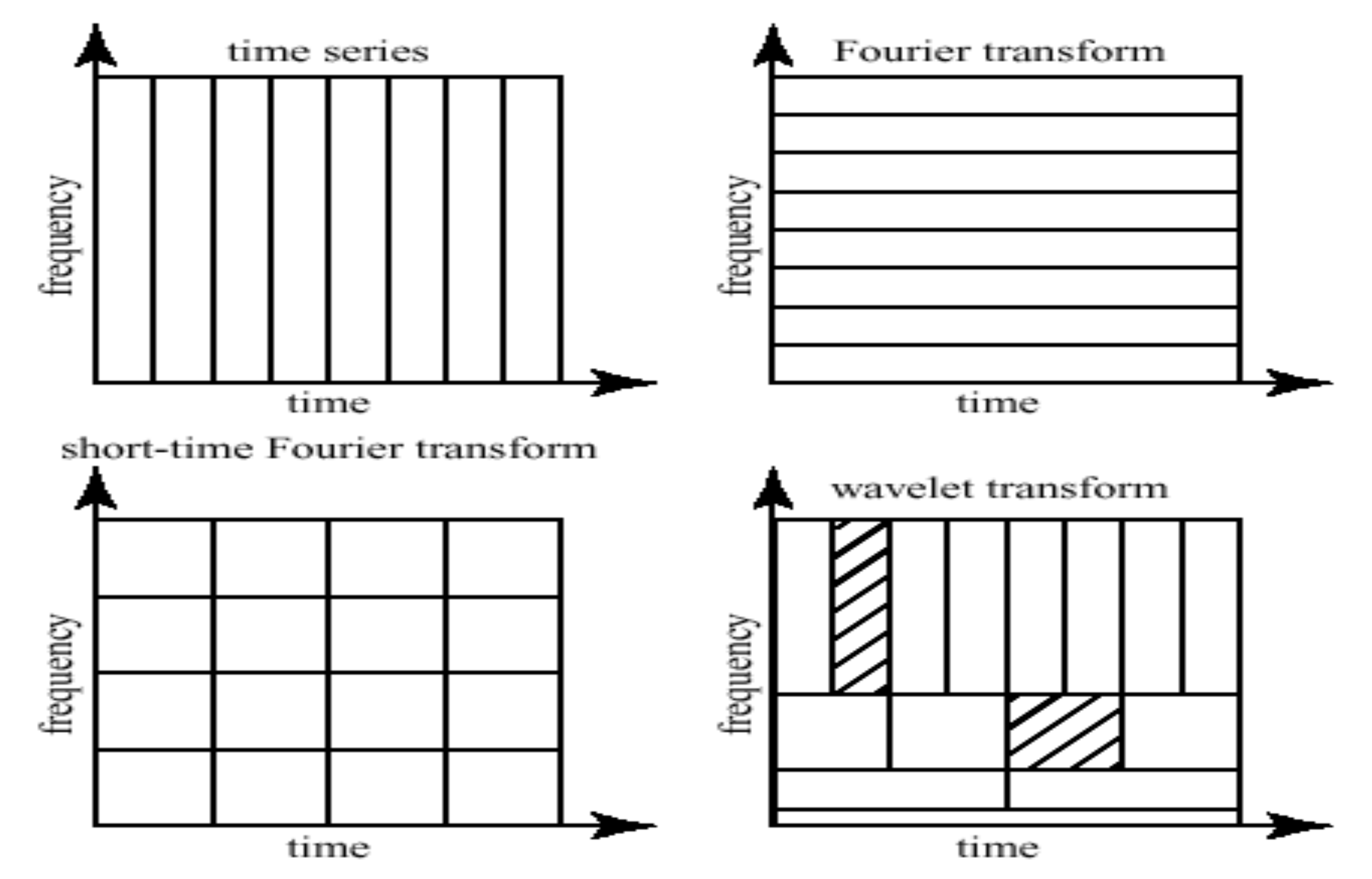

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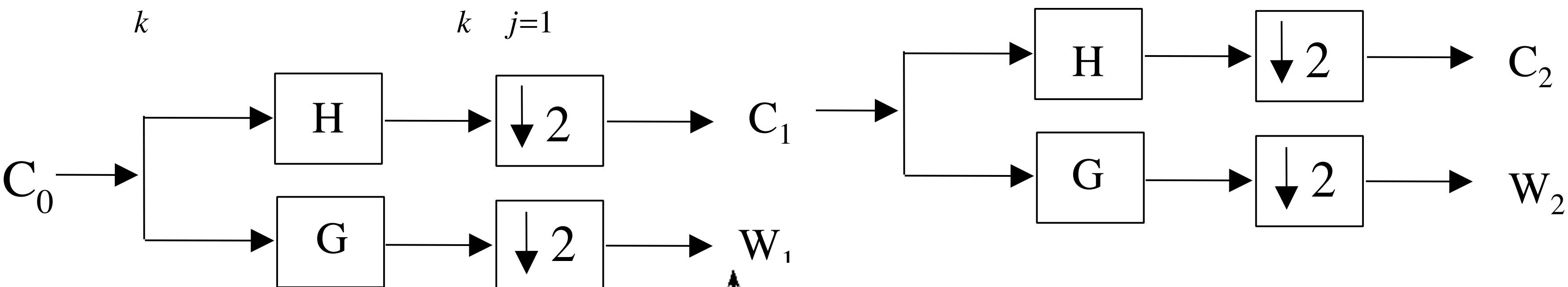
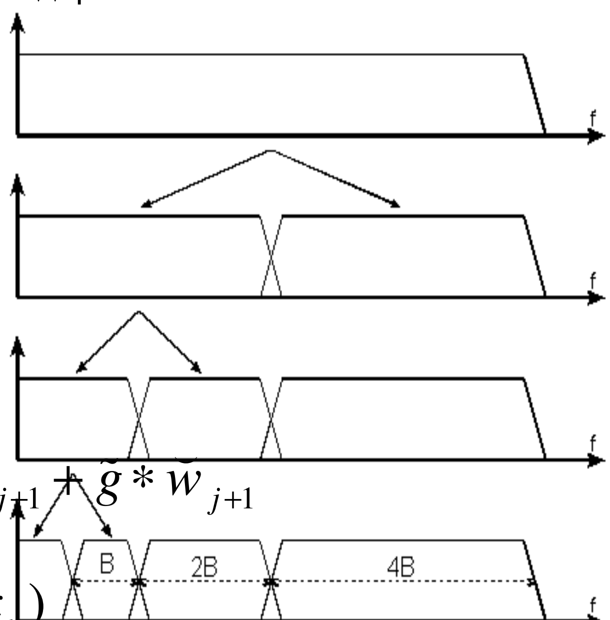
The Classical Orthogonal Wavelet Transform in 1D (OWT)

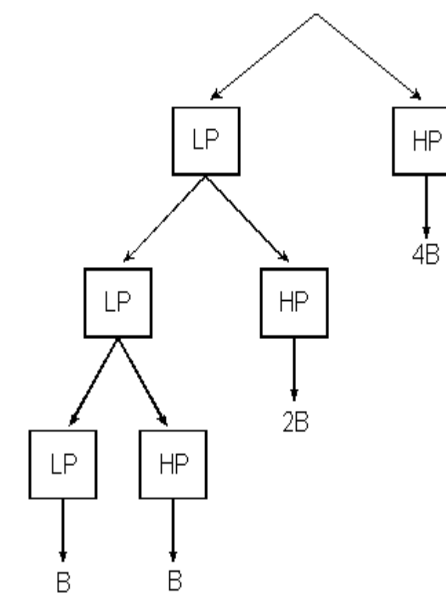
$$s_l = \sum_k c_{J,k}\phi_{J,l}(k) + \sum_k \sum_{j=1}^{J} \psi_{j,l}(k) w_{j,k}$$



$$c_{j+1,l} = \sum_h h_{k-2l} c_{j,k} = (\bar{h} * c_j)_{2l}$$

$$w_{j+1,l} = \sum_h g_{k-2l} c_{j,k} = (\bar{g} * c_j)_{2l}$$



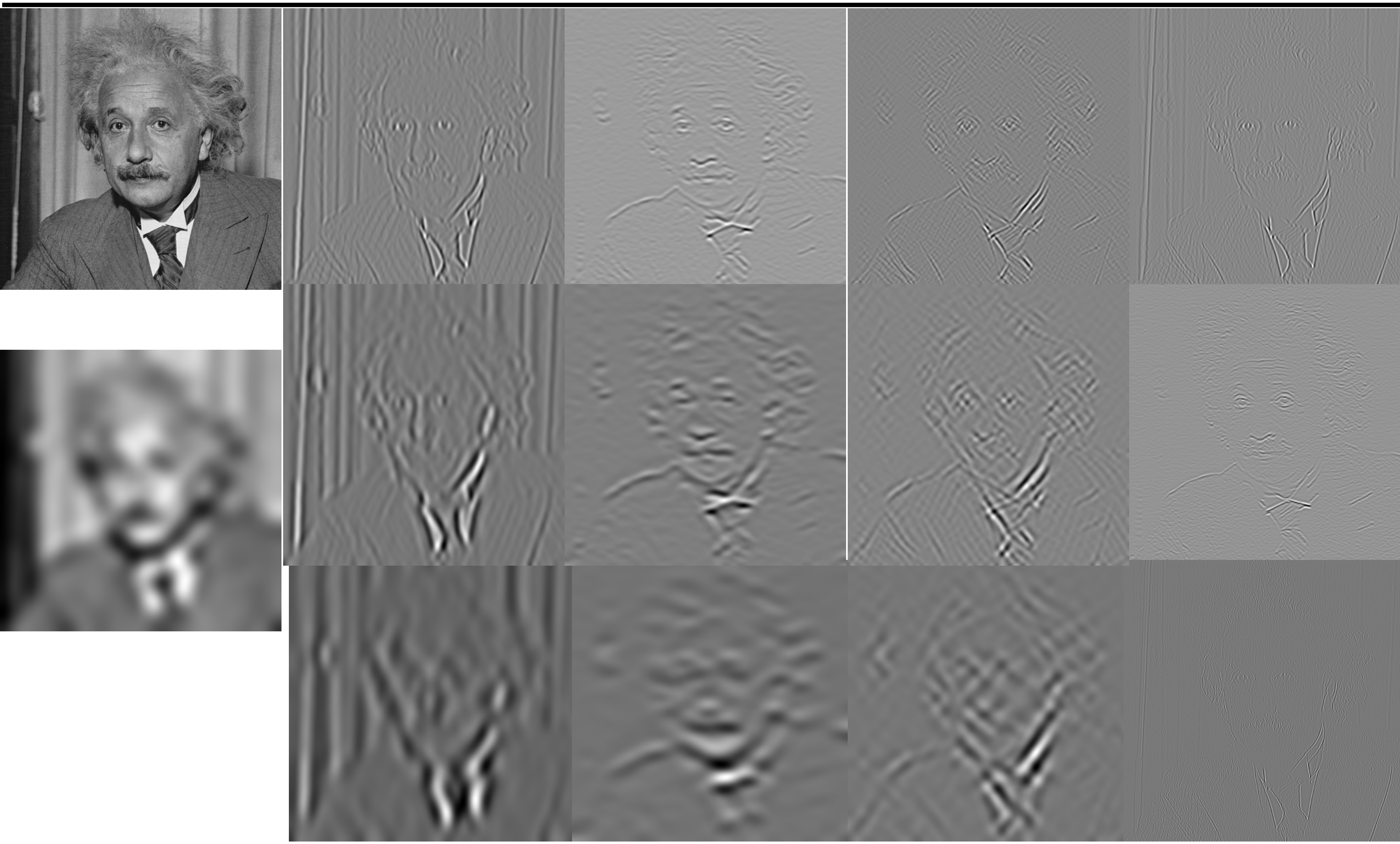


Reconstruction:

$$c_{j,l} = \sum_k \tilde{h}_{k+2l} c_{j+1,k} + \tilde{g}_{k+2l} w_{j+1,k} = \tilde{h} * \breve{c}_{j+1} + \tilde{g} * \breve{w}_{j+1}$$

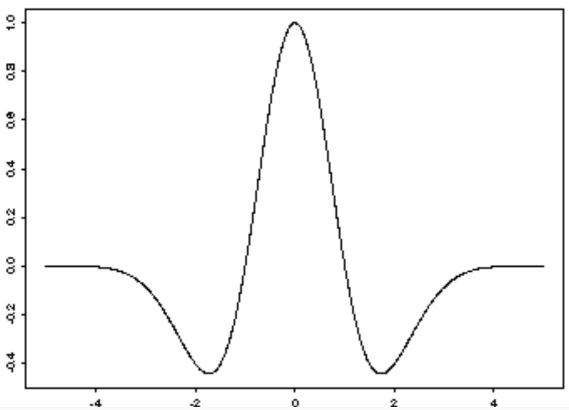
$$\breve{x} = (x_1, 0, x_2, 0, x_3, \ldots, 0, x_j, 0, \ldots, x_{n-1}, 0, x_n)$$

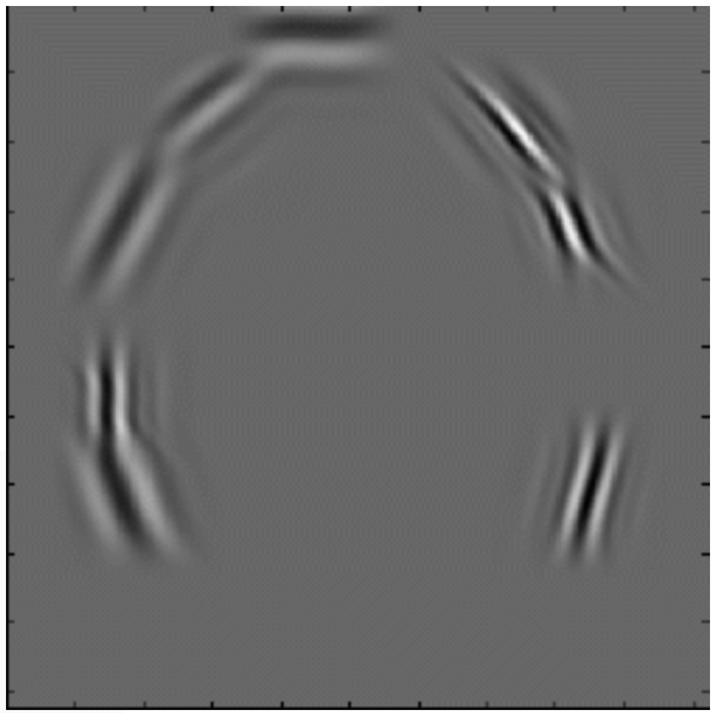
Undecimated Isotropic Wavelet Transform Has the Sum Of Its Components as its Inverse

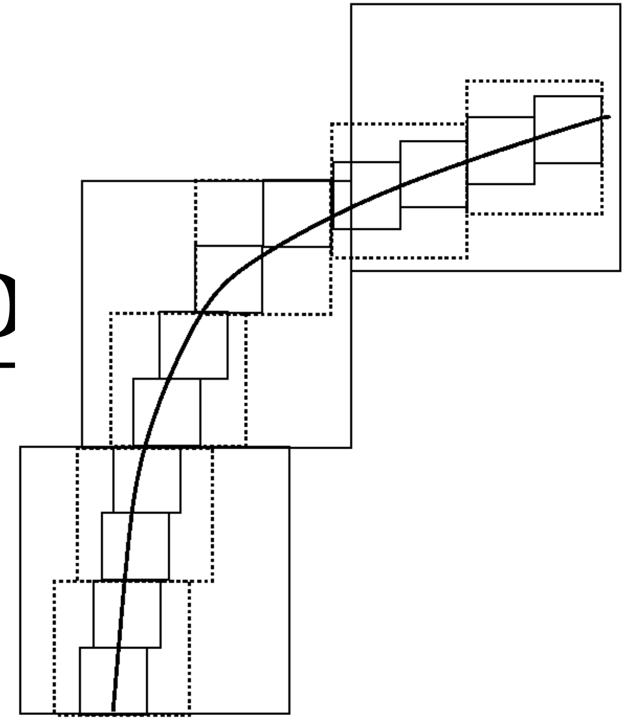


Curvelets Obey Parabolic Scaling (length ~ width^2): Sparsely Represent Edges in 2D

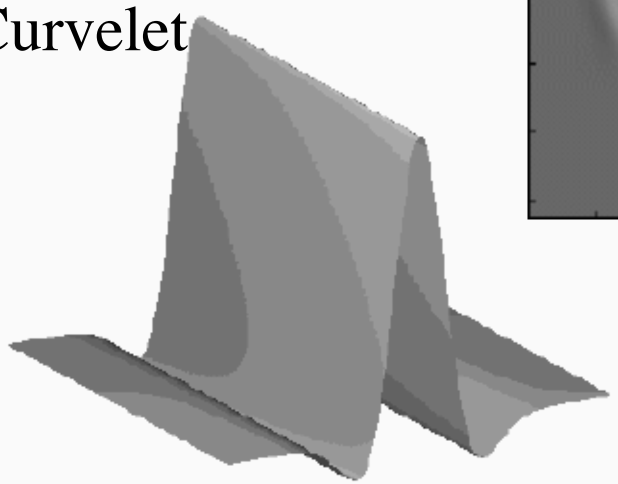
Wavelet

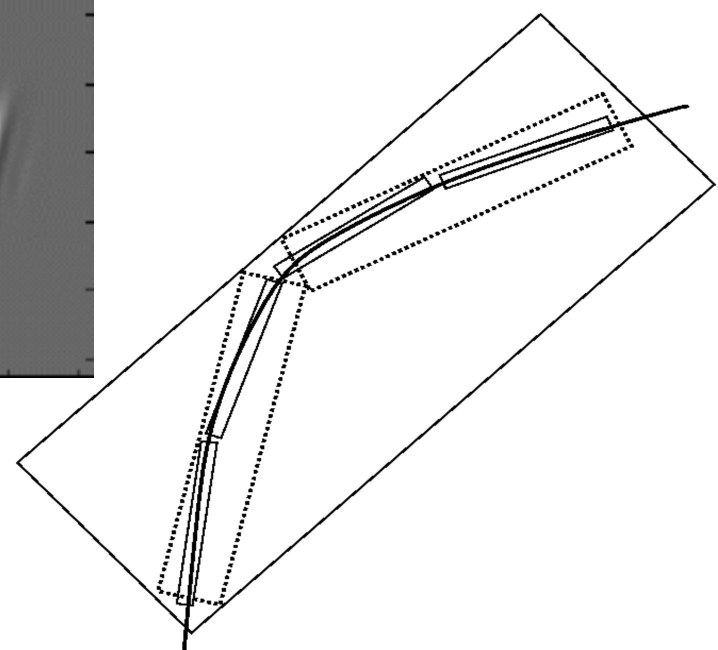




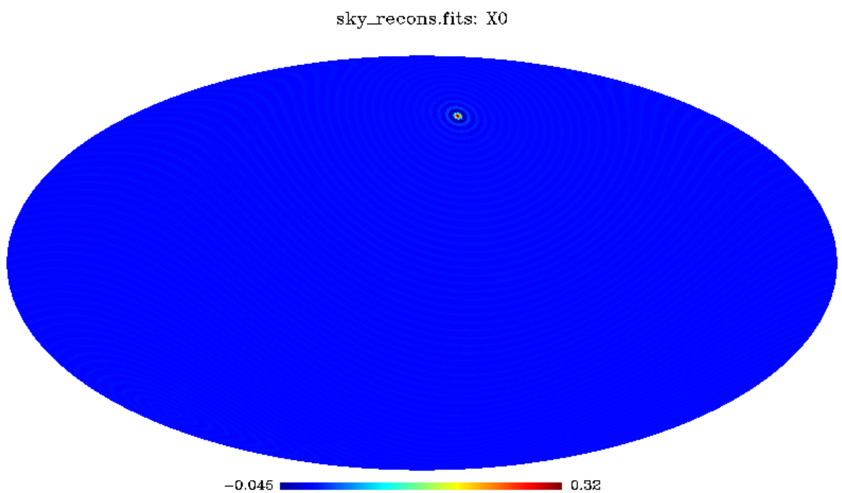


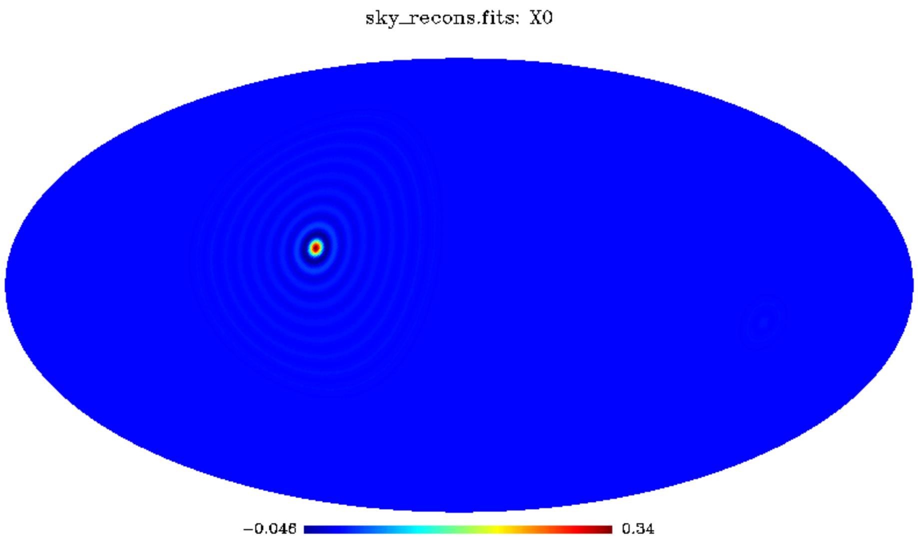
Curvelet

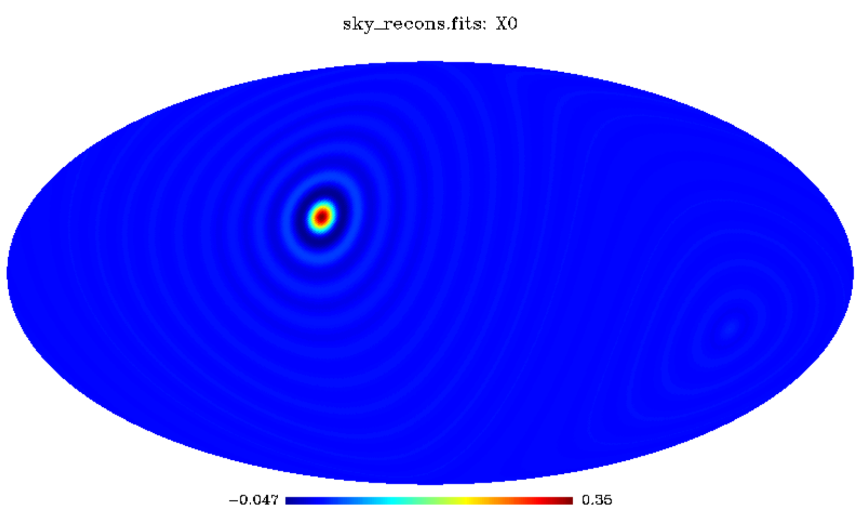




The Isotropic Undecimated Wavelet Transform on the Sphere







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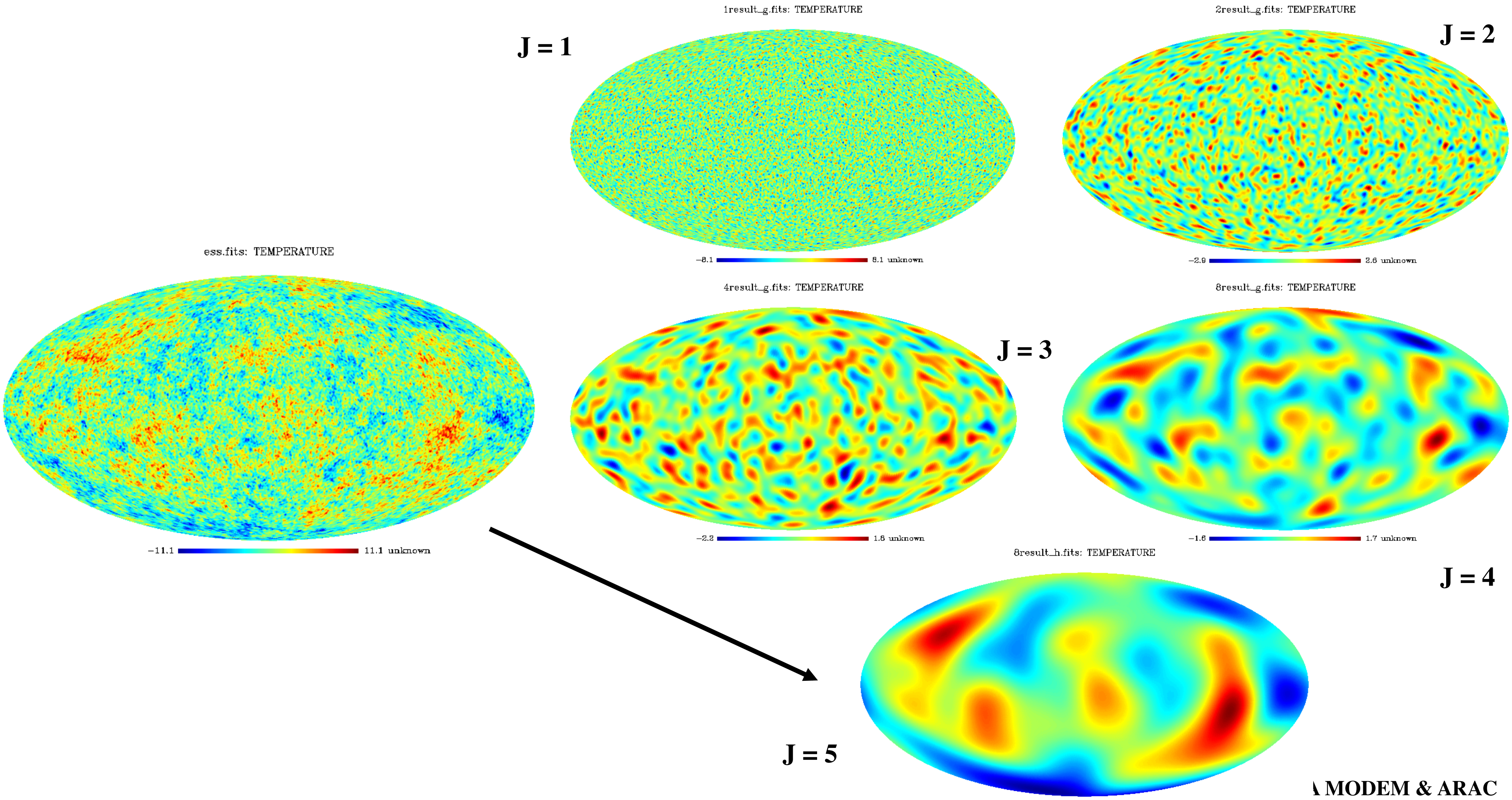
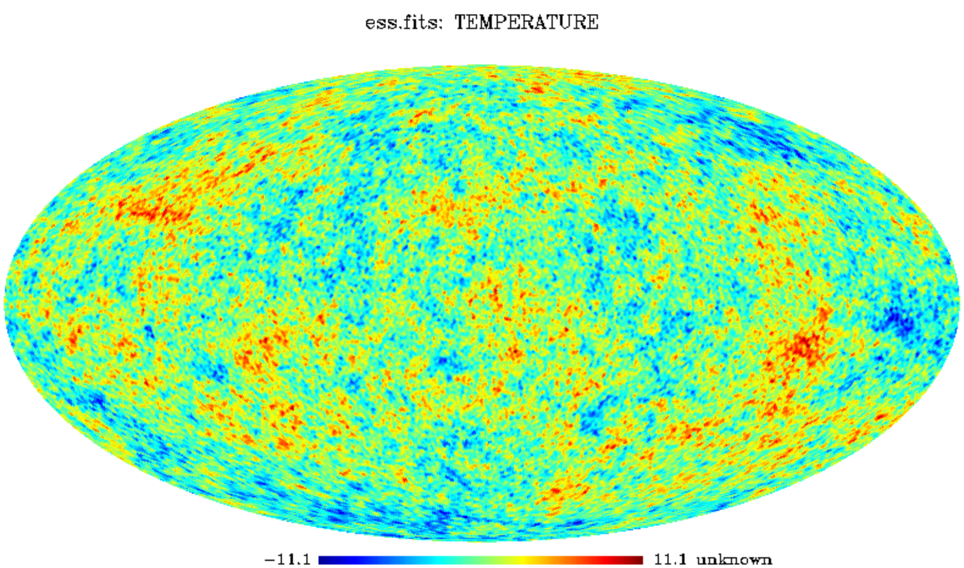
The Isotropic Undecimated Wavelet Transform of CMB



The Isotropic *Pyramidal* Wavelet Transform on the Sphere



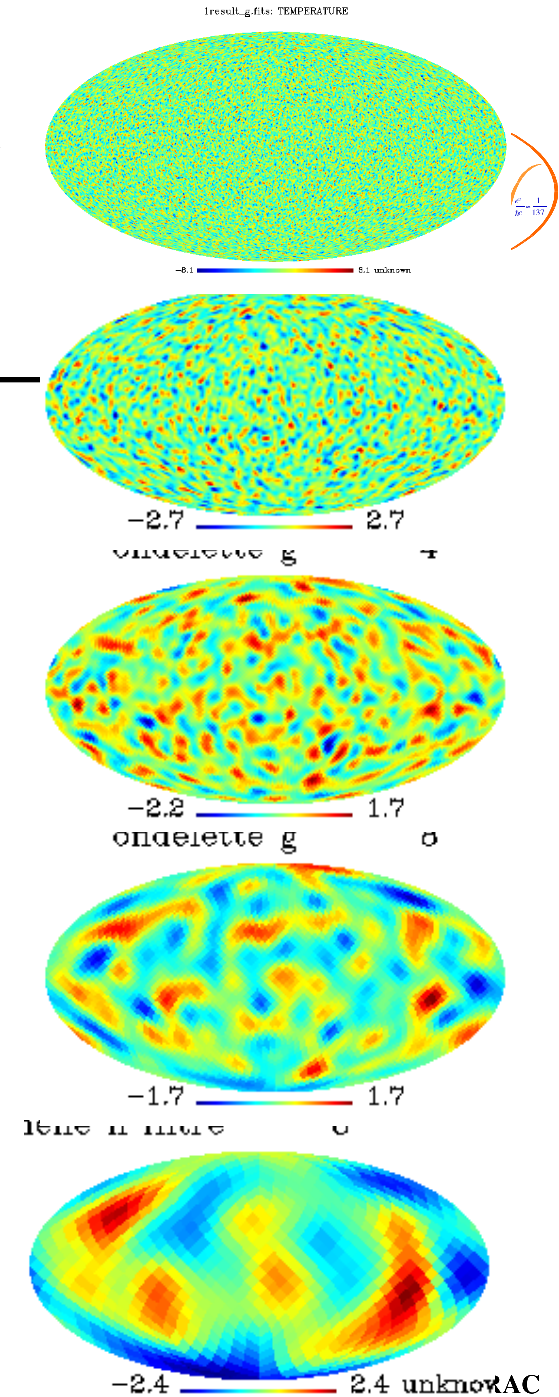
j=1

j=2

j=3

j=4

j=5



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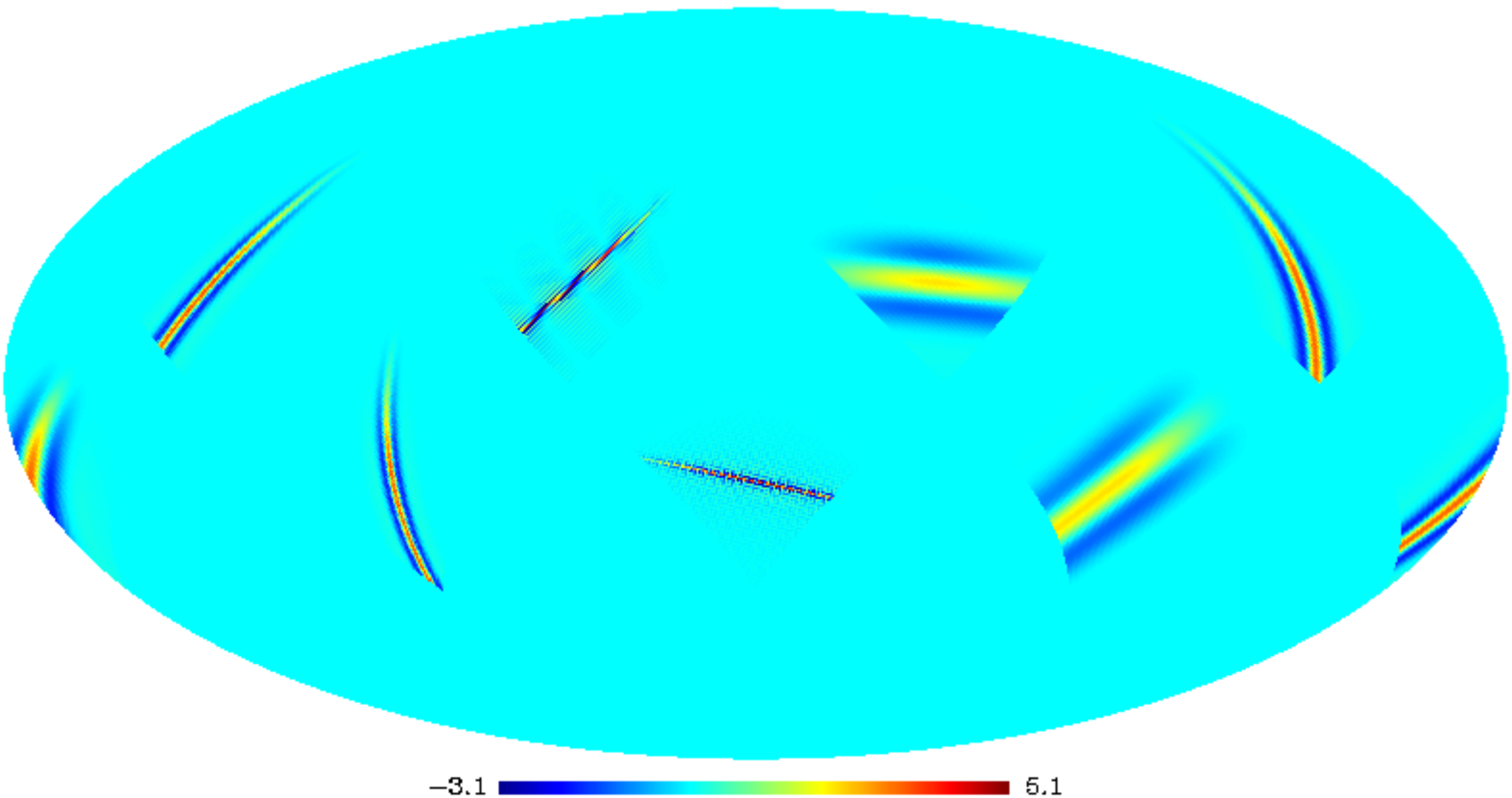
Ridgelets on the Sphere

SSR−Ridgelet Transform on the Sphere



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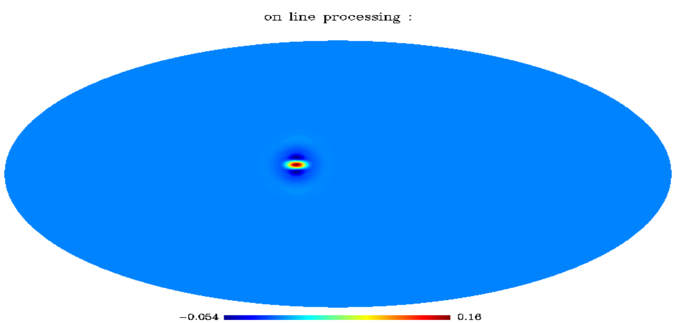
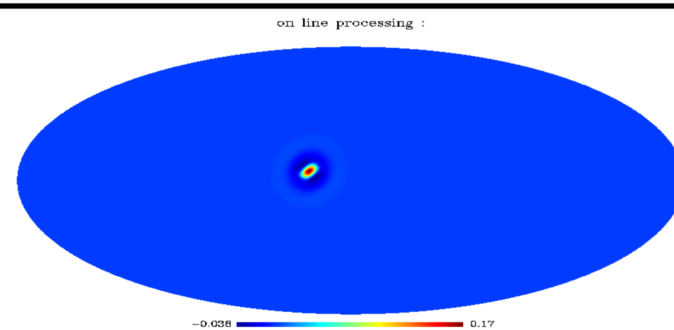
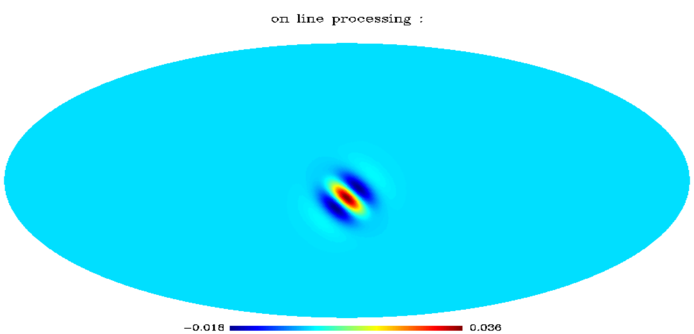
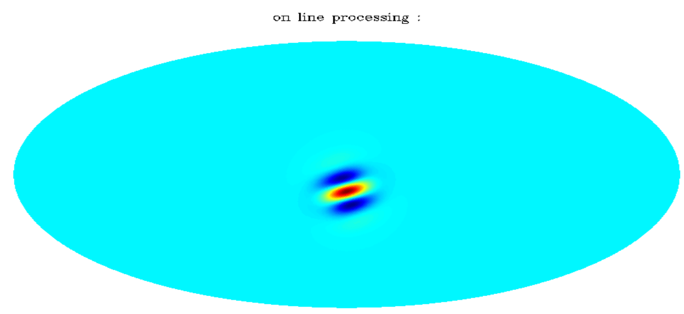
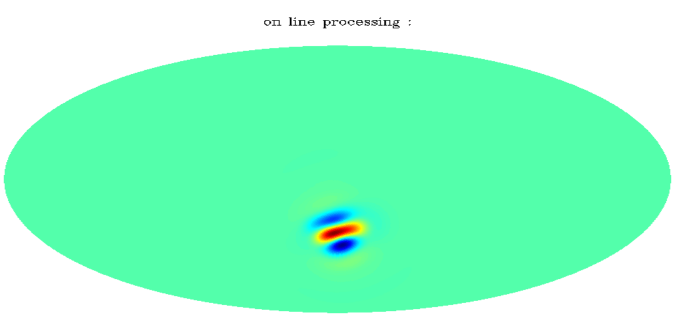
Curvelets on the Sphere

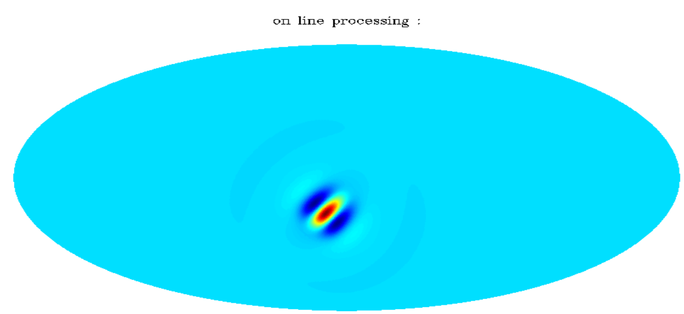












What Does an ICF Spherical Target Surface Look Like (Stitched Together from AFM Traces)?

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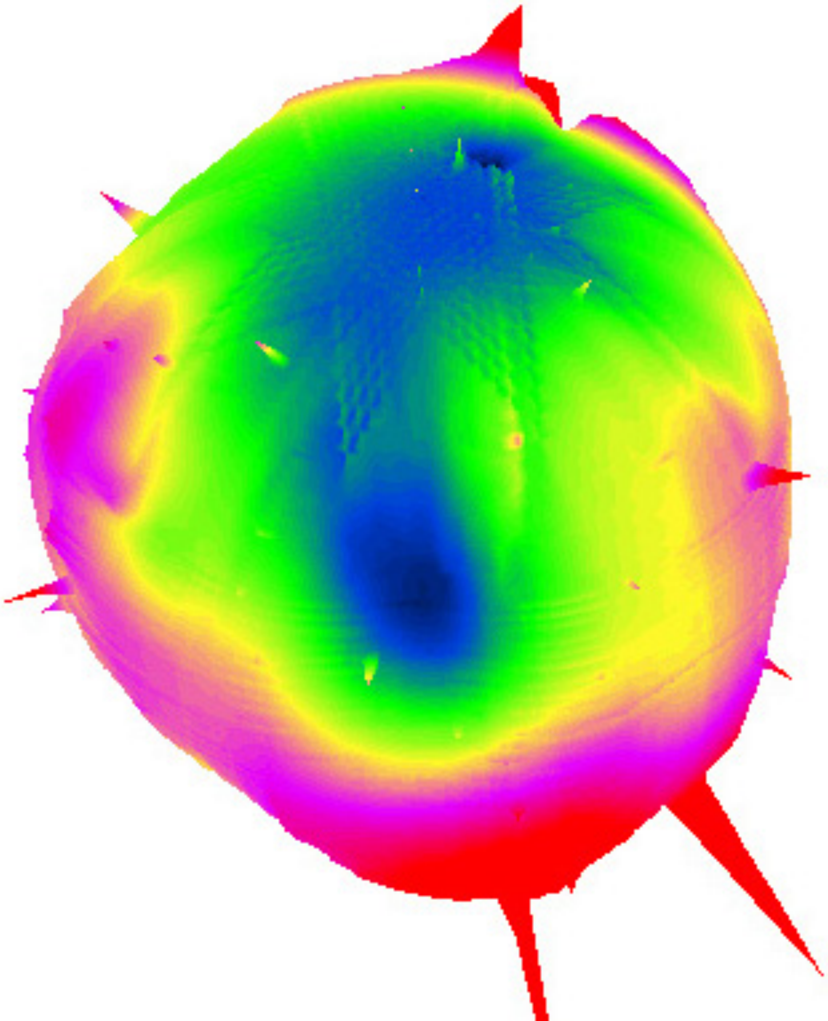


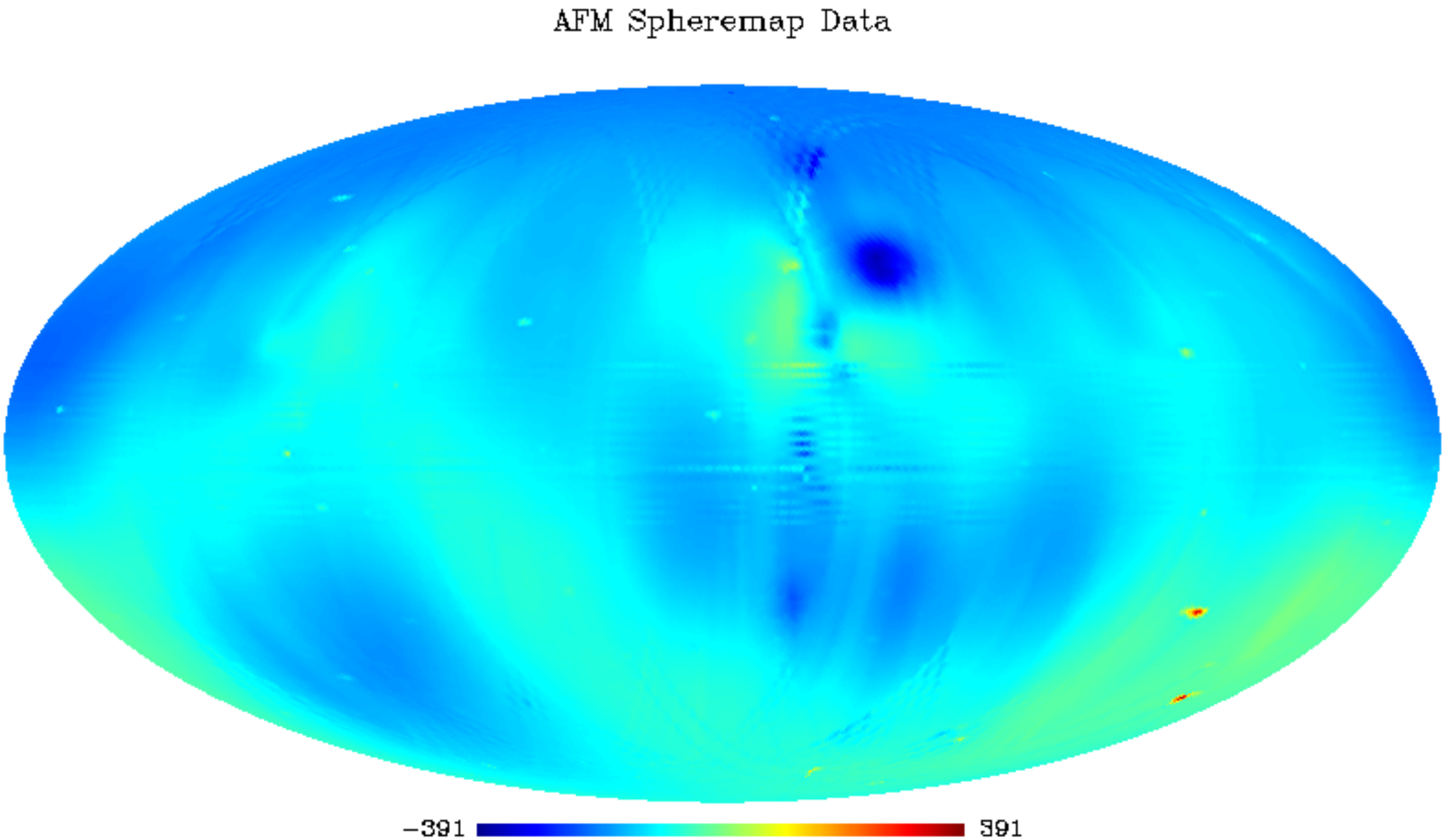
• 300 x exaggerated fine scale spikes, just for visualization clarity. 990 μm Direct Drive ICF target

• Ultimate goal, identify what kind of imperfections, flaws, manufacturing errors with what kind of statistical properties, will likely cause a failure mode in the implosion dynamics.

• Can't inspect entire surface of every target for a functioning laser fusion facility where The rep. rate is 5-10 Hz!

• Can't inspect entire target when it is encased in a radiation (hohlraum) cage even with little slits or holes.

AFM Full Spheremap Data in Mollweiede Format



MODEM in Action Local Gaussians Bumps + Lines + Noise Are Successfully Separated

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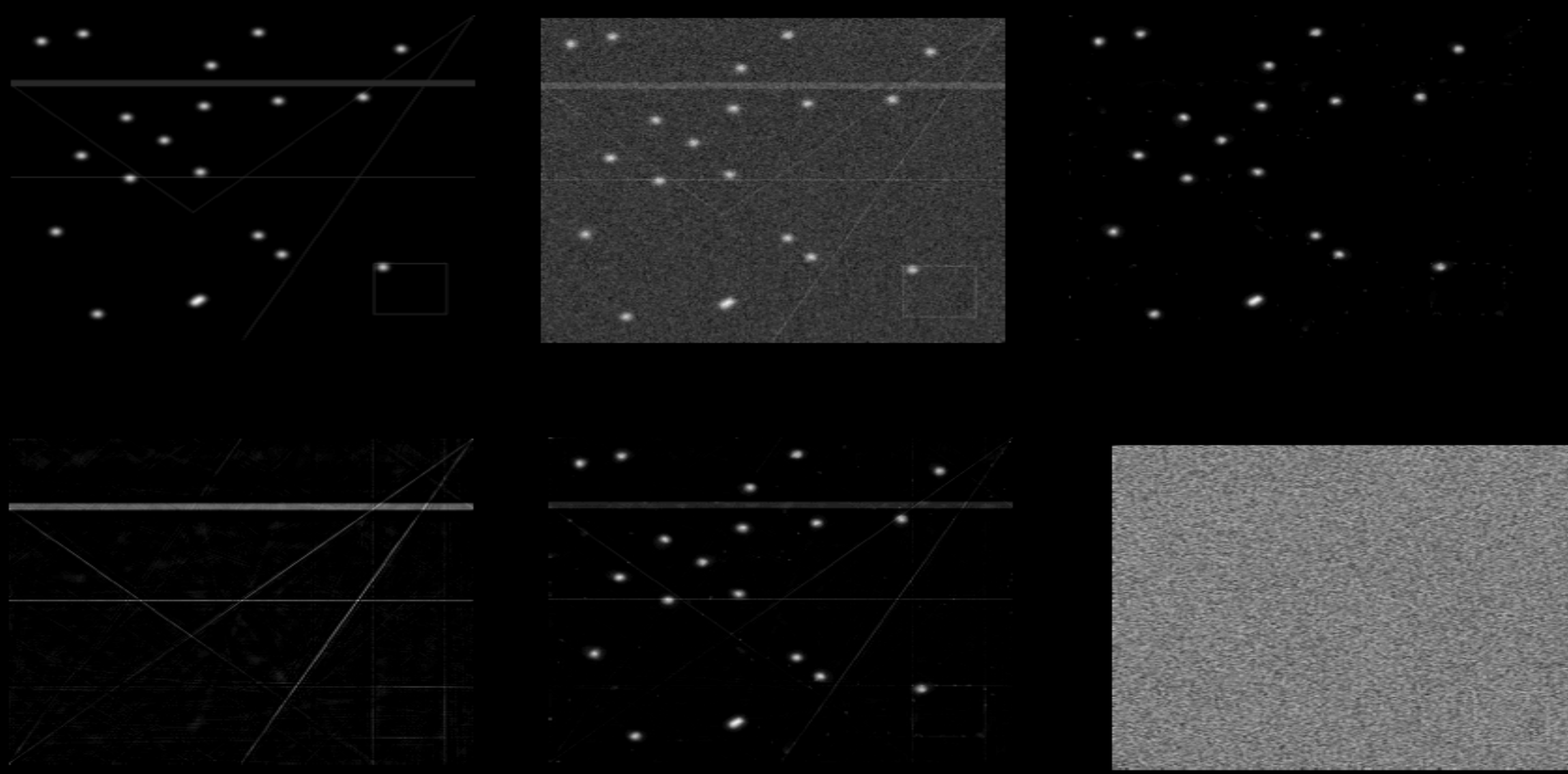
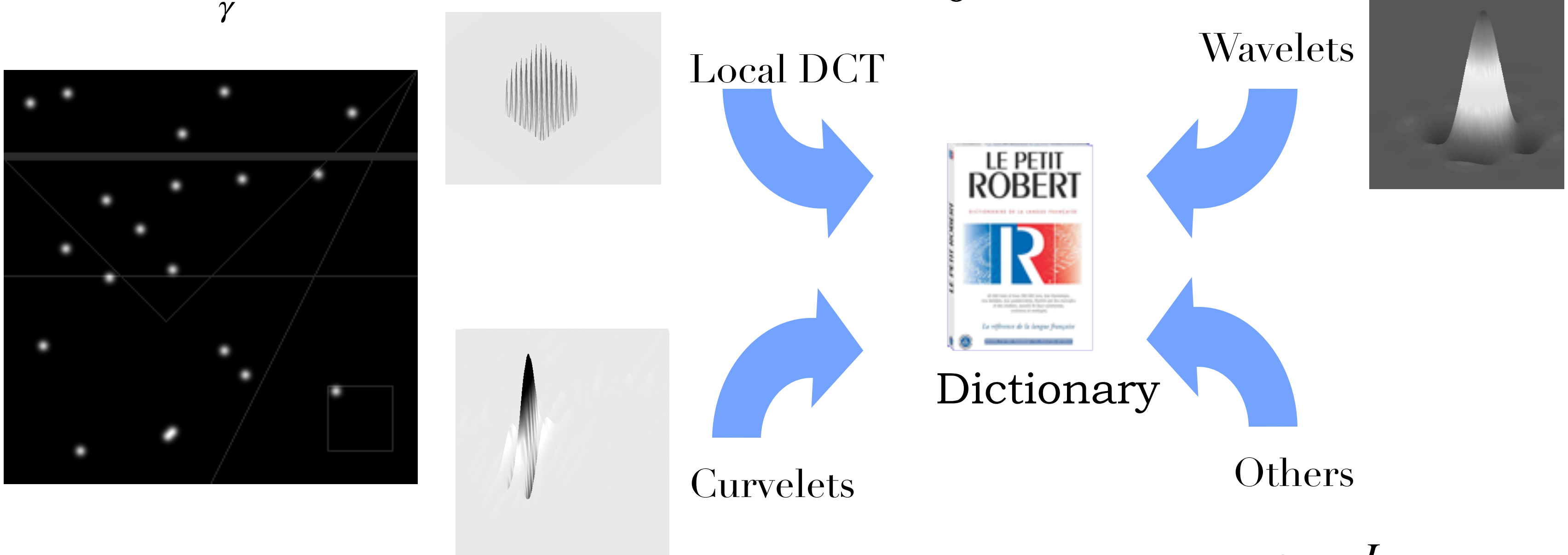


The Idea Behind Morphological Diversity Extraction (MODEM)

A dictionary D is defined as a collection of waveforms $(\phi_\gamma)_{\gamma \in \Gamma}$. The goal is to obtain a representation of a signal s with a linear combination of a small number of basis functions so that:

$$s = \sum_{\gamma} \alpha_\gamma \phi_\gamma \quad \text{==> Minimizes} \quad \|\alpha\|_0 \quad \text{subject to} \quad s = \phi\alpha$$



$$\phi = [\phi_1, \ldots, \phi_L], \quad \alpha = \{\alpha_1, \ldots, \alpha_L\}, \quad s = \phi\alpha = \sum_{k=1}^{L} \phi_k \alpha_k$$

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Alternate Theoretically Tractable Approaches: Morphological Component Analysis (MCA)

•***Redundant Multiscale Transforms and their Application for Morphological Component Analysis***, *Advances in Imaging and Electron Physics, 132, 2004.*
•***Image Decomposition Via the Combination of Sparse Representation and a Variational Approach***, *IEEE Trans. on Image Proces., 14, 10, pp 1570--1582, 2005*

$$MIN_{s_1,\ldots,s_L} \sum_{k=1}^{L} \left\| T_k s_k \right\|_p \quad \text{subject to} \quad \left\| s - \sum_{k=1}^{L} s_k \right\|_2^2 < \varepsilon$$

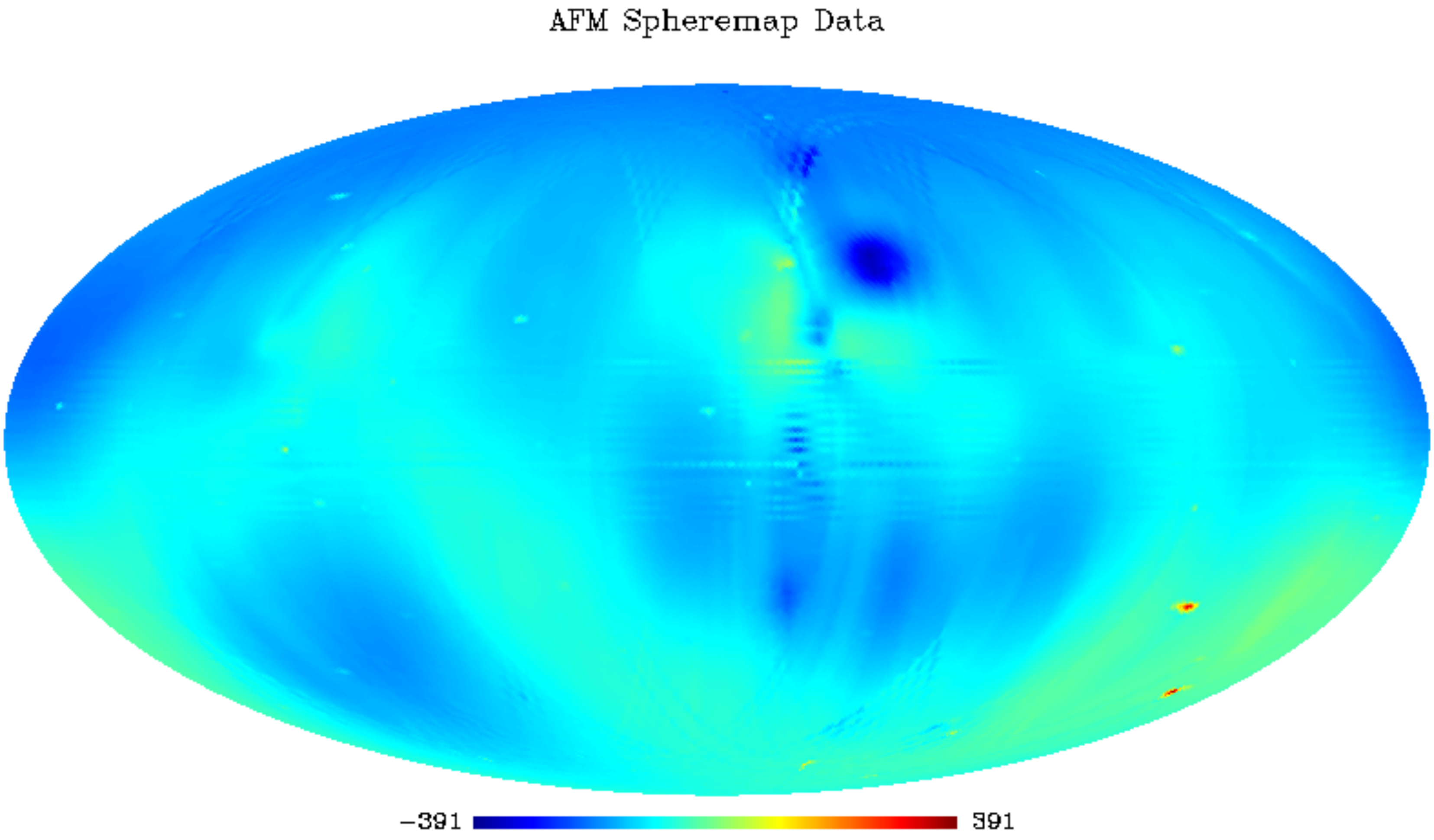
Interpolation of Missing Data

•***Simultaneous Cartoon and Texture Image Inpainting using Morphological Component Analysis (MCA),*** *ACHA, 2005.*
•***Inpainting and Zooming using Sparse Representations***, The Computer Journal , *in press.*

$$MIN_{s_1,\ldots,s_L} \sum_{k=1}^{L} \left\| T_k s_k \right\|_p \quad \text{subject to} \quad \left\| M\left(s - \sum_{k=1}^{L} s_k\right) \right\|_2^2 < \varepsilon$$

Where M is the mask: M(i,j) = 0 ==> missing data
M(i,j) = 1 ==> good data

AFM Full Spheremap Data in Mollweiede Format



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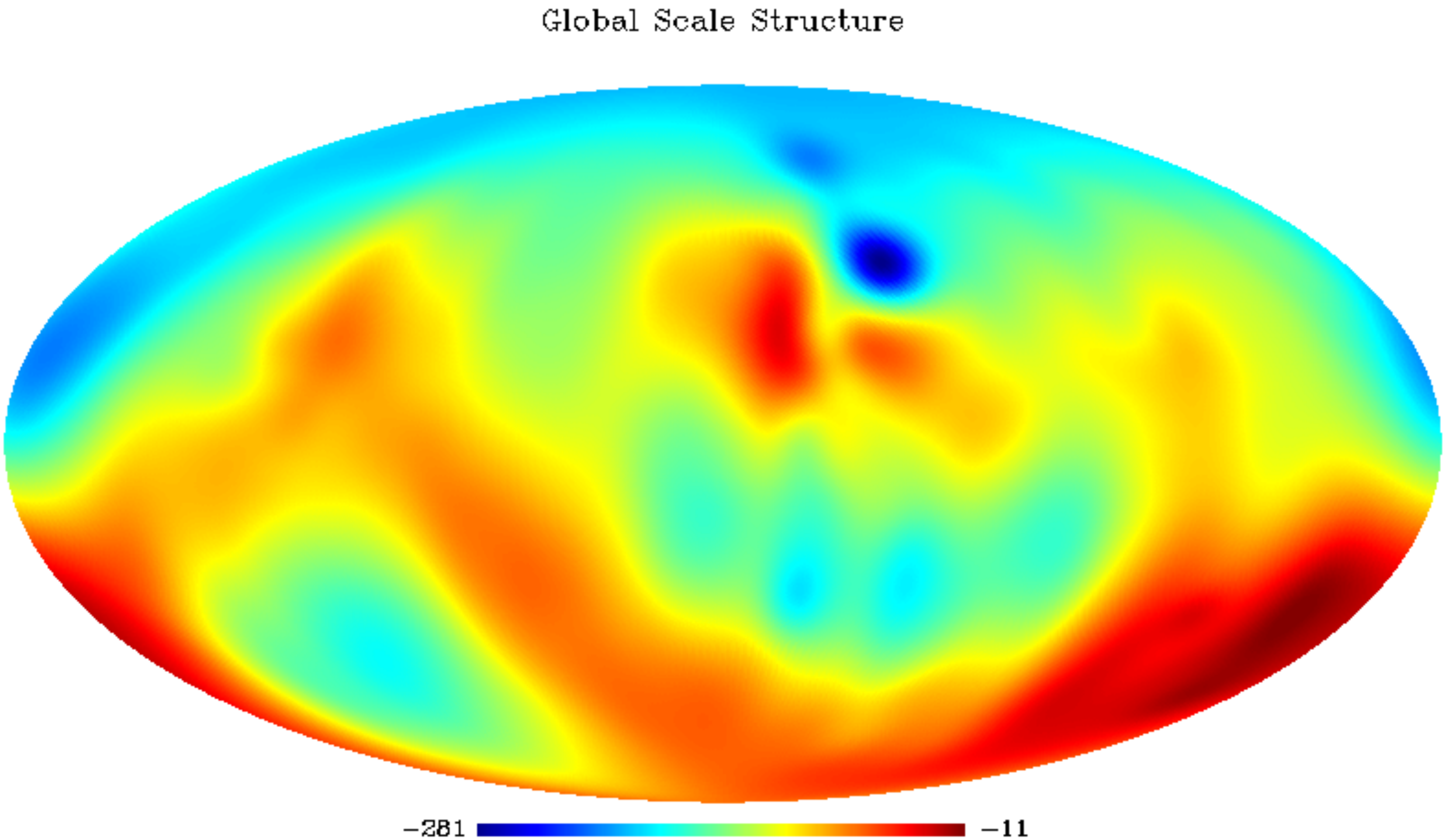
Separating the Global Features in the AFM Data Leads to This



Almost All the Artifacts Contained in the Stitched AFM Data Are Separated in This Image

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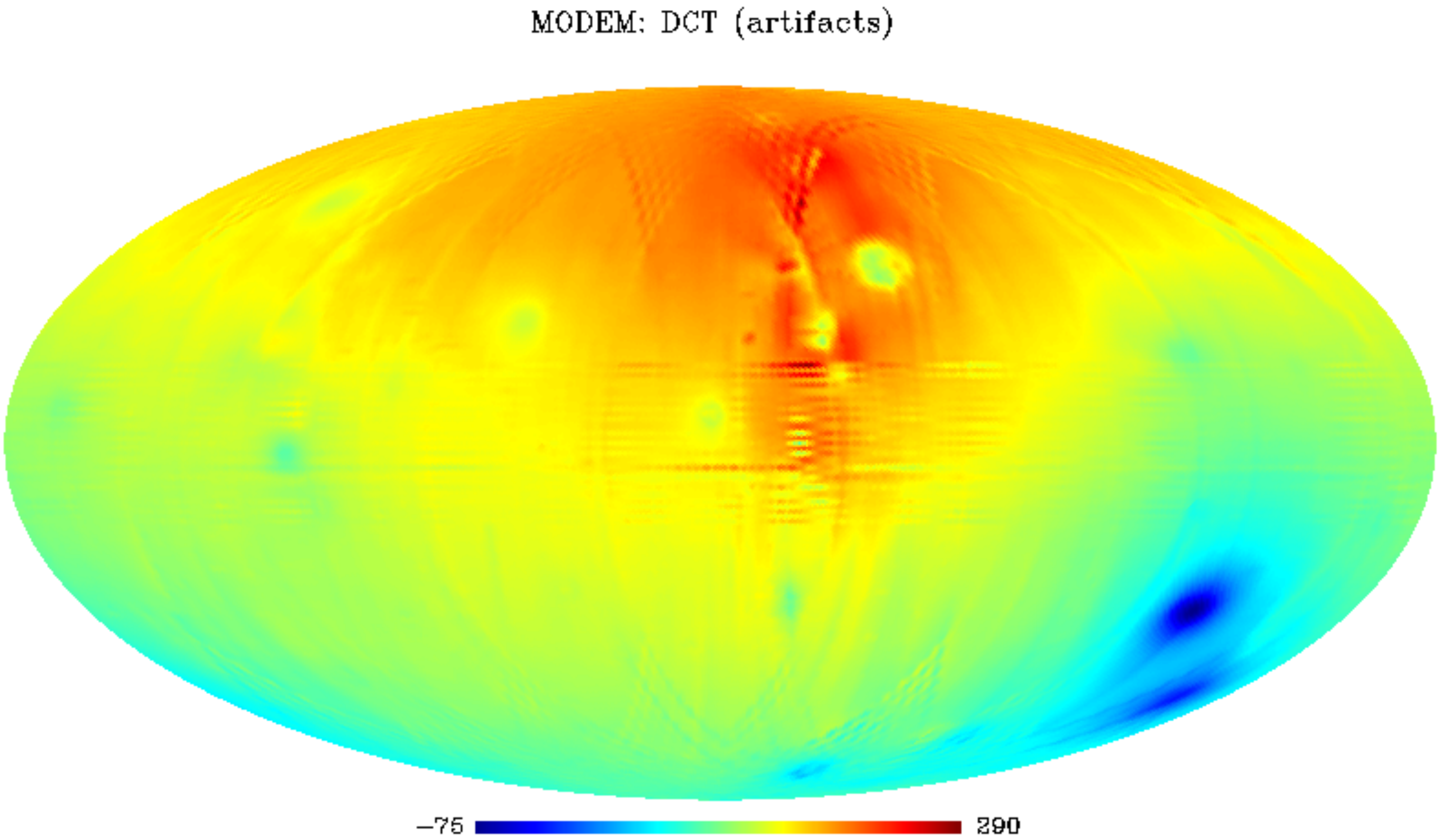
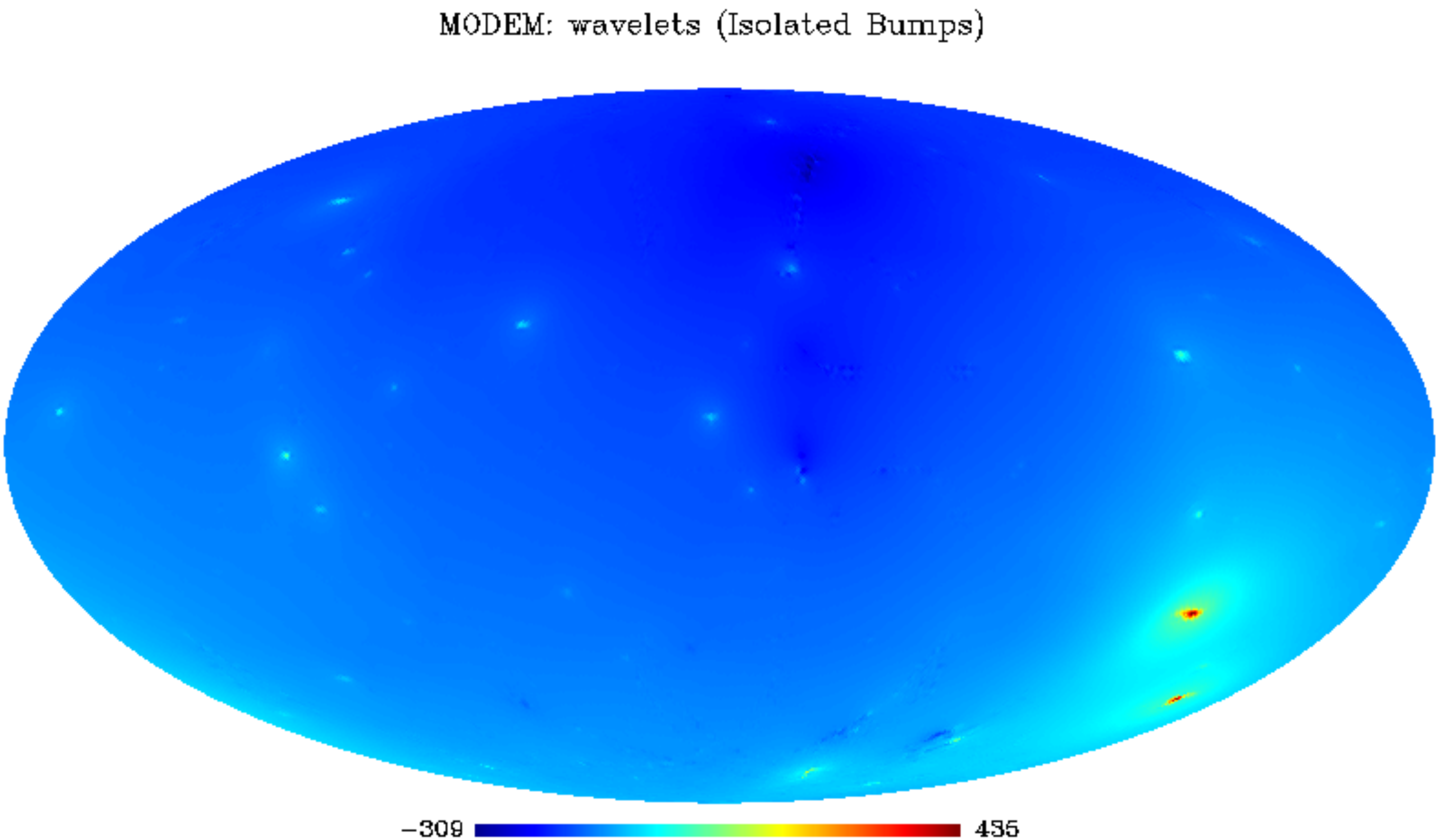
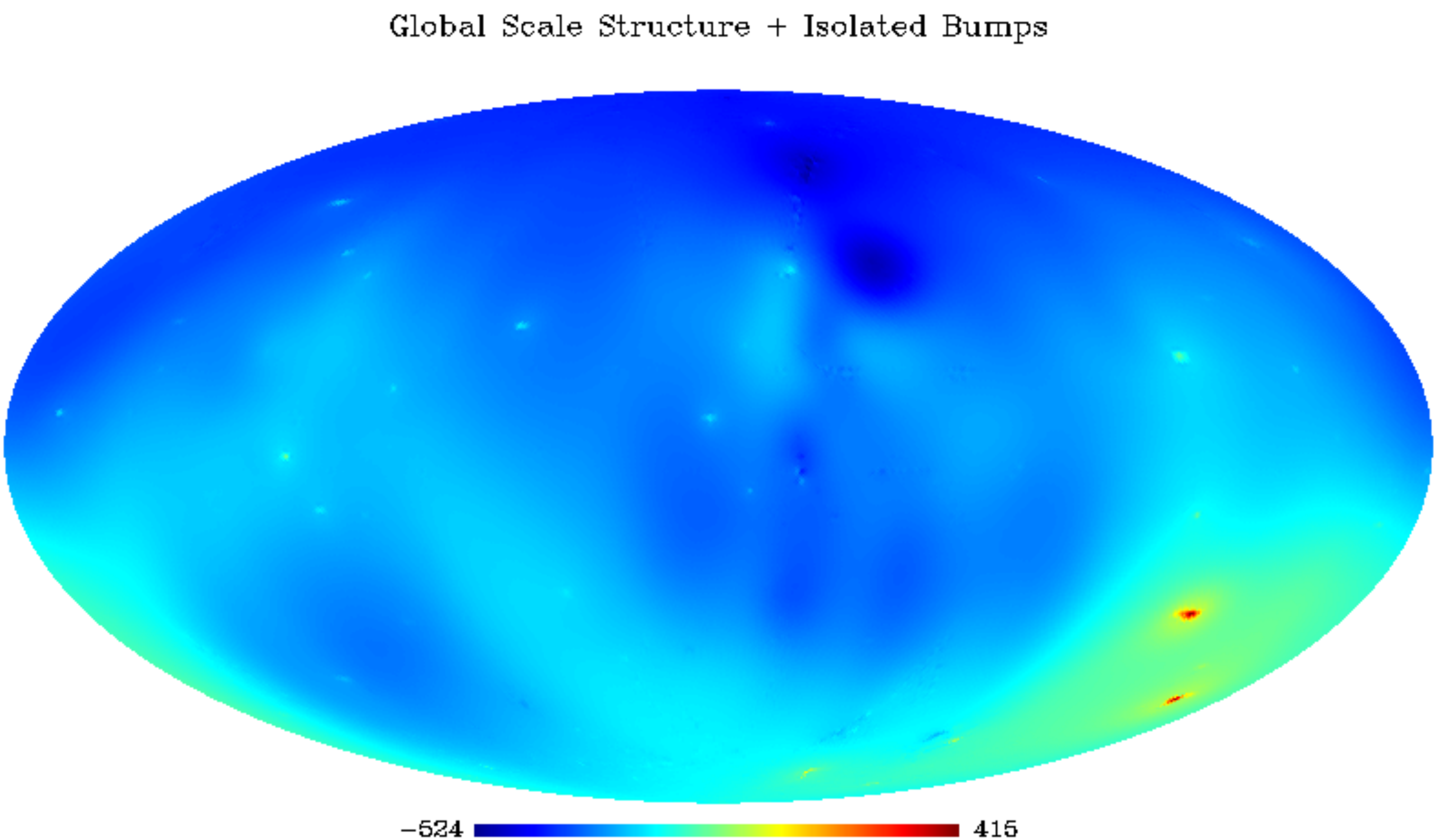


The Important Localized Structures Have Been Isolated Using MODEM on Spherical Surface AFM Data

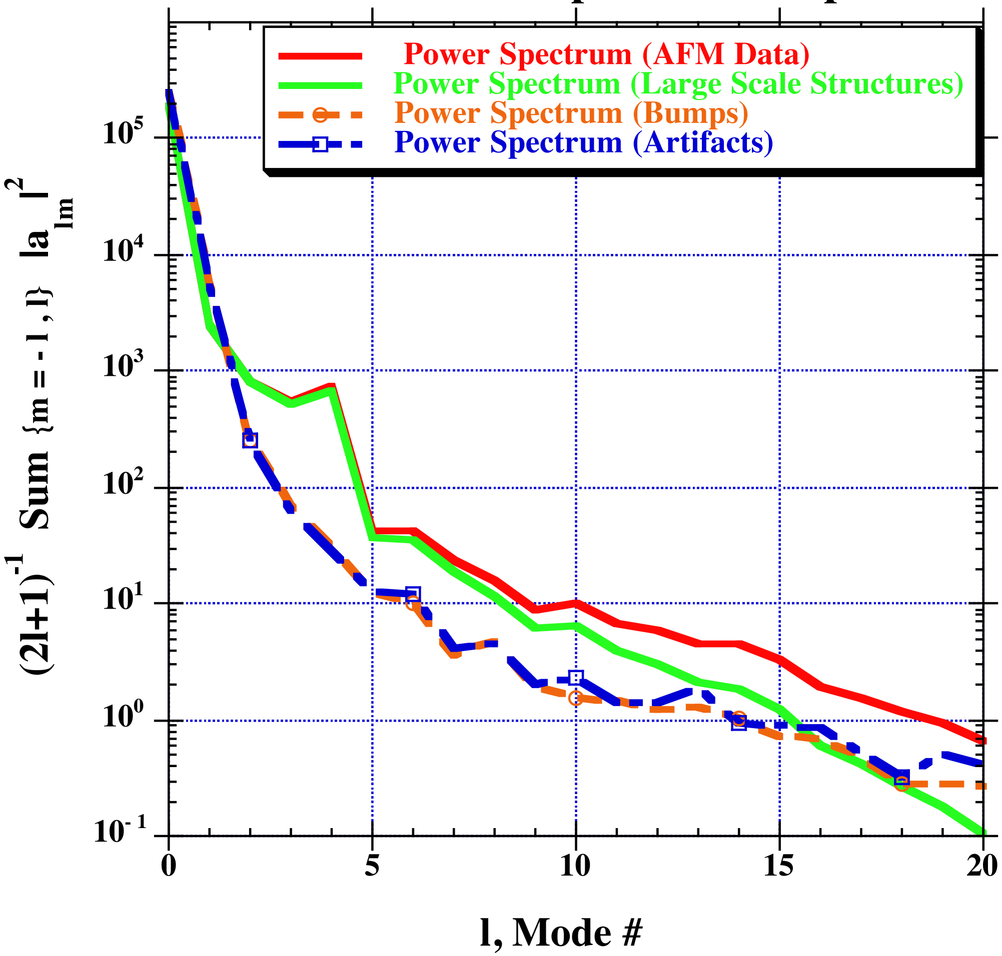


Global Features + Localized Bumps = Spheremap Data - Artifacts



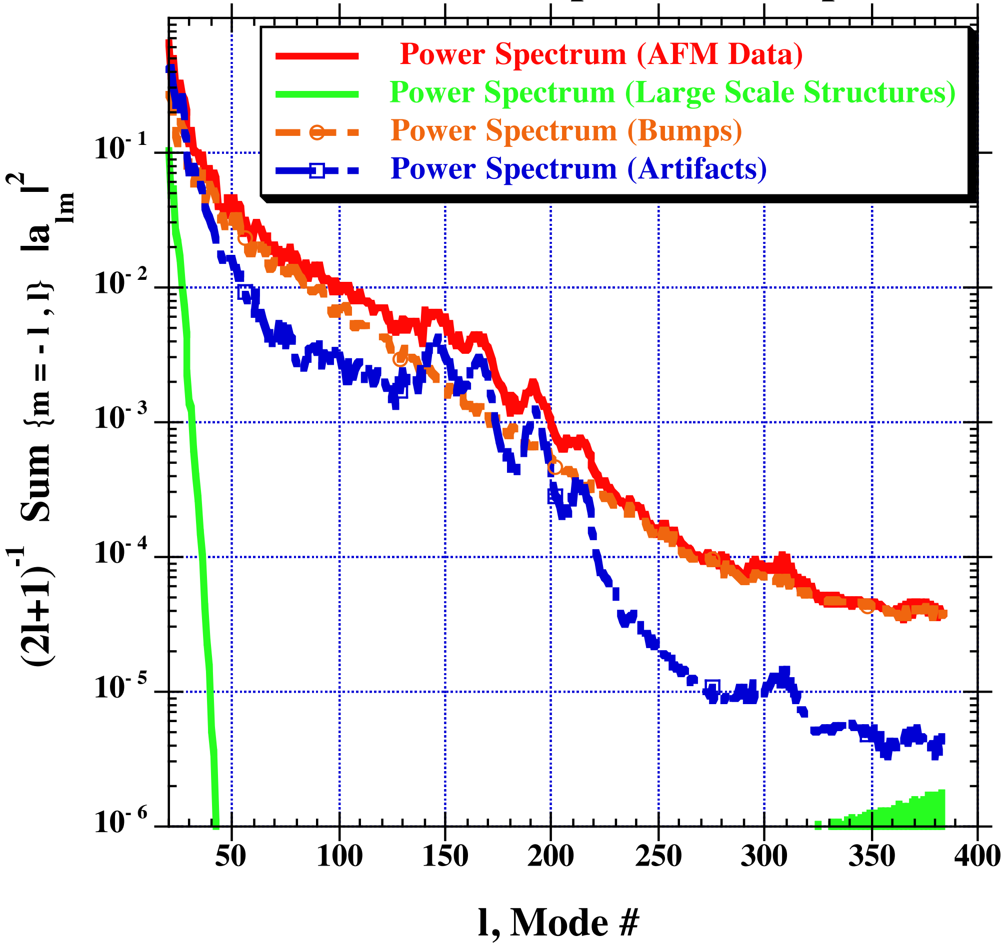
Let's Look at the Partition of the Feature Specific Spherical Harmonic Power Density: Low Modes First

Low Mode Power Spectrum of AFM Data and MODEM Feature Separated Components

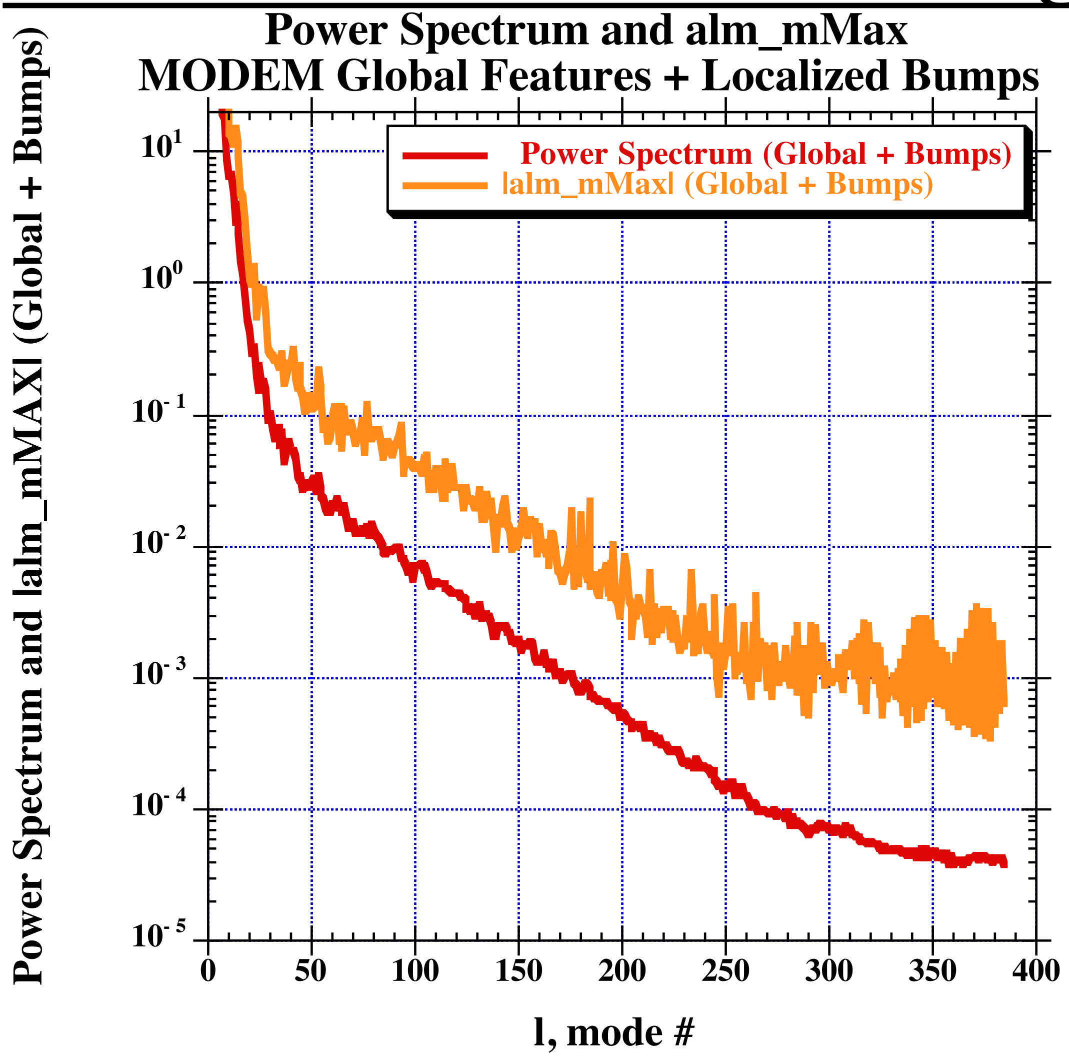


Let's Look at the Breakdown of the Feature Specific Spherical Harmonic Power Density: Higher Modes

High Mode Power Spectrum of AFM Data and MODEM Feature Separated Components



There is up to an Order of Magnitude Variation in the "m" Coefficients at a Given "l" For the High Modes



So Phase Matters!

Integrated Hydro Simulation Inferred (Haan) Instability Growth Factors at Maximum Velocity for Outer Surface Perturbations

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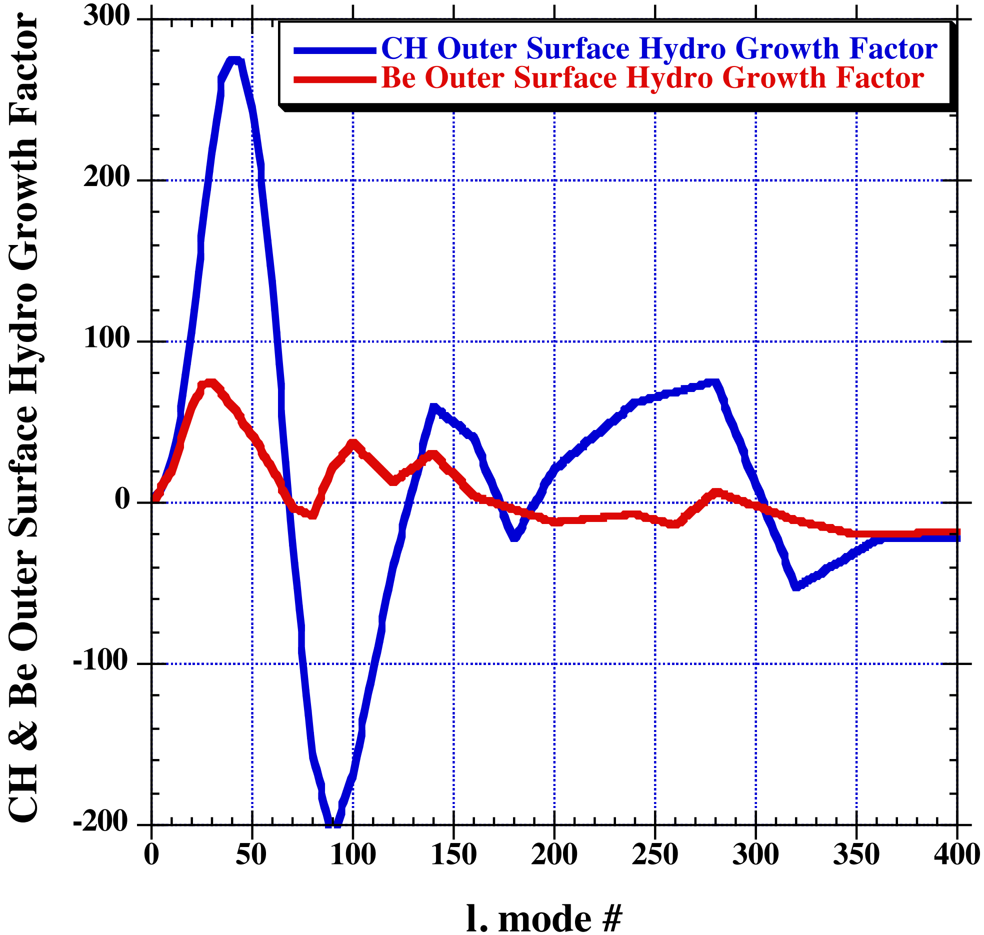
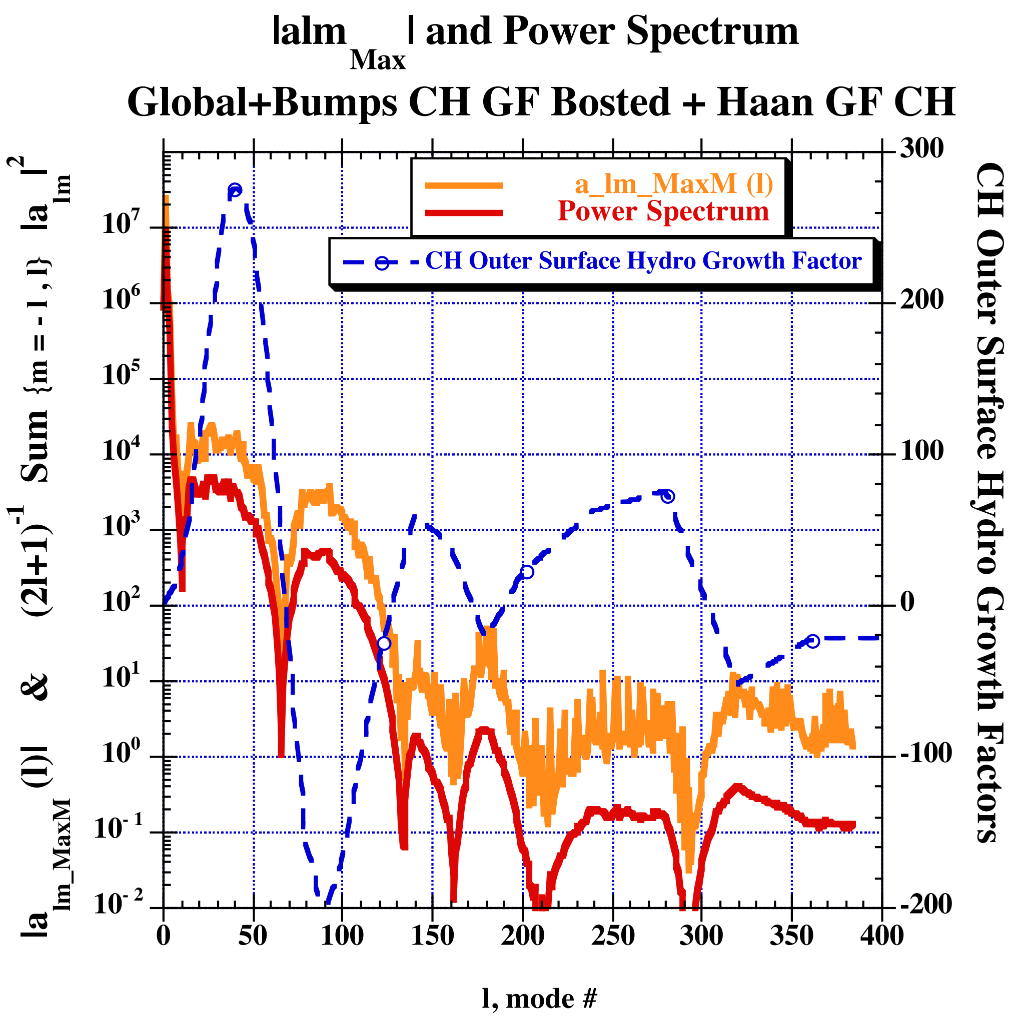
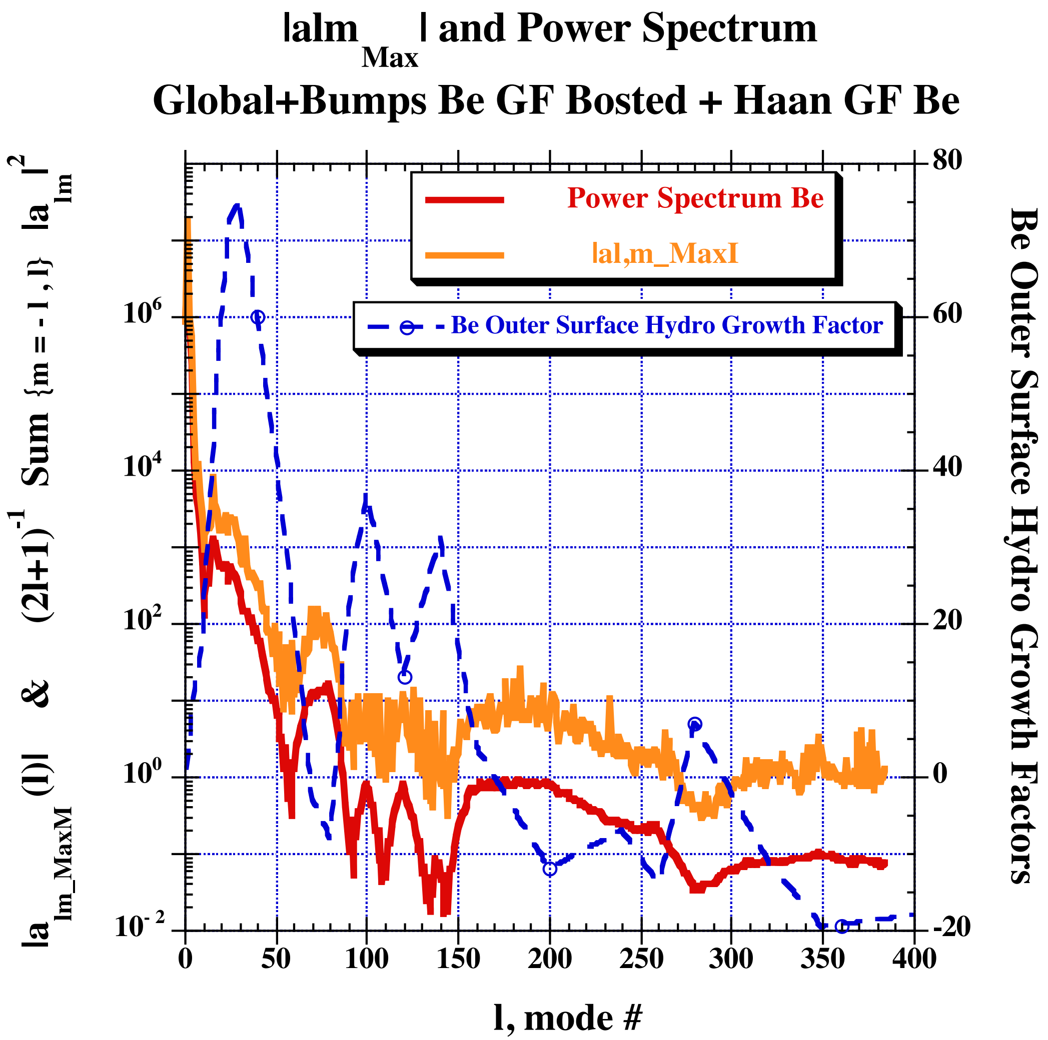
CH & Be Haan Integrated Hydro Growth Factors at Maximum Velocity



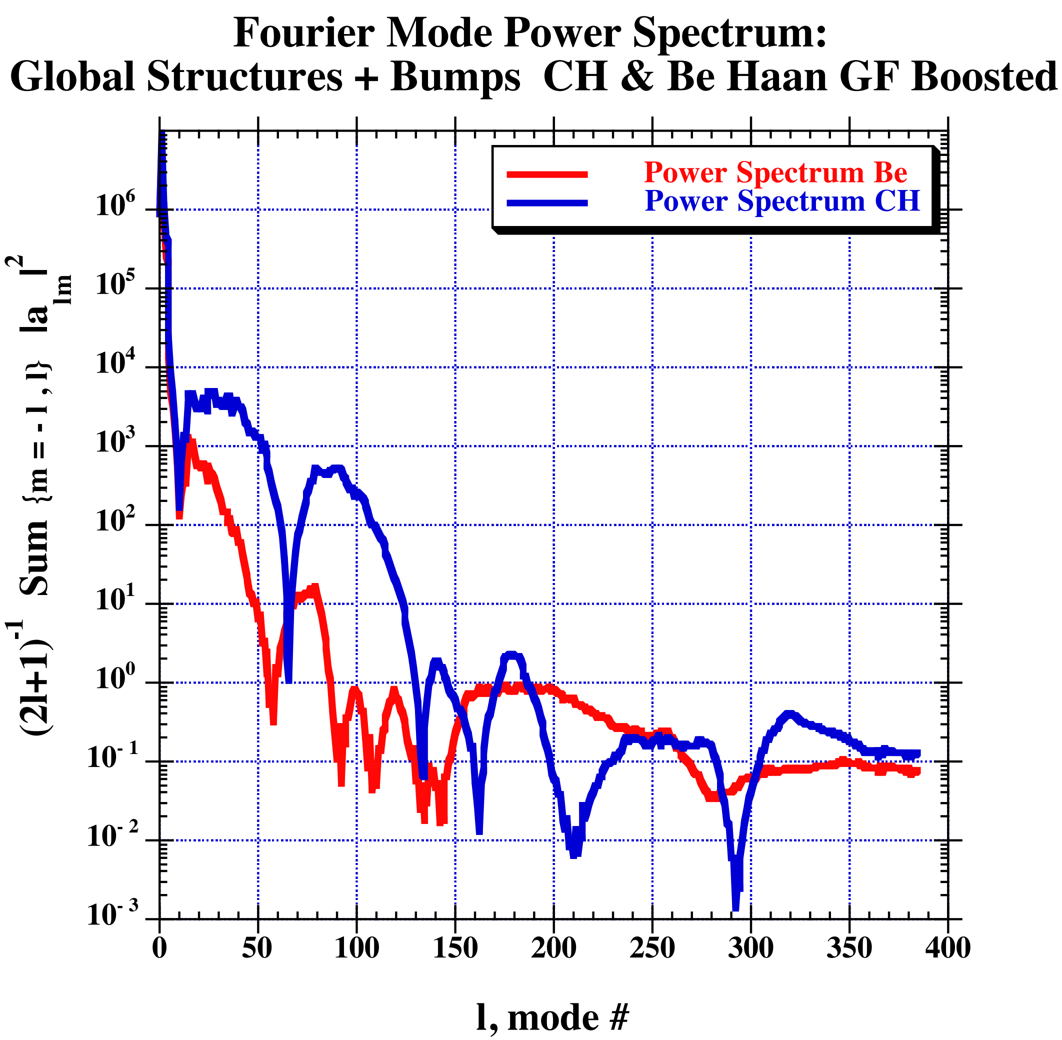
Final Power Spectra After Implosion, at Peak Velocity, for CH Outer Layers



Final Power Spectra After Implosion, at Peak Velocity, for Be Outer Layers



A Comparison of the Final Power Spectra for Be and CH Layers



"Diamond" Should Beat Both of These

Reconstruction of the Outer Surfaces After Implosion at Peak Velocity for a CH Layer

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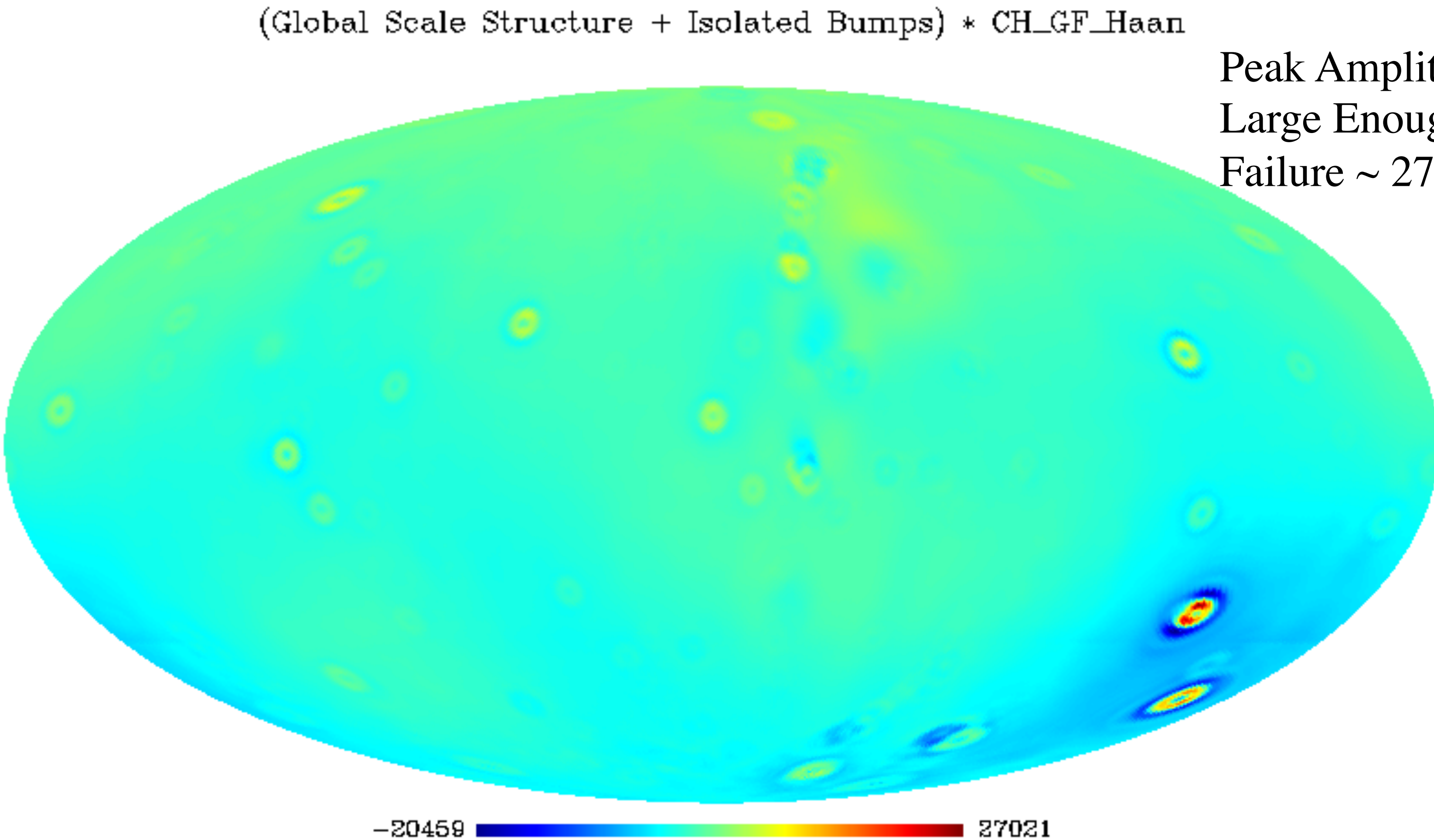
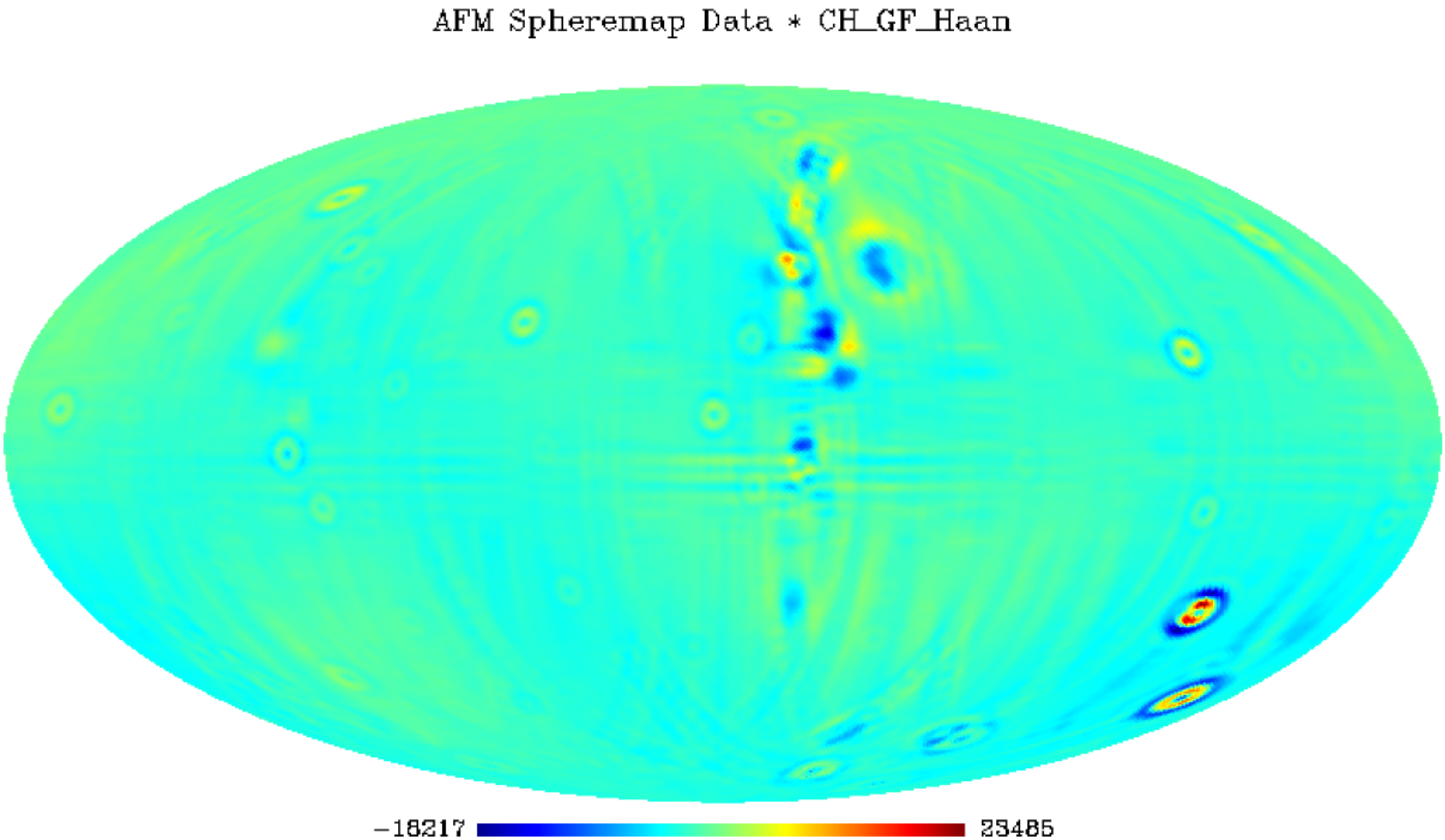
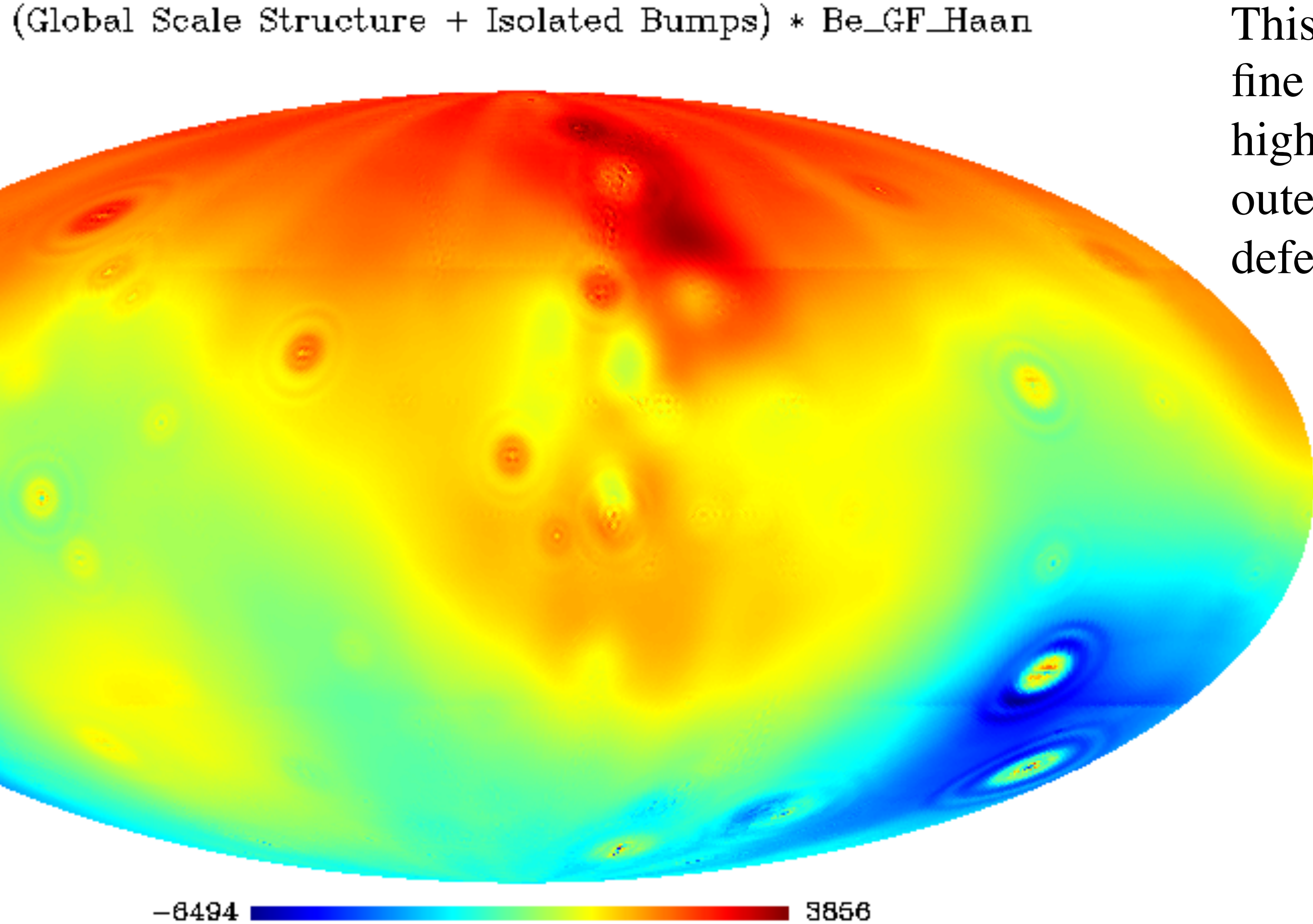


Peak Amplitudes Are Large Enough to Cause Failure ~ 27 μm

Reconstruction of the Outer Surfaces After Implosion at Peak Velocity for a CH Layer (Without MODEM Separation)

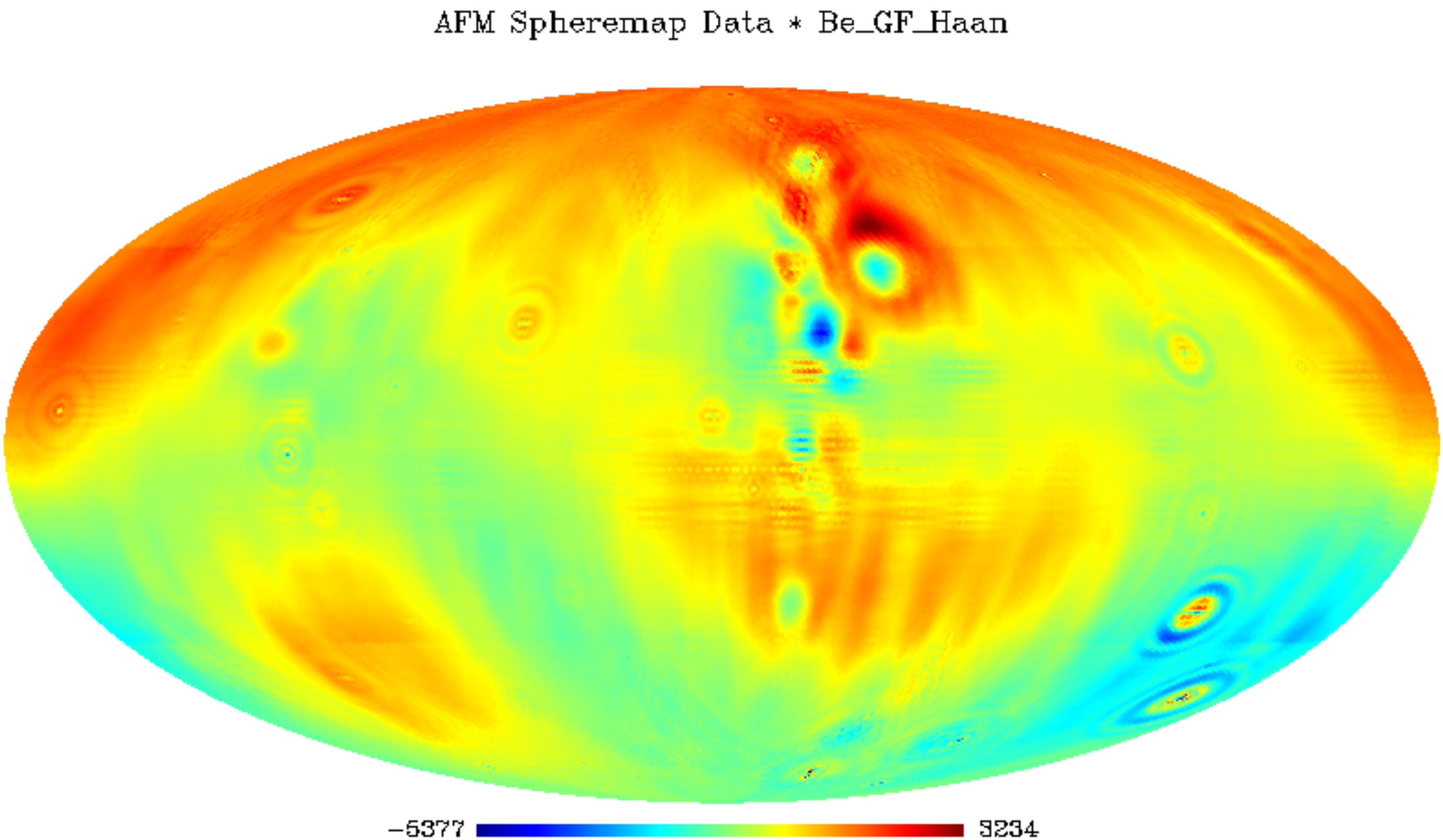


Reconstruction of the Outer Surfaces After Implosion at Peak Velocity for a Be Layer



This target is fine vis a vis high mode outer surface defects ~ 7 μm

Reconstruction of the Outer Surfaces After Implosion at Peak Velocity for a CH Layer (Without MODEM Separation)



The Detection and Classification Process in MODEM:

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- Identify Isolated Features

- Measure Their Heights (signed and separately)

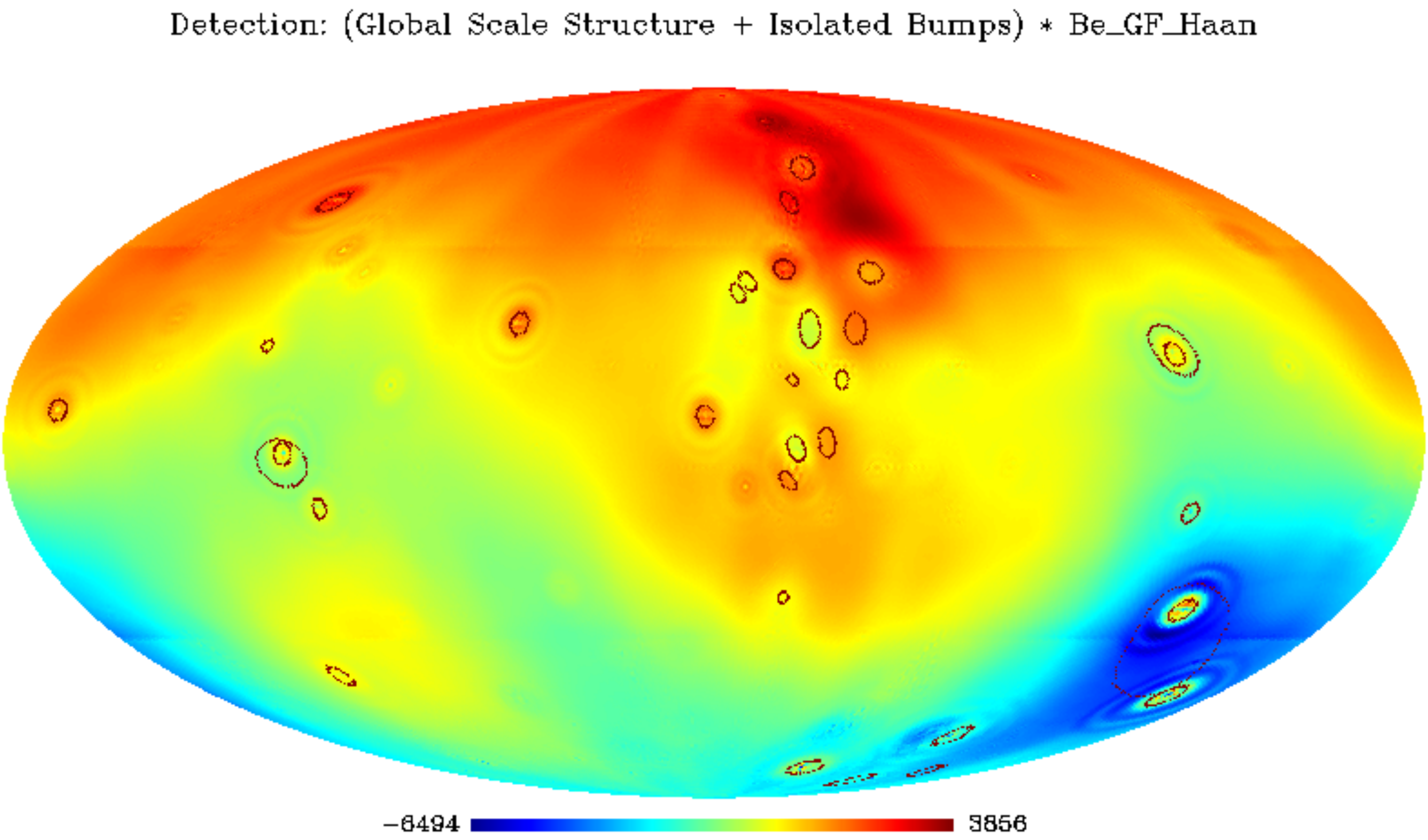
- Measure Their Ellipticity:

(Minor and Major Axes FWHM Lengths)

- Measue their Relative Positions (coordinates) on the Sphere

- Identify Clustering by 2 Pt. Correlation Functions and Compare to a Poisson Process

Here is the Feature Detection for the Be Shell After Implosion



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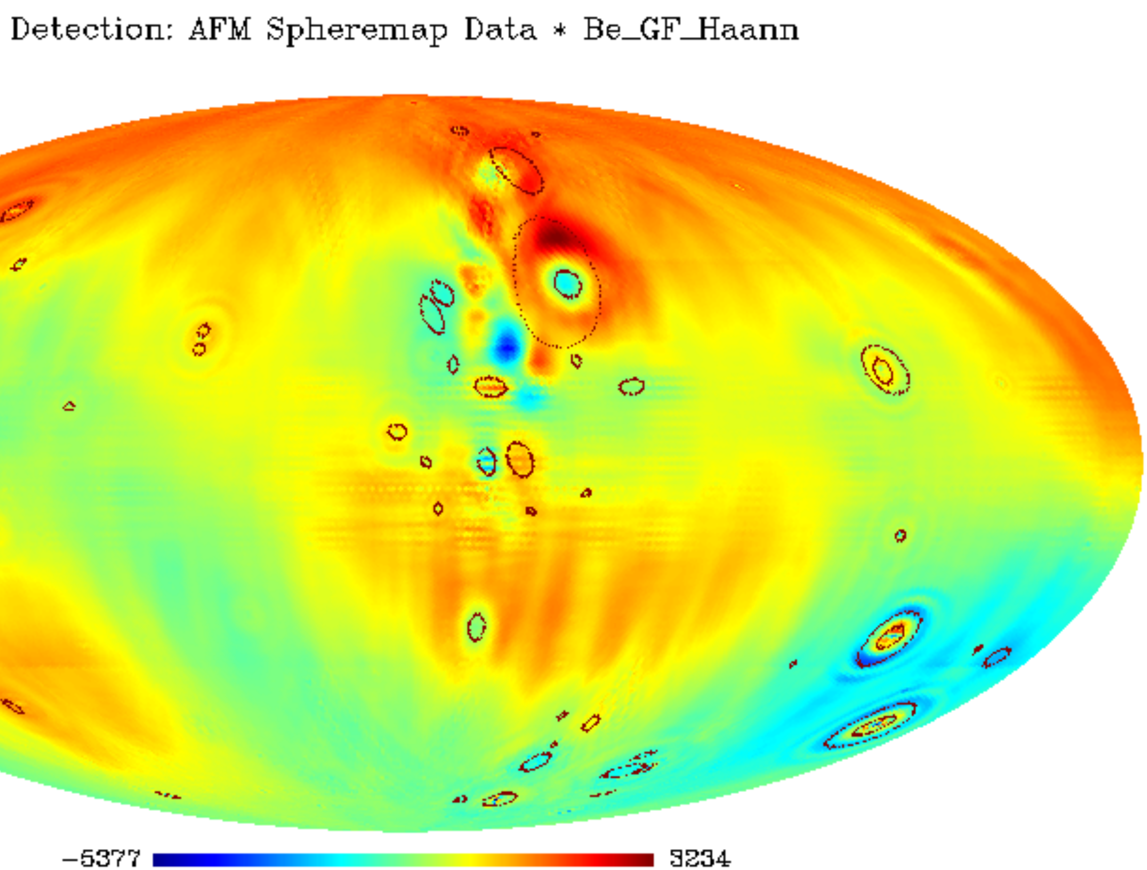
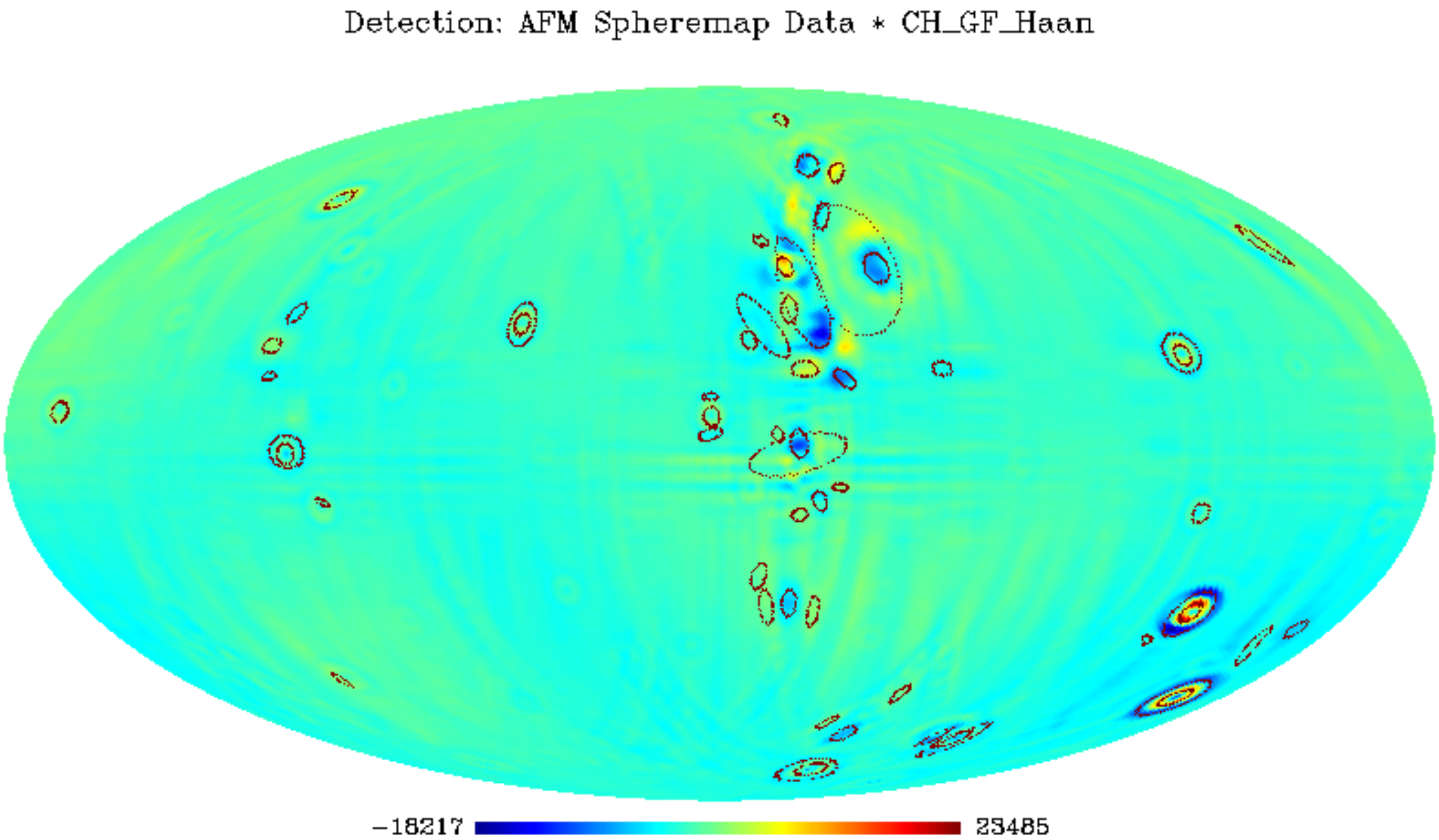
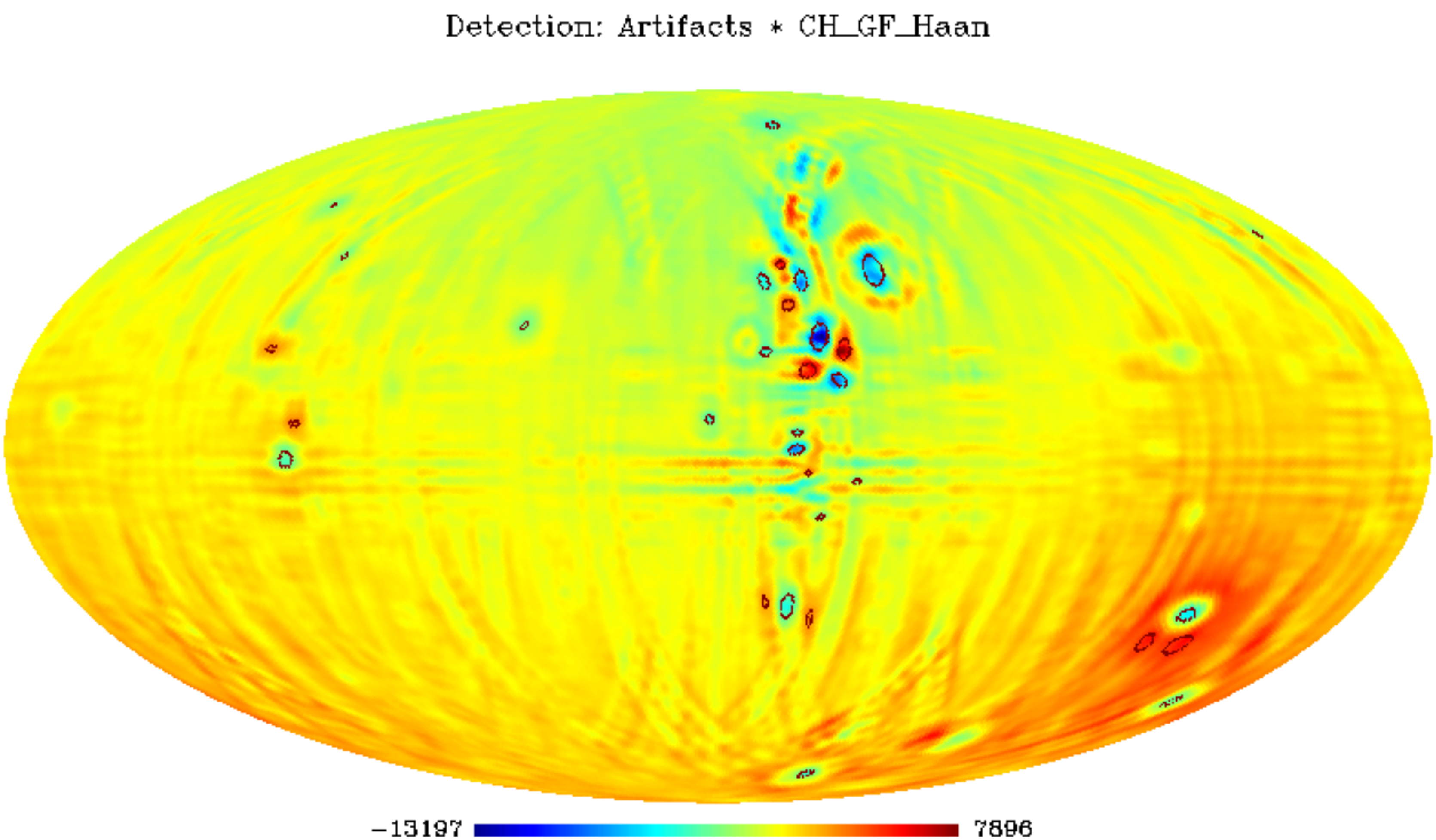
Same Exercise But with Raw Data with Be layer Implosion



Same Exercise with a CH Layer After Implosion at Peak Velocity



Now Just the Artifacts (DCT) for CH



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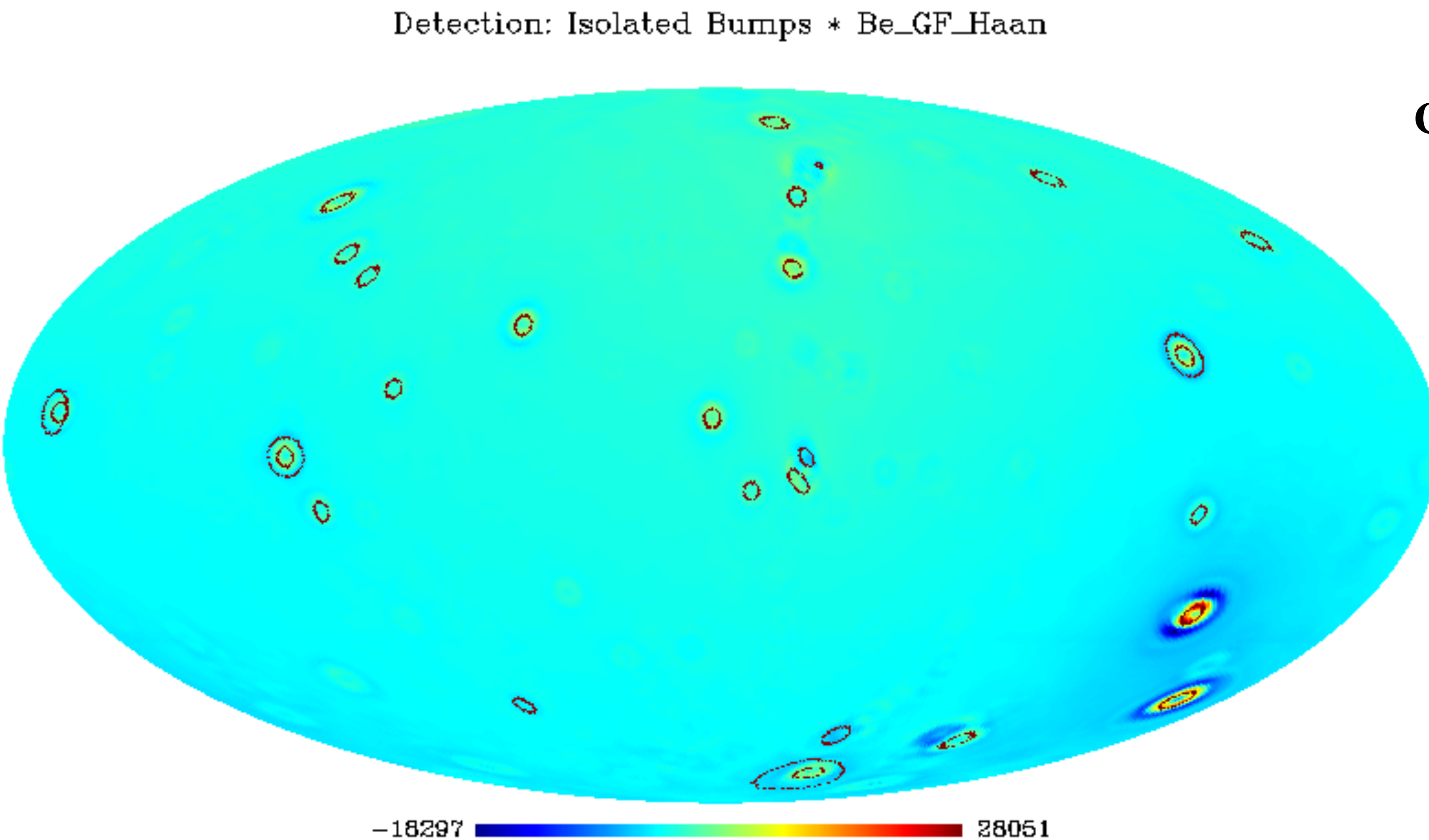
Just the Isolated Bumps (WLT) for CH



CH

Its Good to Know that When There Aren't Any, It Does Not Find Any (here it found 5)

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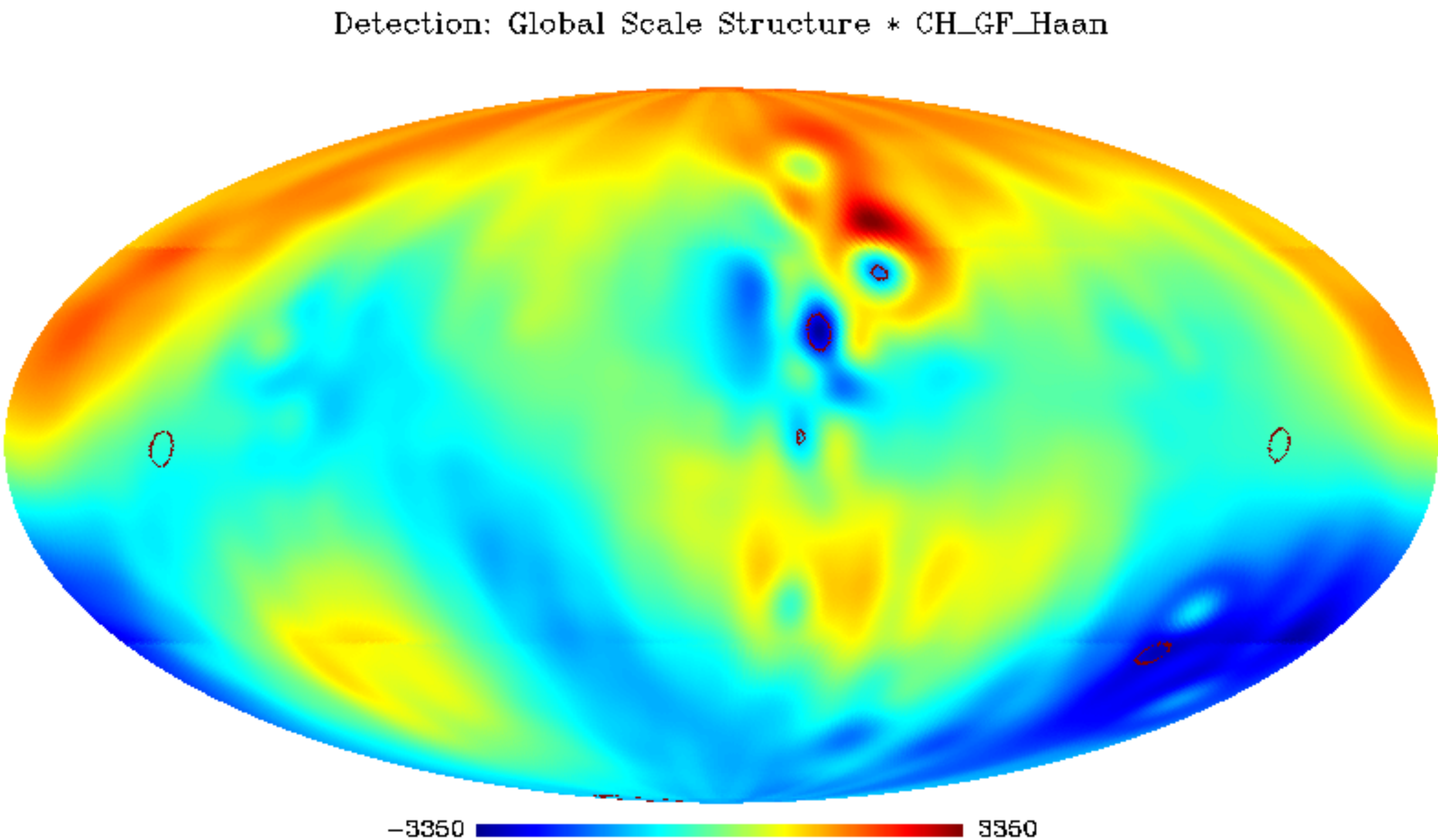
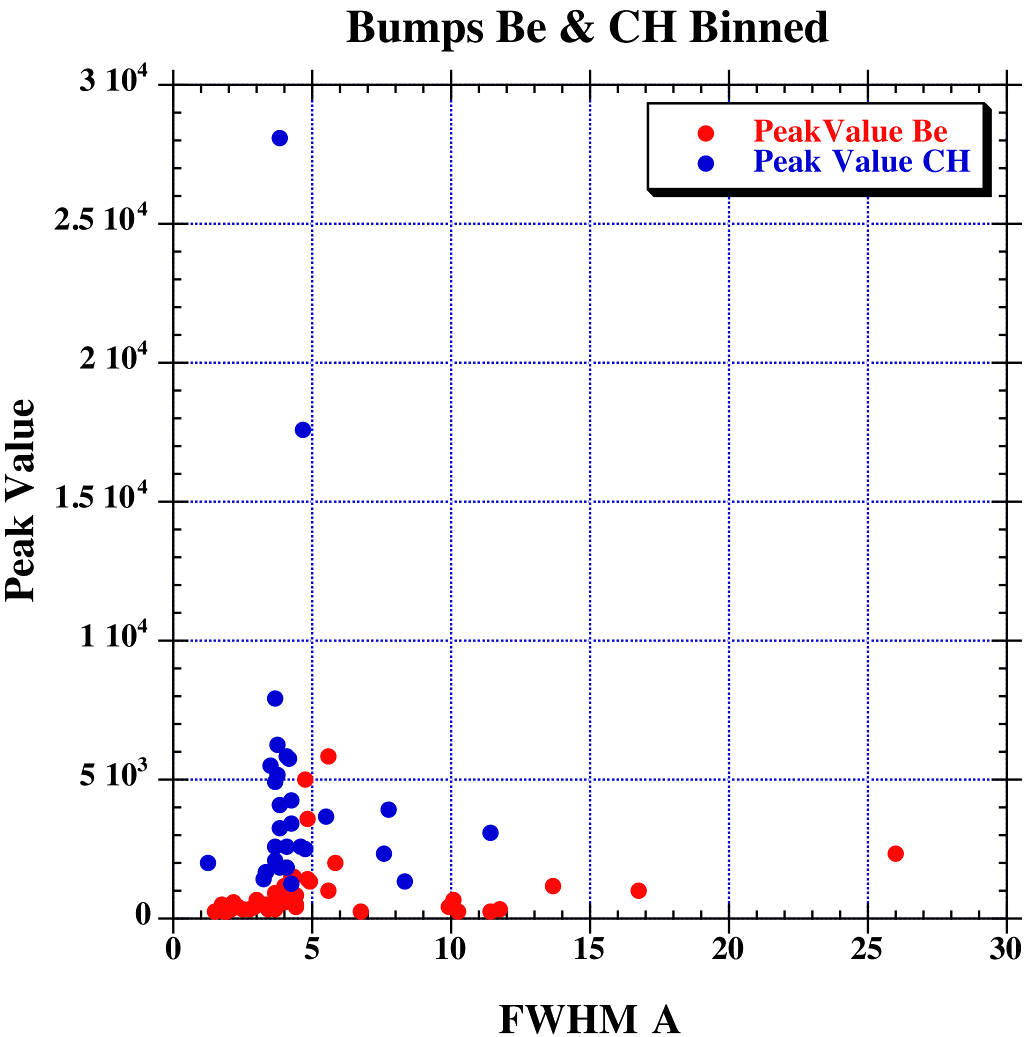
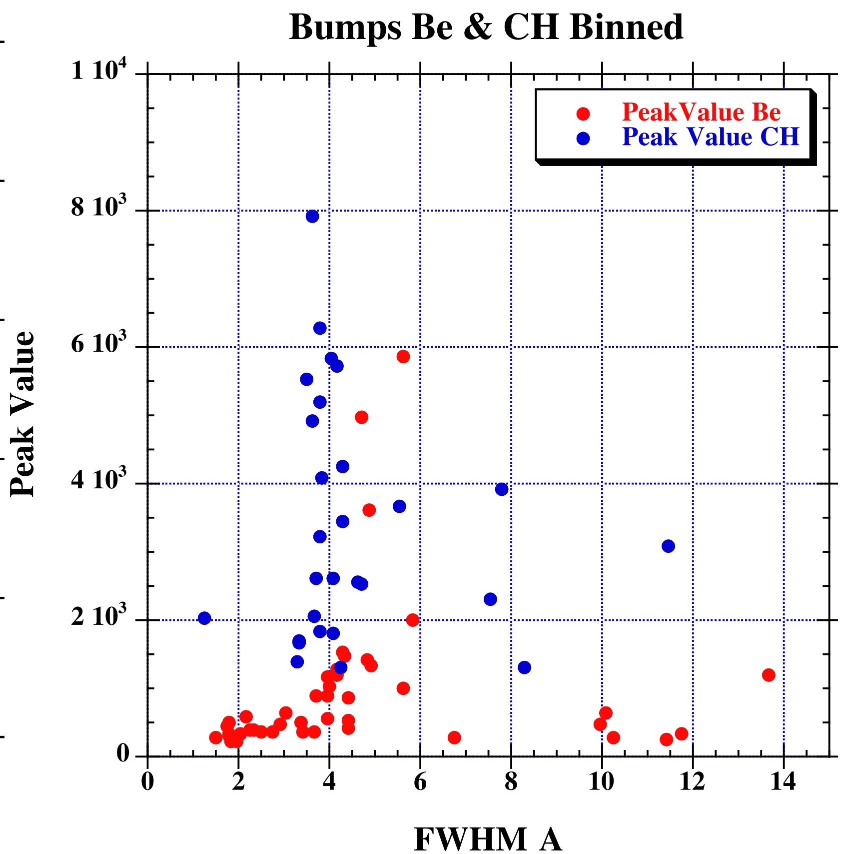
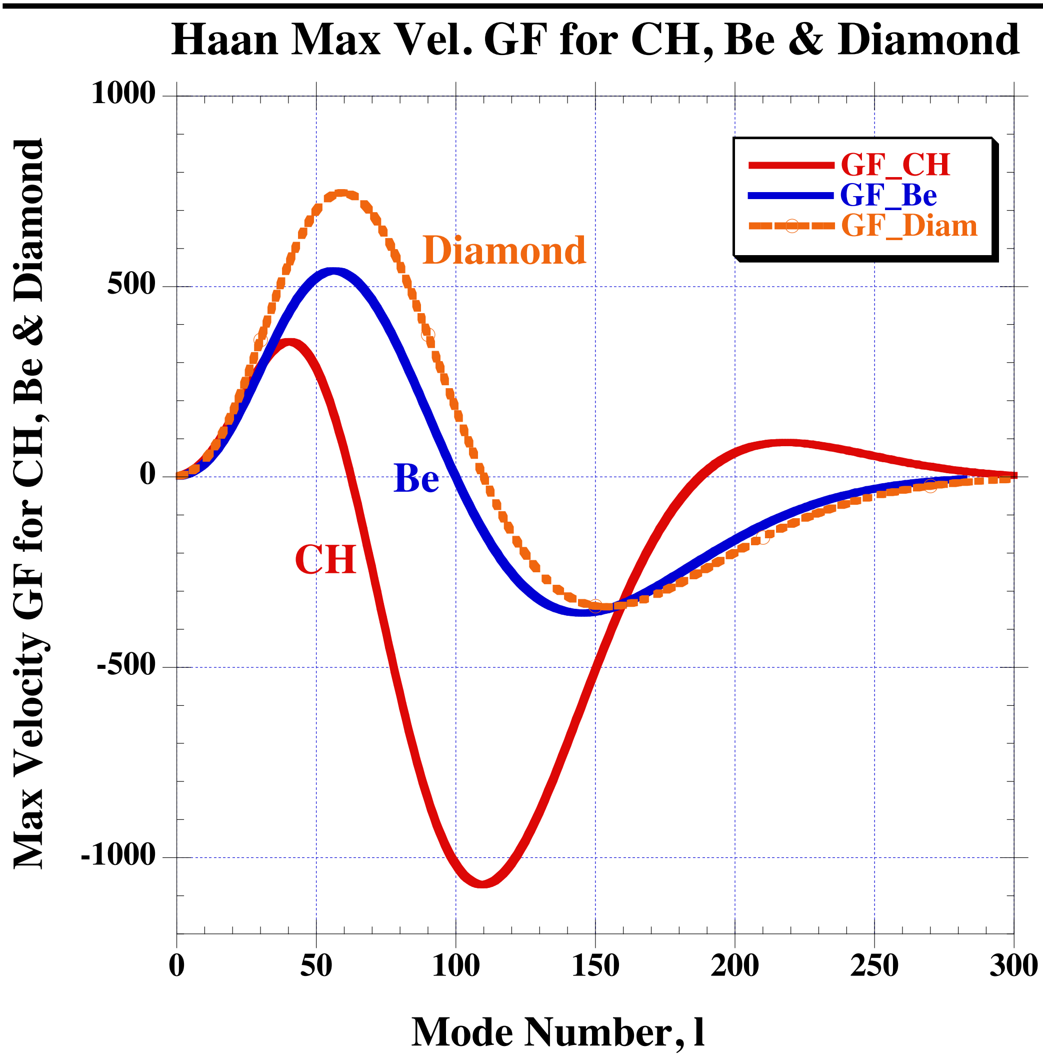


The Size vs Height Distribution of the Isolated Bumps on the CH and Be Layer Targets After Implosion at peak Velocity





Distinguishing Diamond, CH & Be: Integrated Hydrodynamic Haan Growth Factors



Make sure perturbations do not grow to be larger than the shell diameter at maximum implosion velocity to ensure target structural integrity

Features of Localized Bumps in Al Coated GDP-1 Eccentricity & Angle

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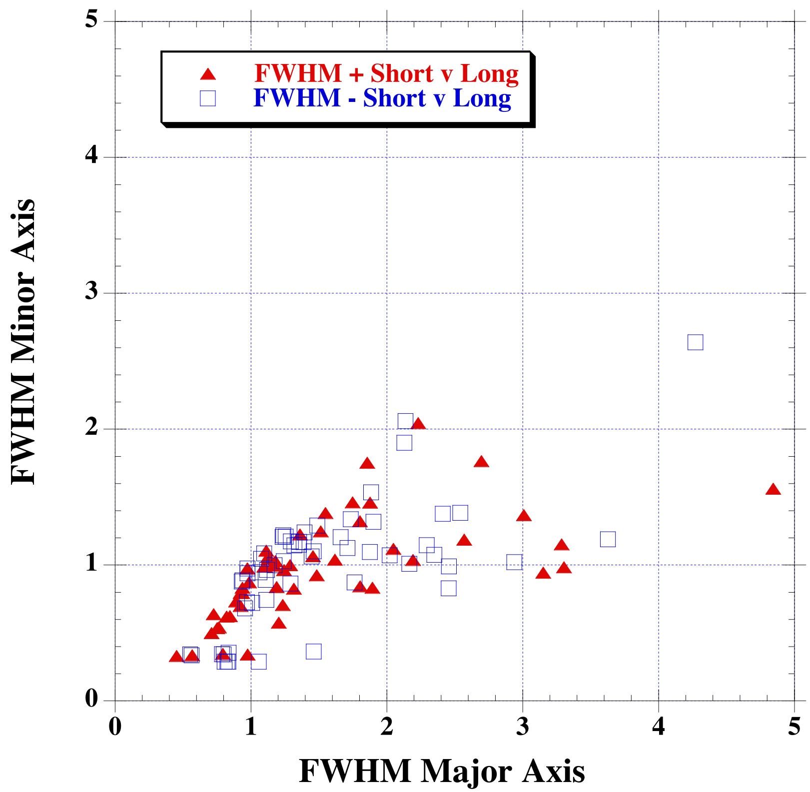
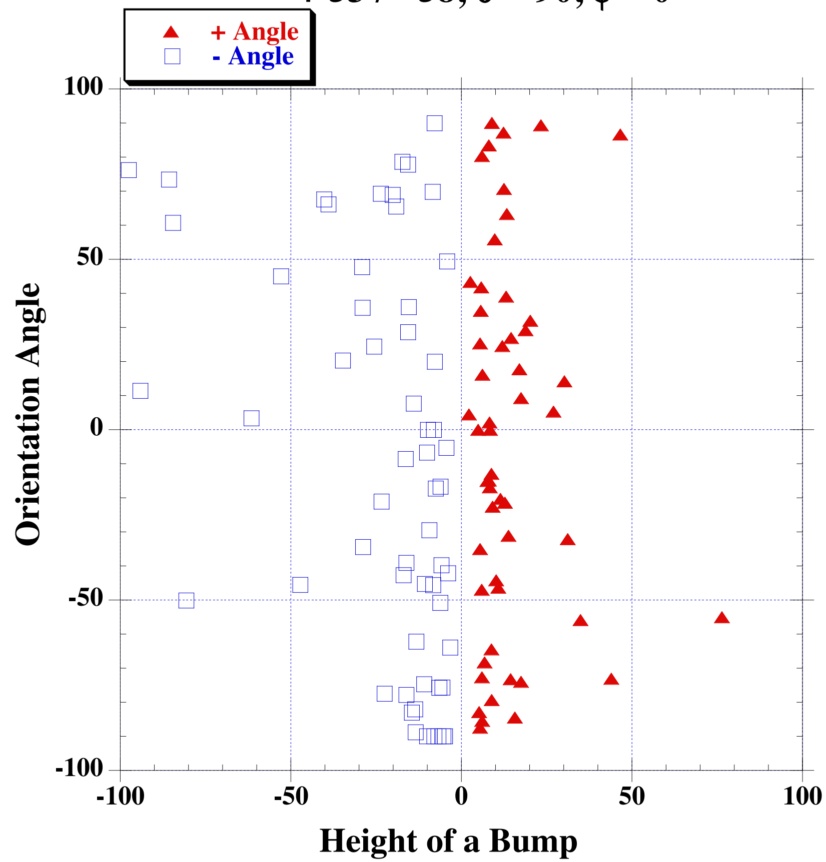
Bump Eccentricity AL_GPD_1
+ 53 / - 58, θ= 90, ϕ = 0

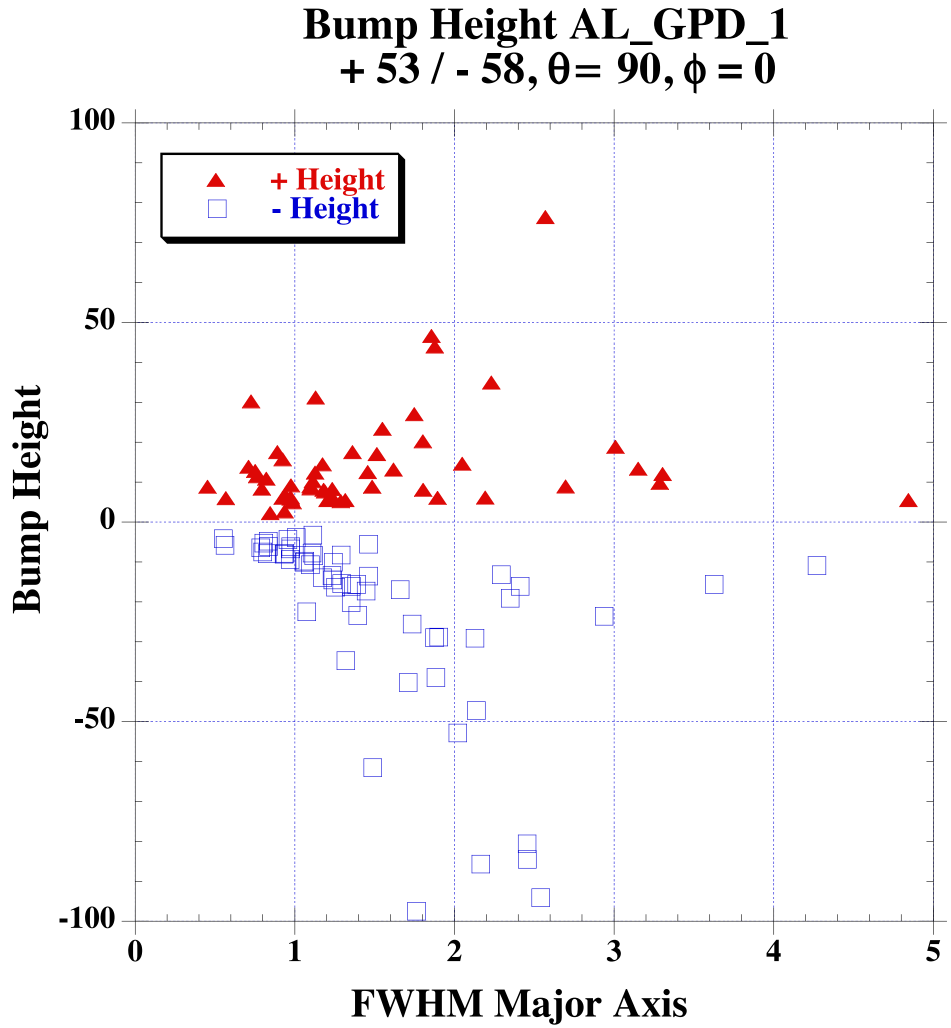


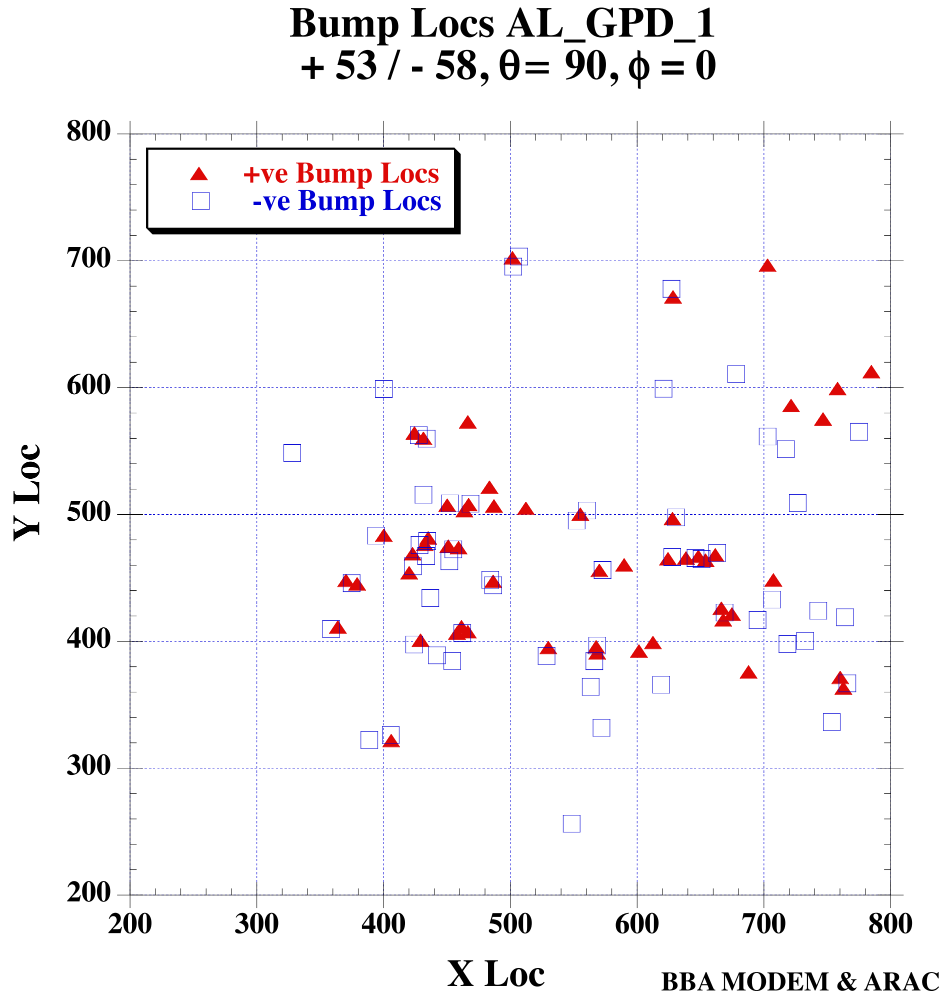
Bump Orientation Angle v Height AL_GPD_1
+ 53 / - 58, θ = 90, ϕ = 0



Features of Localized Bumps in Al Coated GDP-1 H v L & Locs

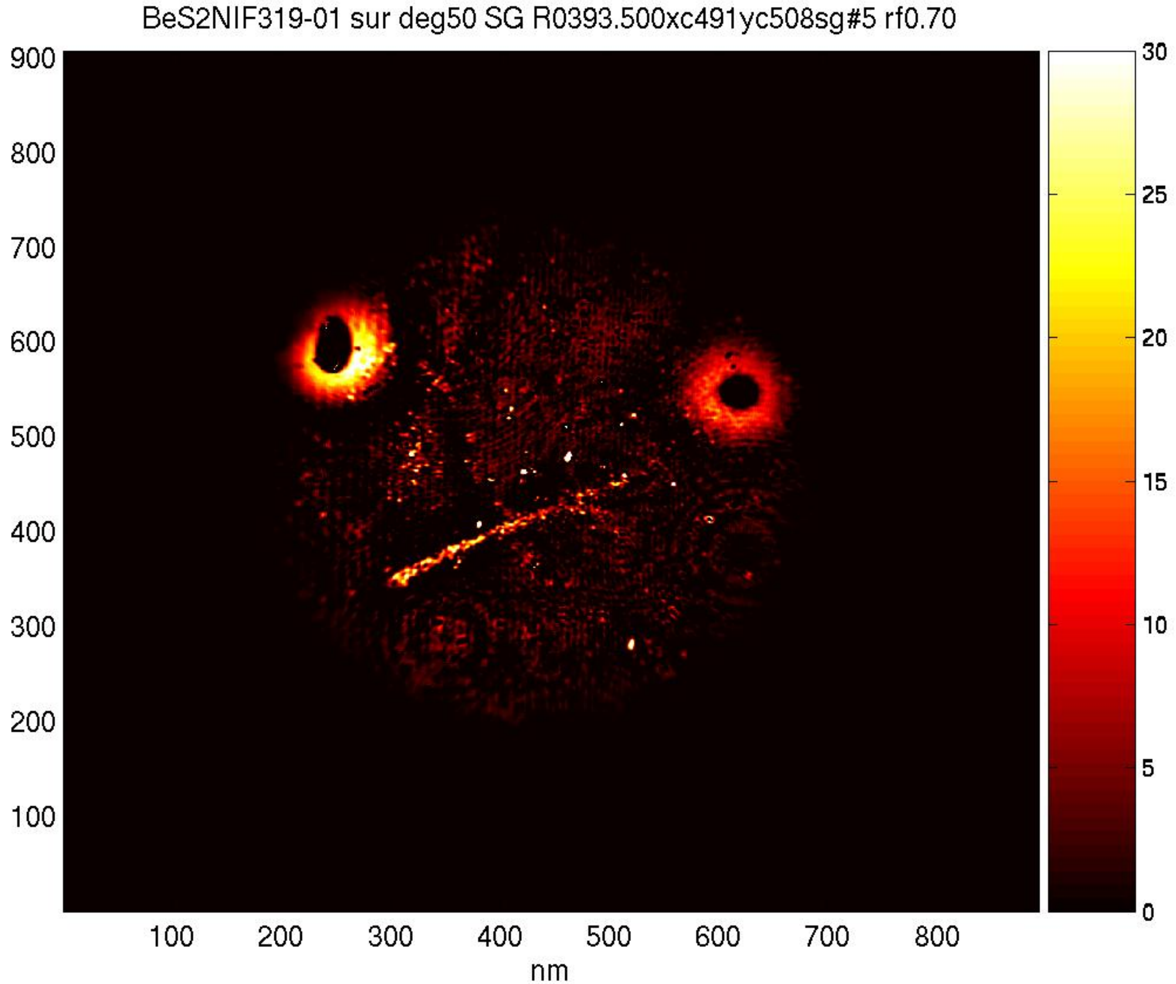
Polymath Research Inc.





Be Target Surface Image Obtained by Spherical Diffractive Interferometry

BeS2NIF319-01_sur_deg50_SG_R0393.500xc491yc508sg#5_rf0.70



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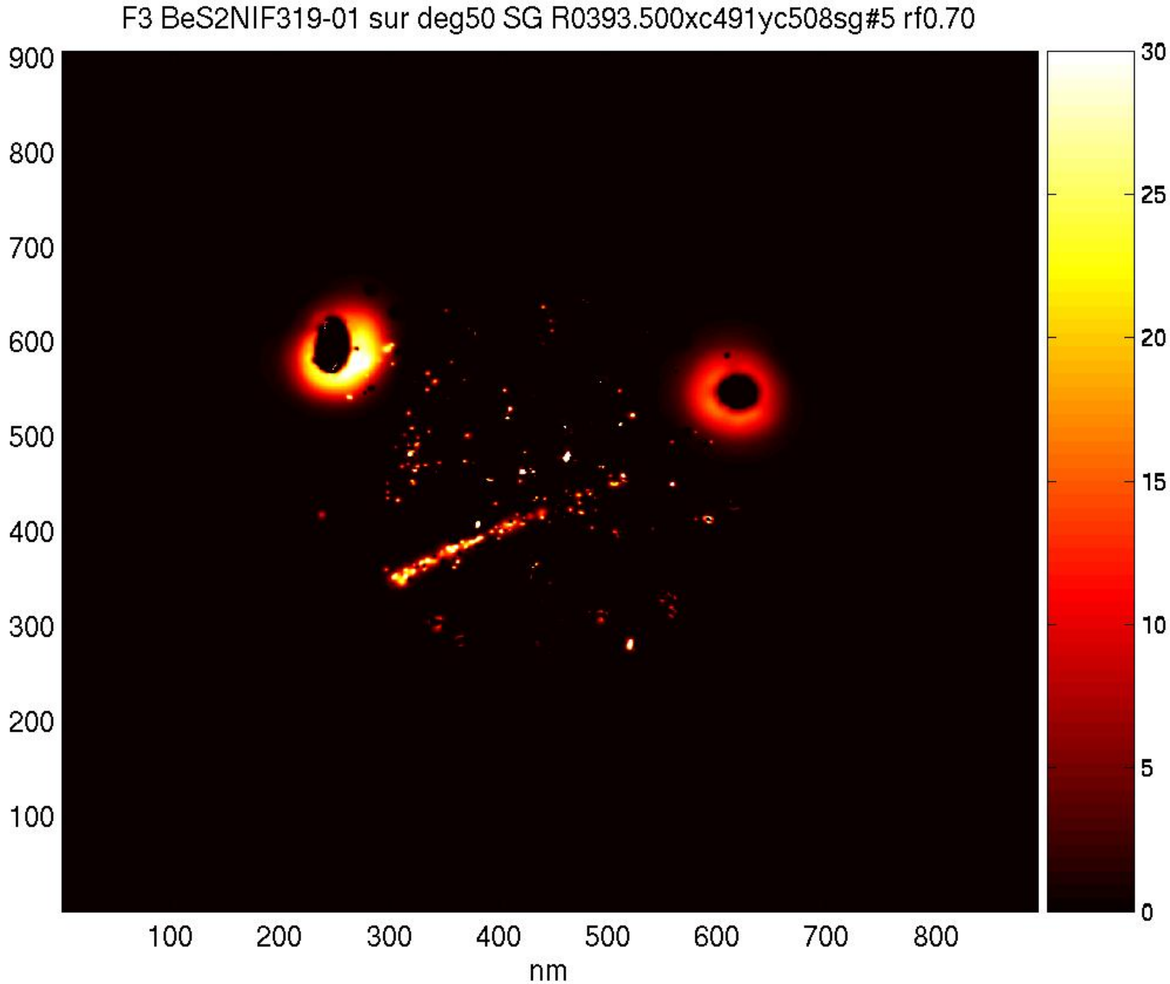
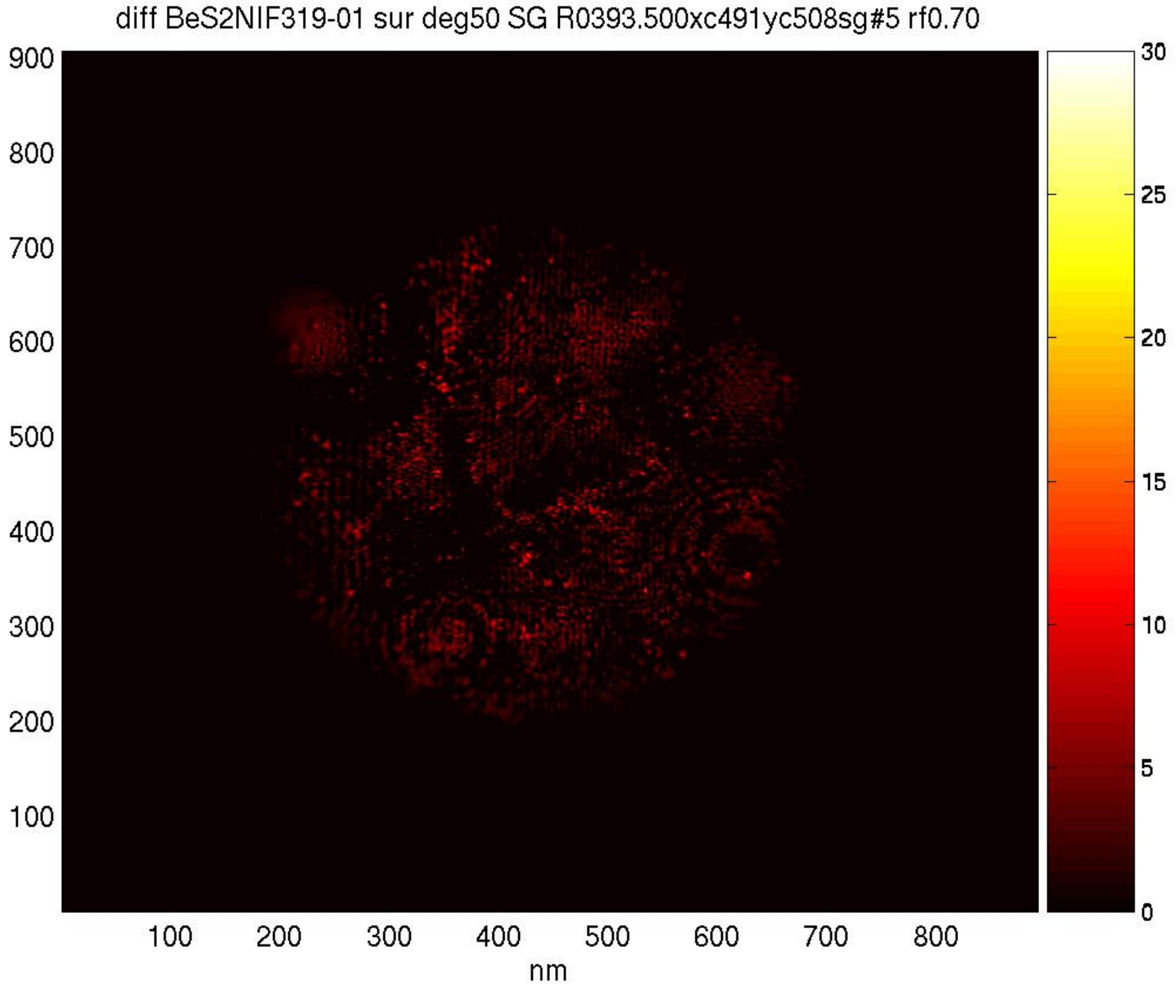
The Localized Structures Can Be Isolated Using MODEM Techniques

BeS2NIF319-01_sur_deg50_SG_R0393.500xc491yc508sg#5_rf0.70



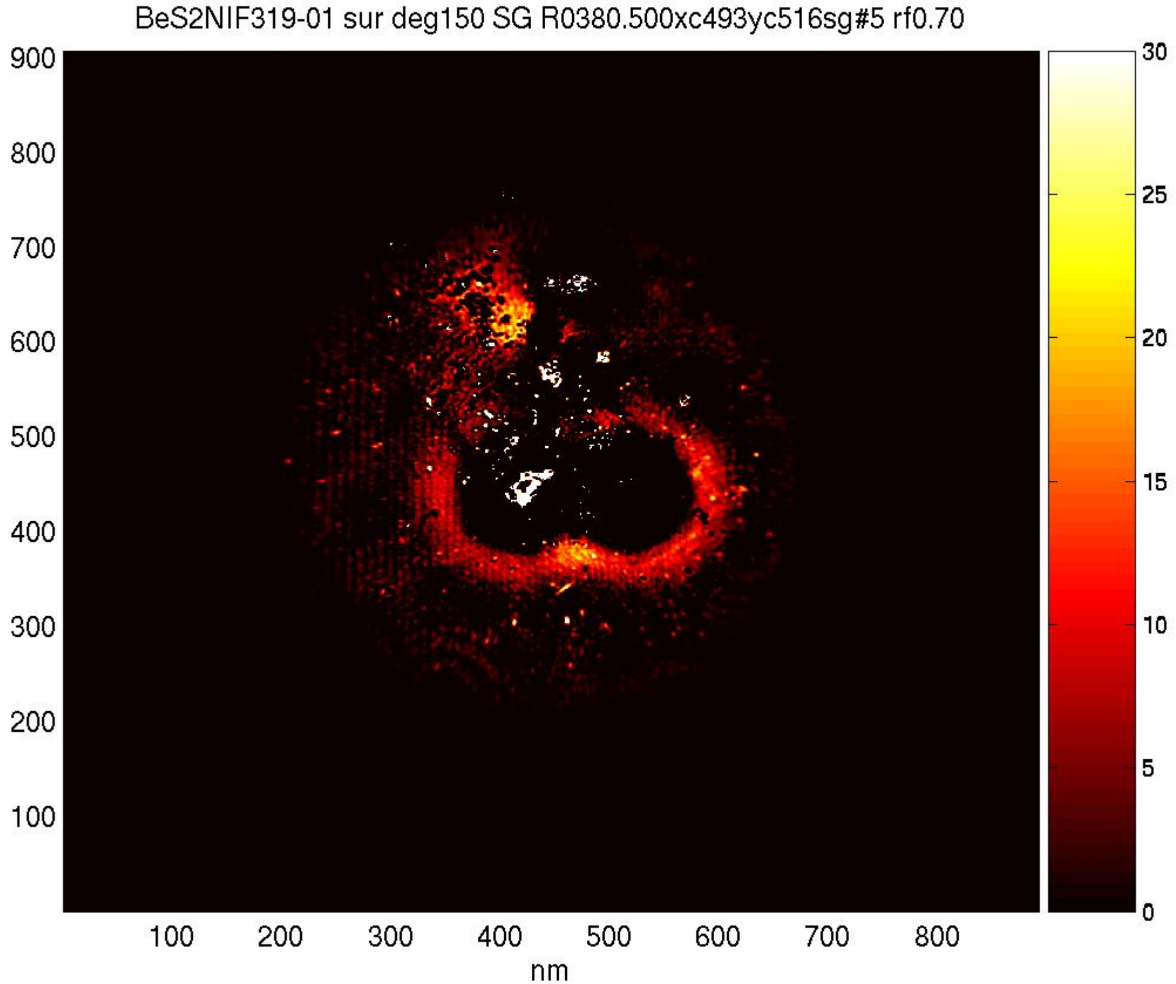
The Residue Can Also Be Extracted Using MODEM Techniques

BeS2NIF319-01_sur_deg50_SG_R0393.500xc491yc508sg#5_rf0.70

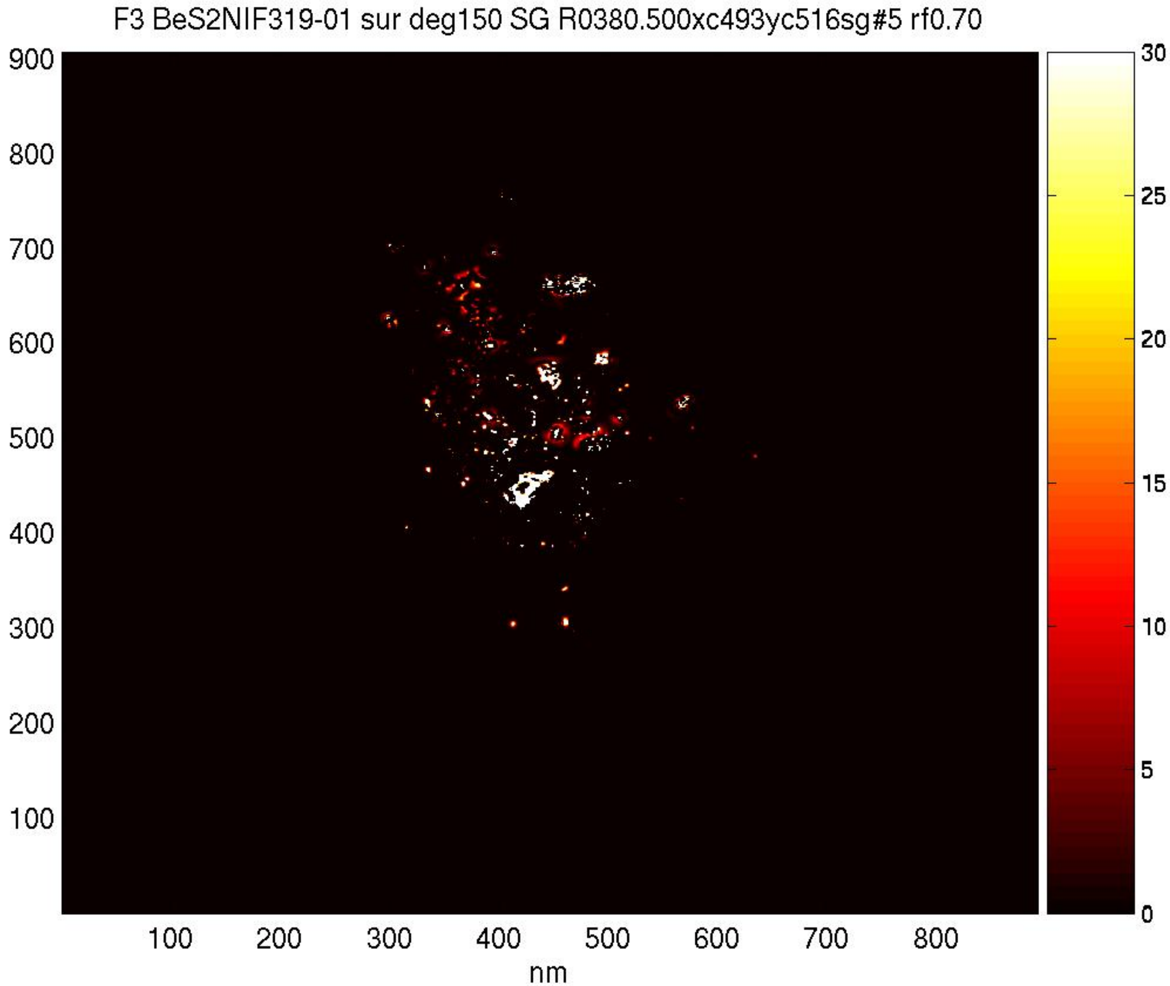


Be Target Surface Image Obtained by Spherical Diffractive Interferometry

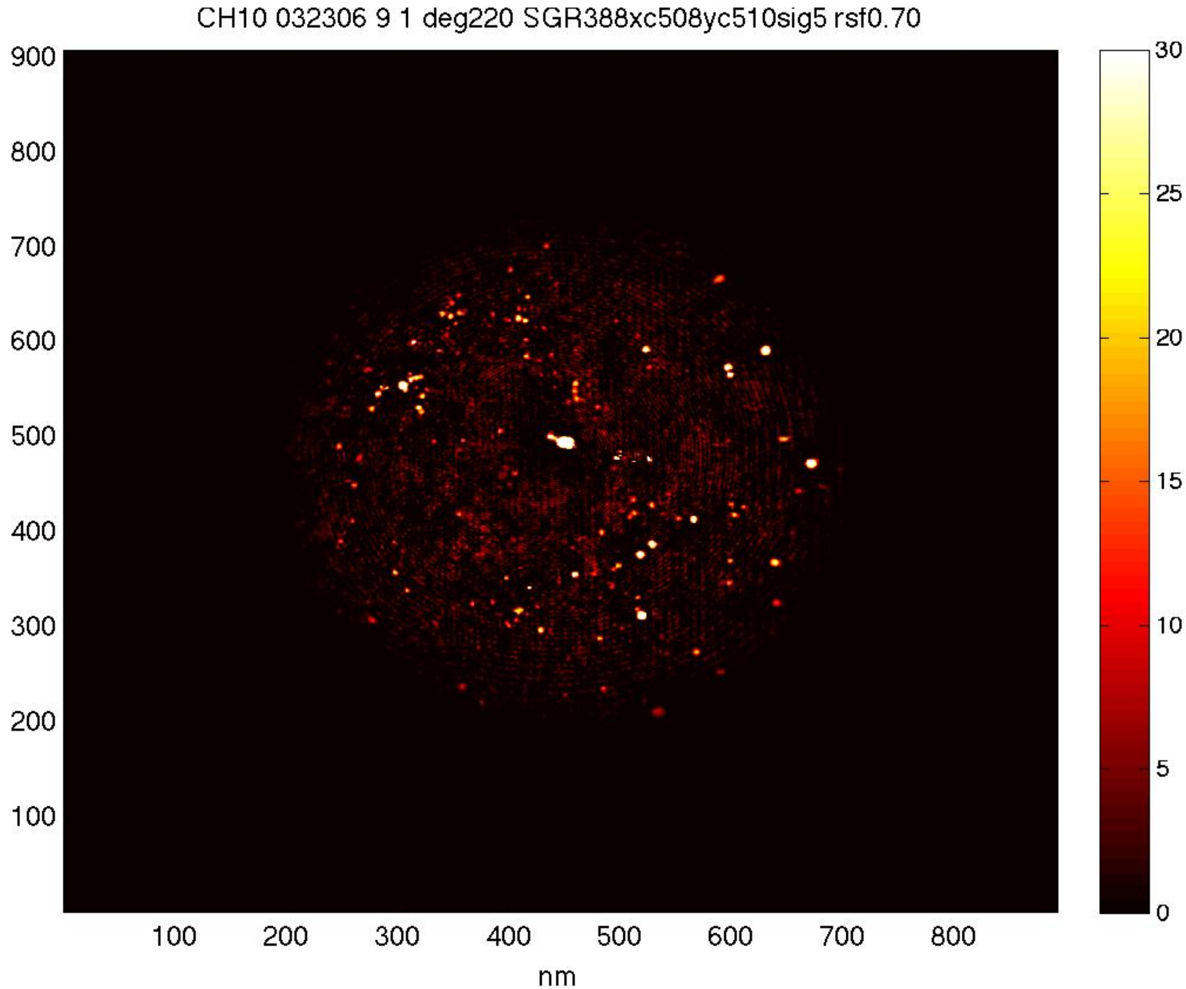
BeS2NIF319-01_sur_deg150_SG_R0380.500xc493yc516sg#5_rf0.70



The Localized Structures Can Be Isolated Using MODEM Techniques



We Have Also Applied MODEM to CH Coated Spheres



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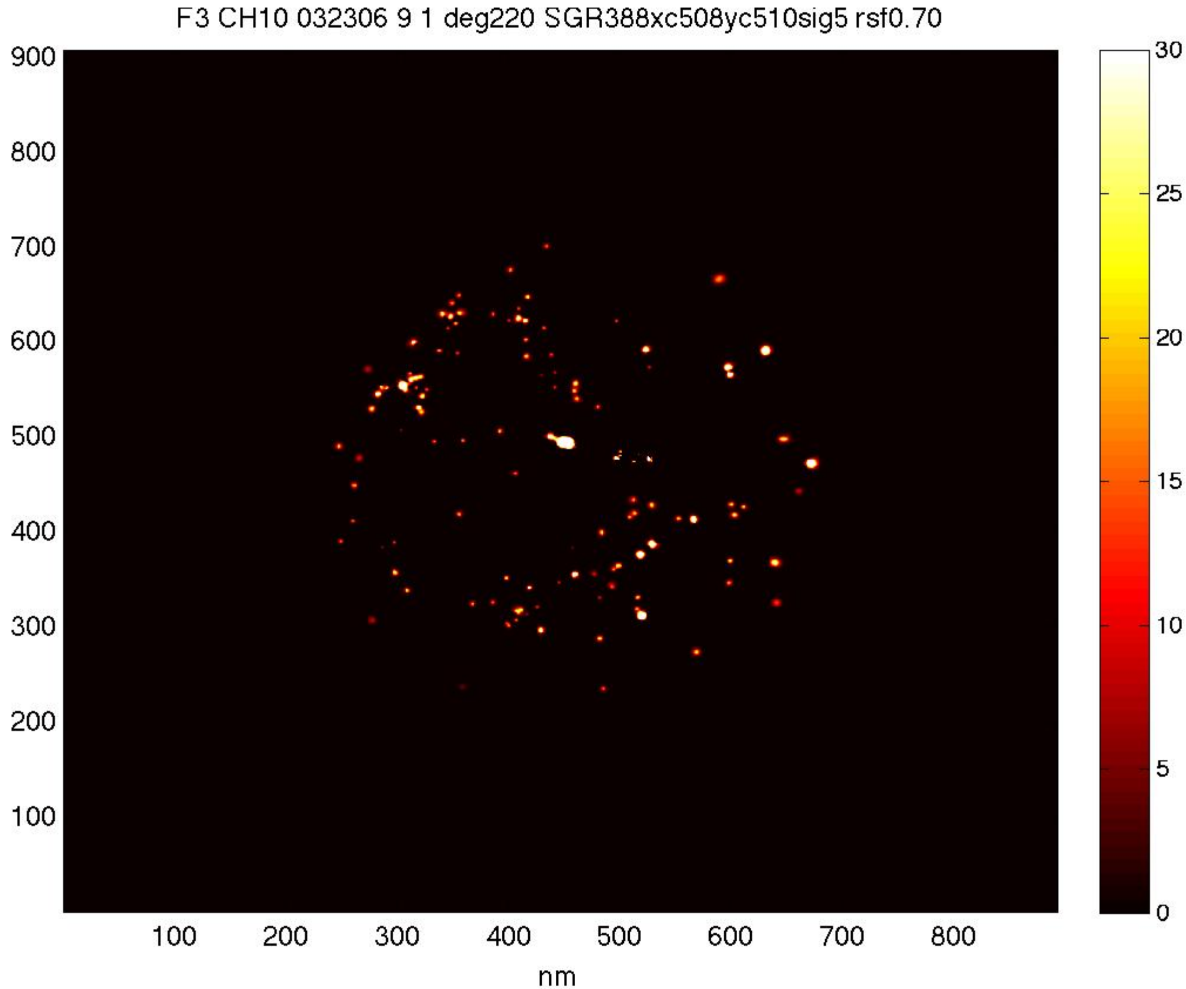
The Desired Localized Structures Are Isolated Here



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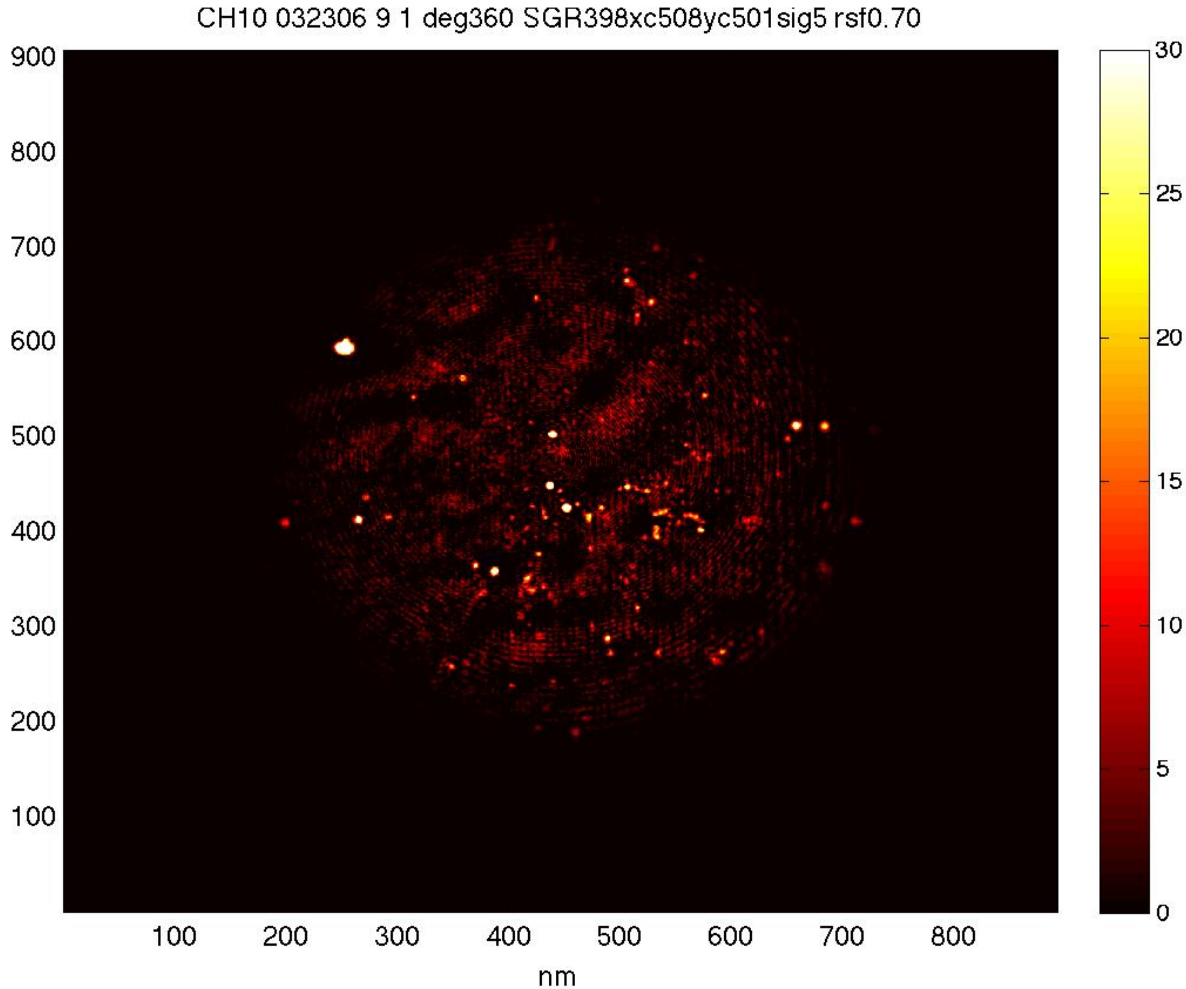
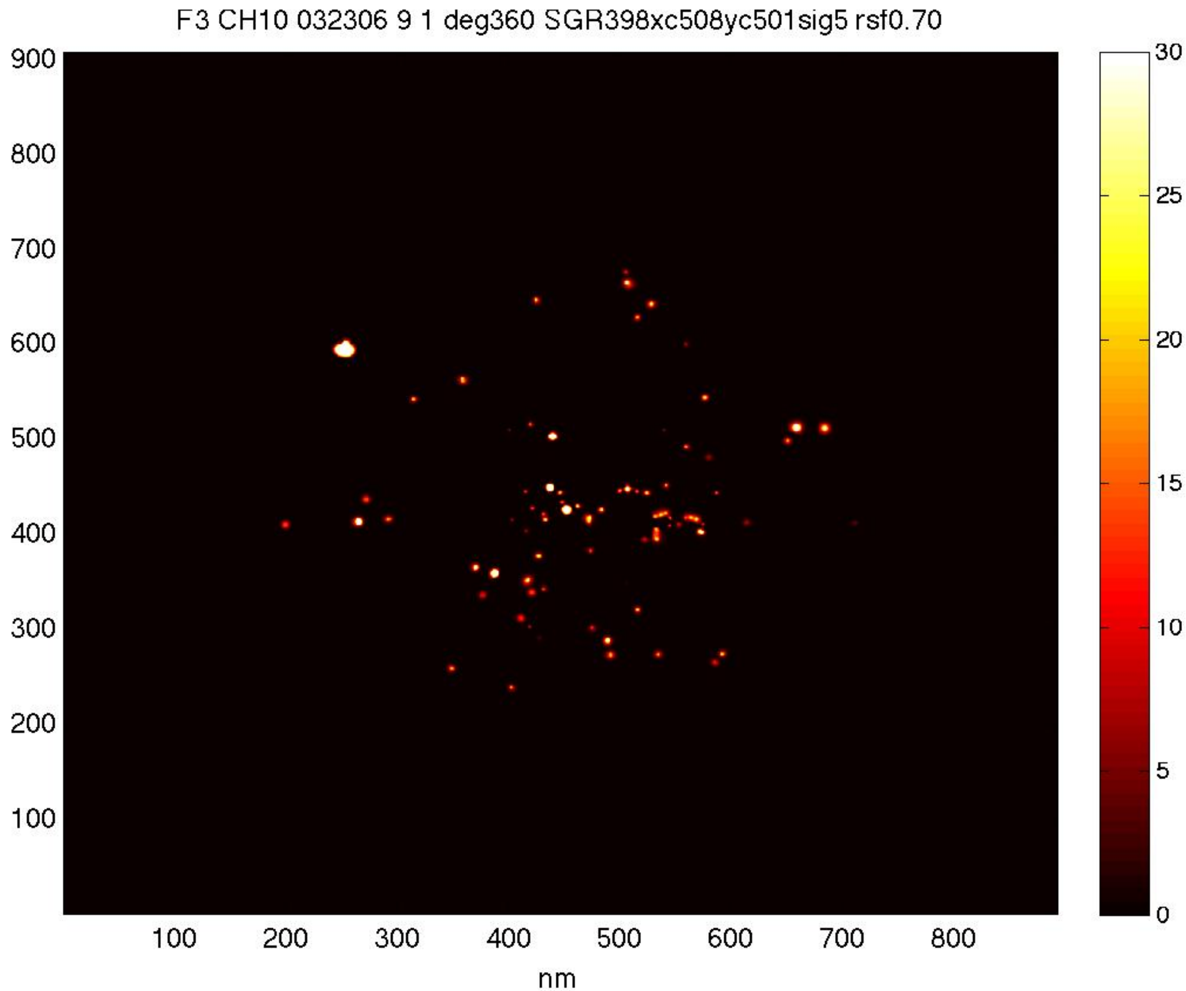
Here is a CH (Plastic) Spherical Shell Piece with LDI the Technique

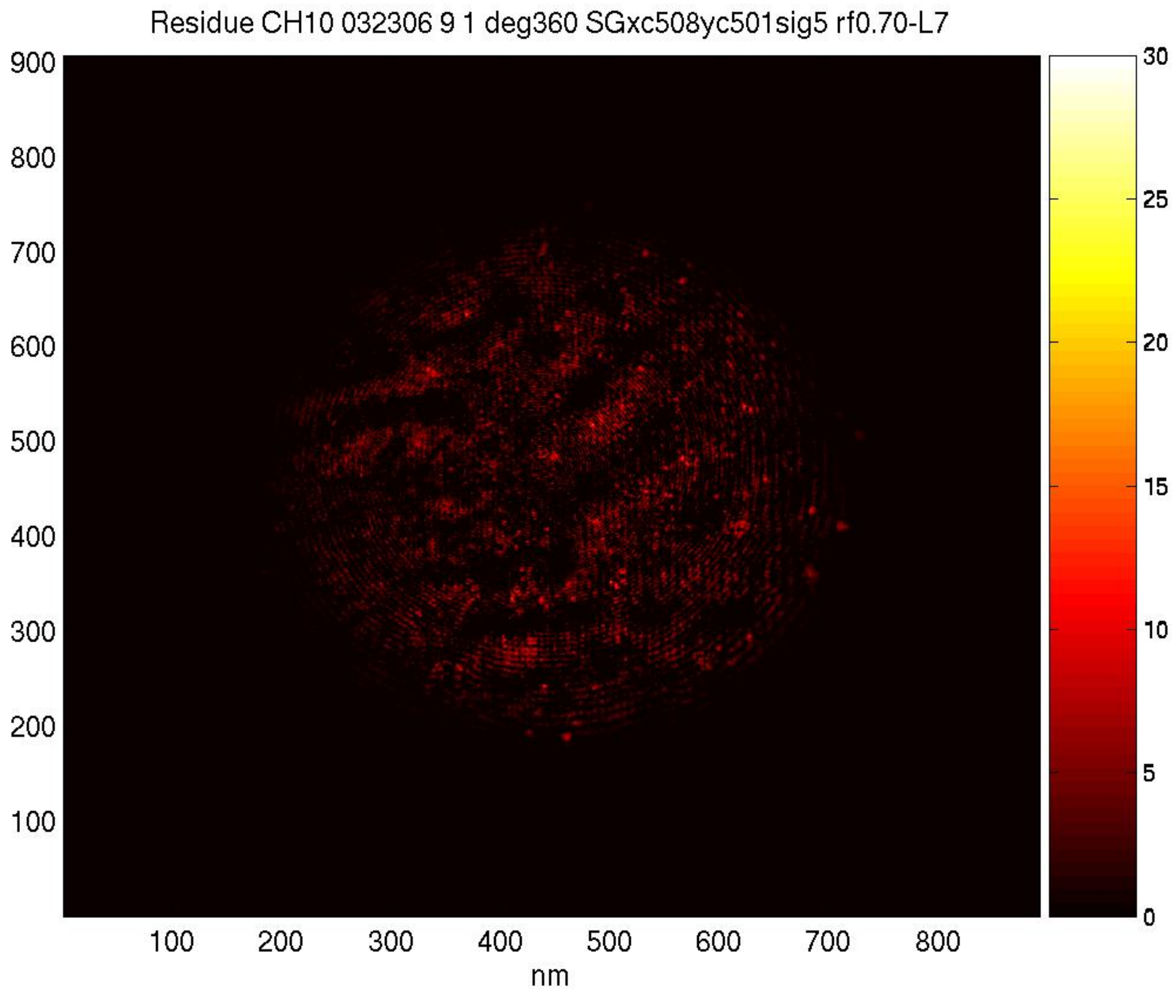


Polymath Research Inc.

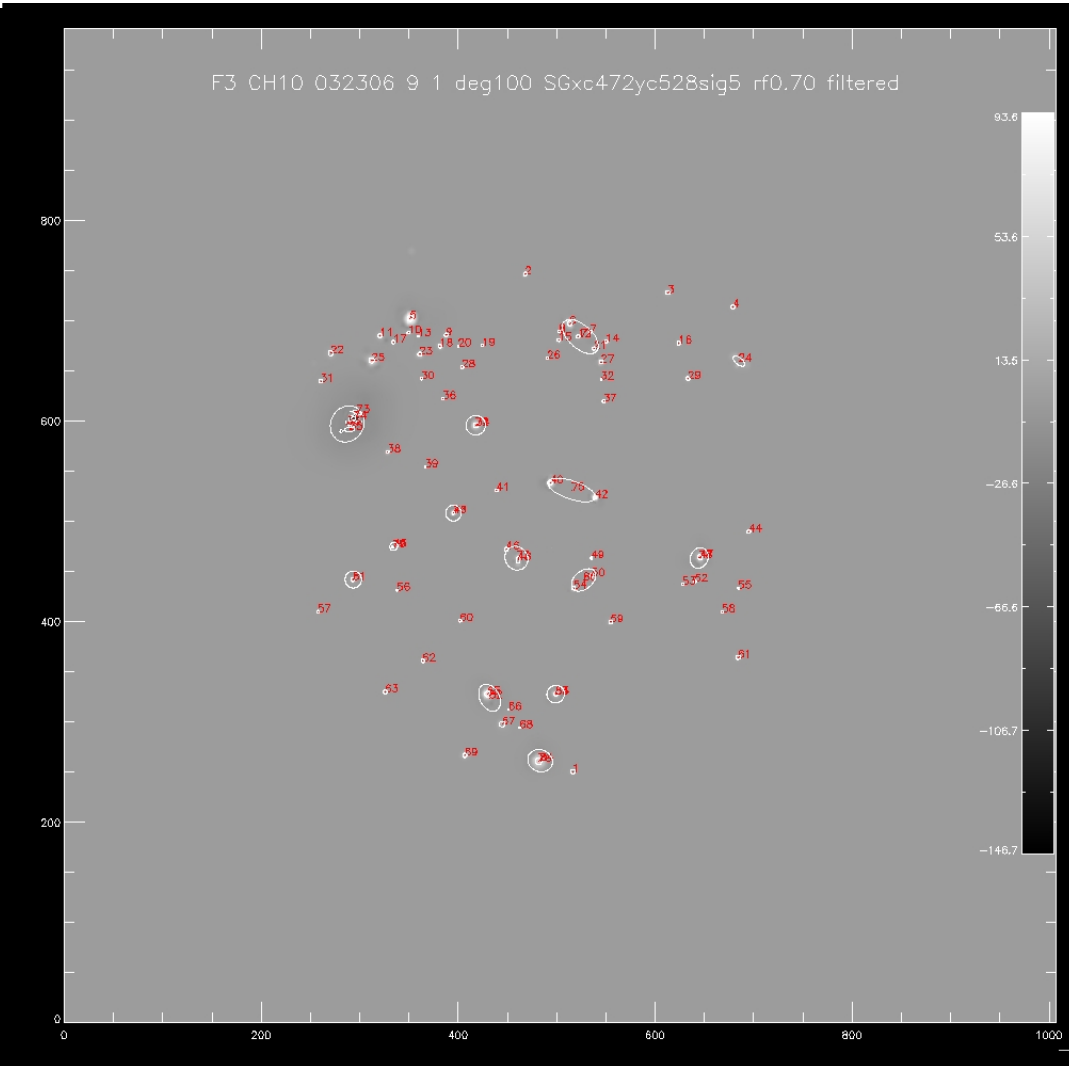
The Desired Localized Structures Are Isolated Here



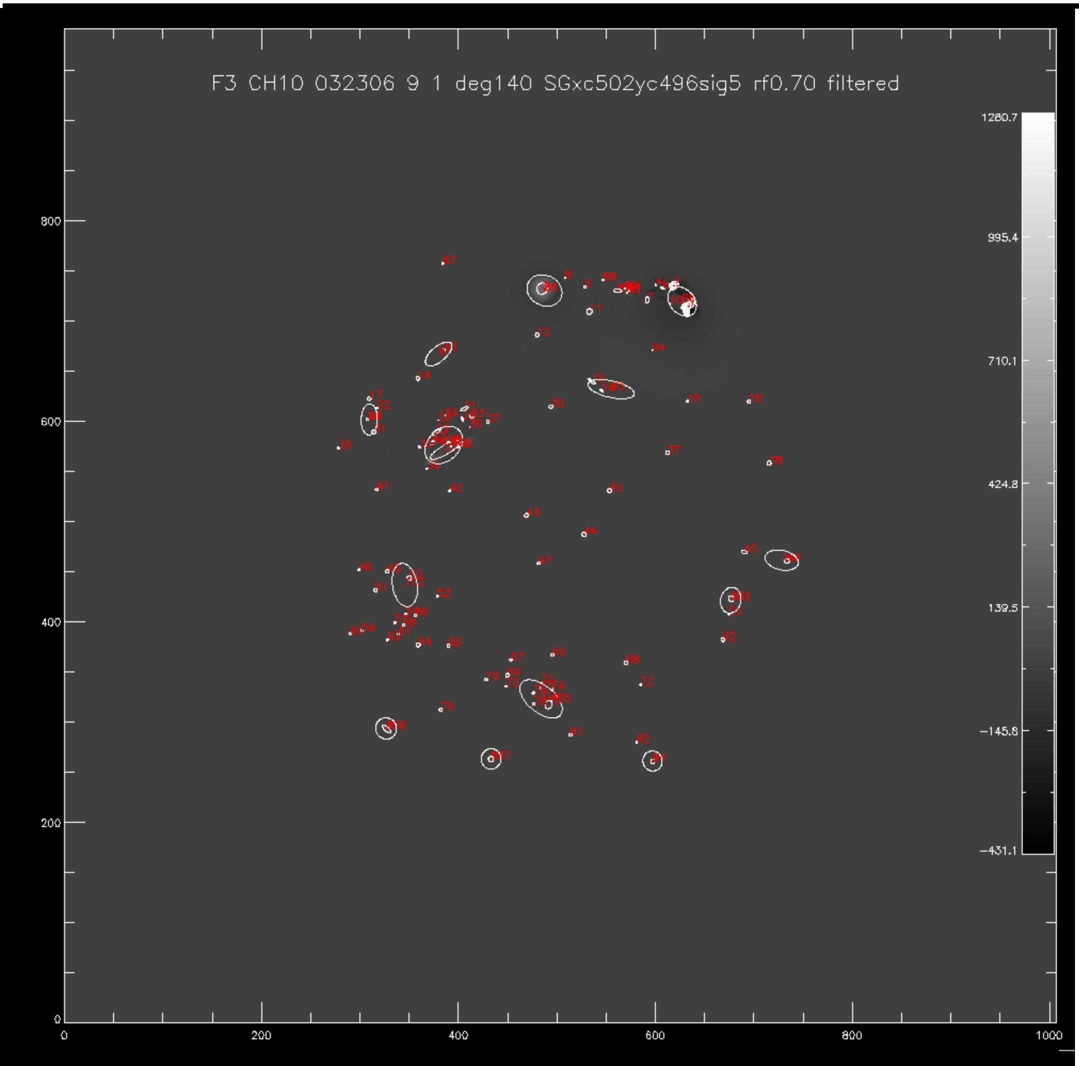
The Successfully Isolated Diffraction Effects off Defects, Dust Particles and Misalignment Artifacts Are:



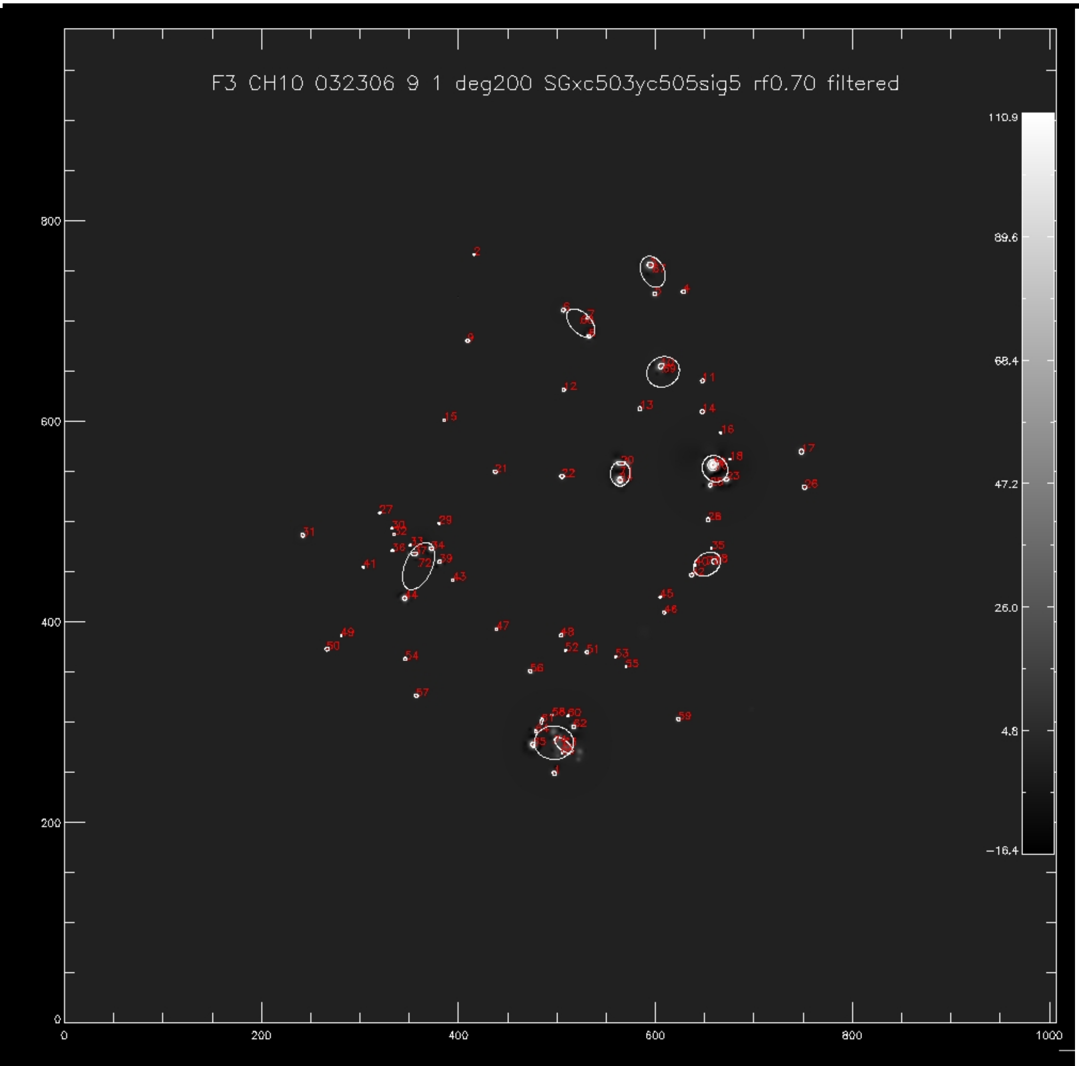
A CH Sequence of 18 Medallions around a Great Circle ~ 20% Area of the Surface of a Sphere (100 Deg)



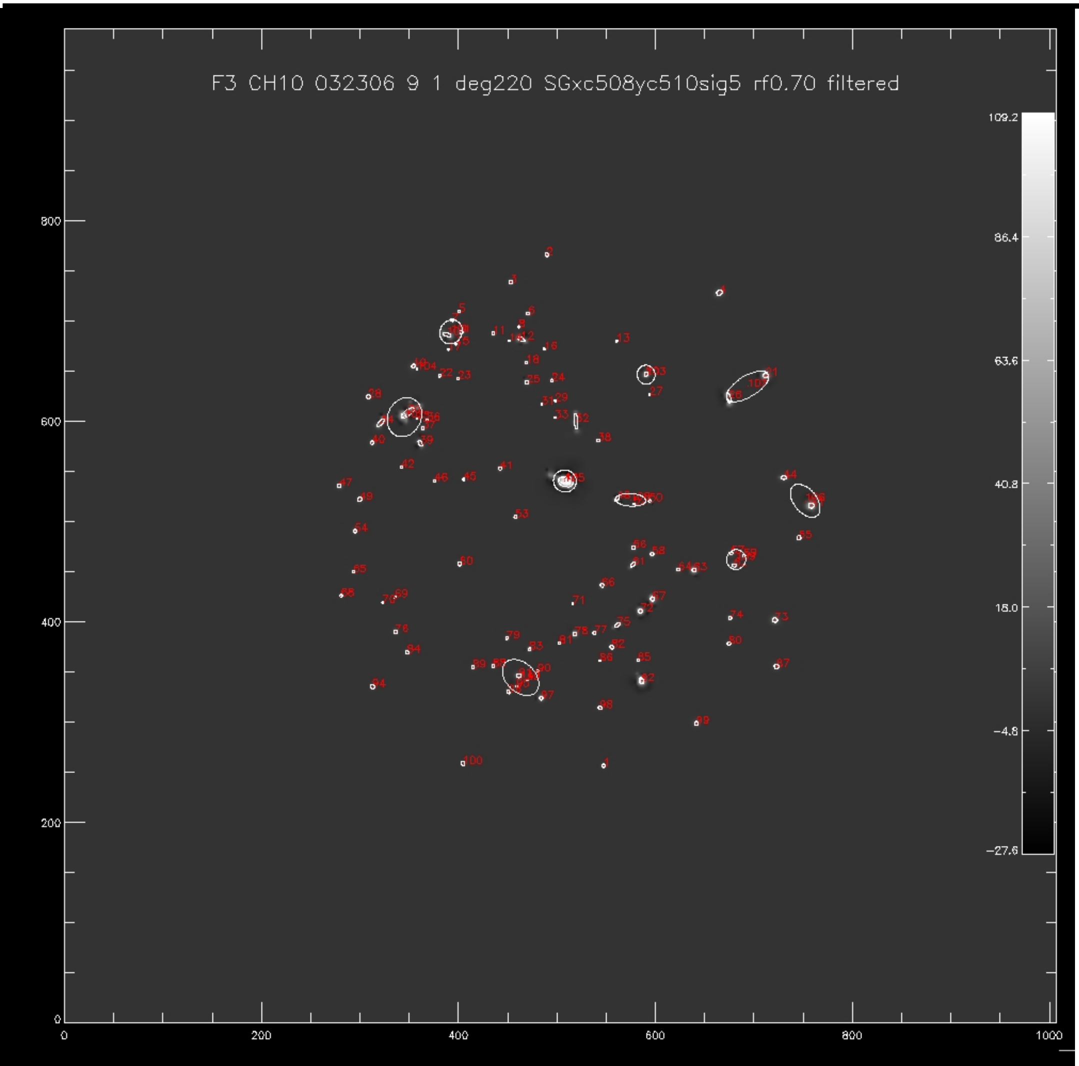
A CH Sequence of 18 Medallions around a Great Circle ~ 20% Area of the Surface of a Sphere (140 Deg)



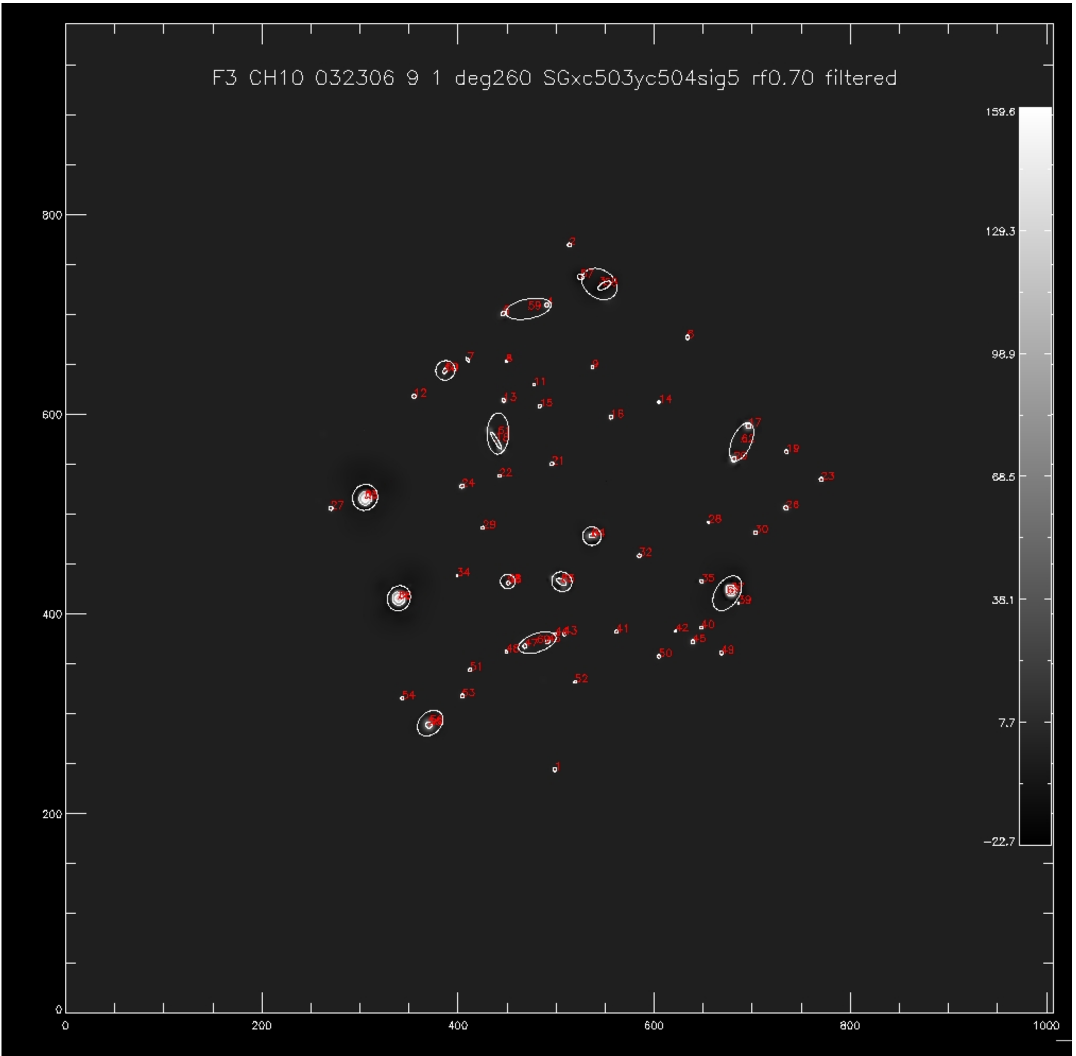
A CH Sequence of 18 Medallions around a Great Circle ~ 20% Area of the Surface of a Sphere (200 Deg)



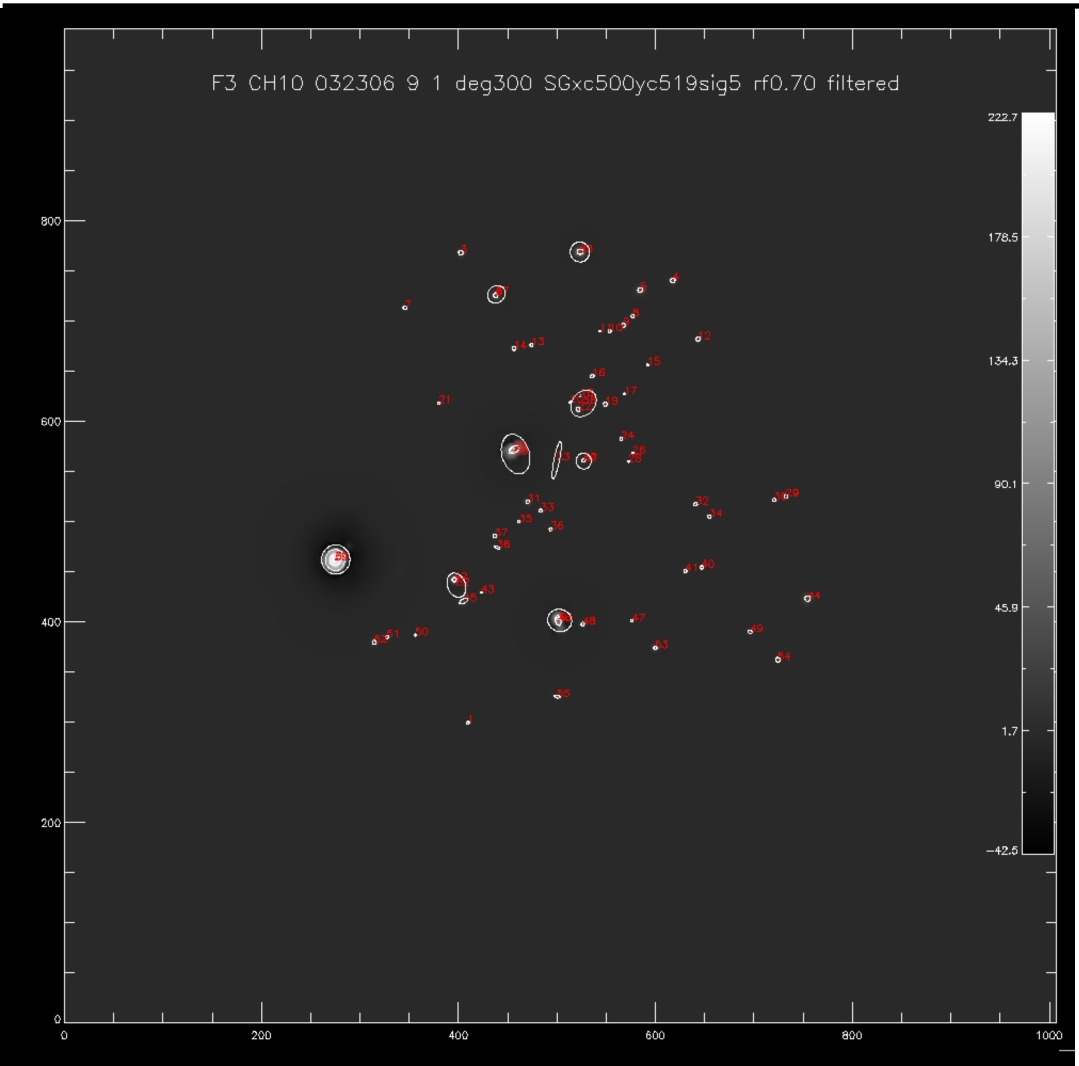
A CH Sequence of 18 Medallions around a Great Circle ~ 20% Area of the Surface of a Sphere (220 Deg)



A CH Sequence of 18 Medallions around a Great Circle ~ 20% Area of the Surface of a Sphere (260 Deg)



A CH Sequence of 18 Medallions around a Great Circle ~ 20% Area of the Surface of a Sphere (300 Deg)



A CH Sequence of 18 Medallions around a Great Circle ~ 20% Area of the Surface of a Sphere (340 Deg)

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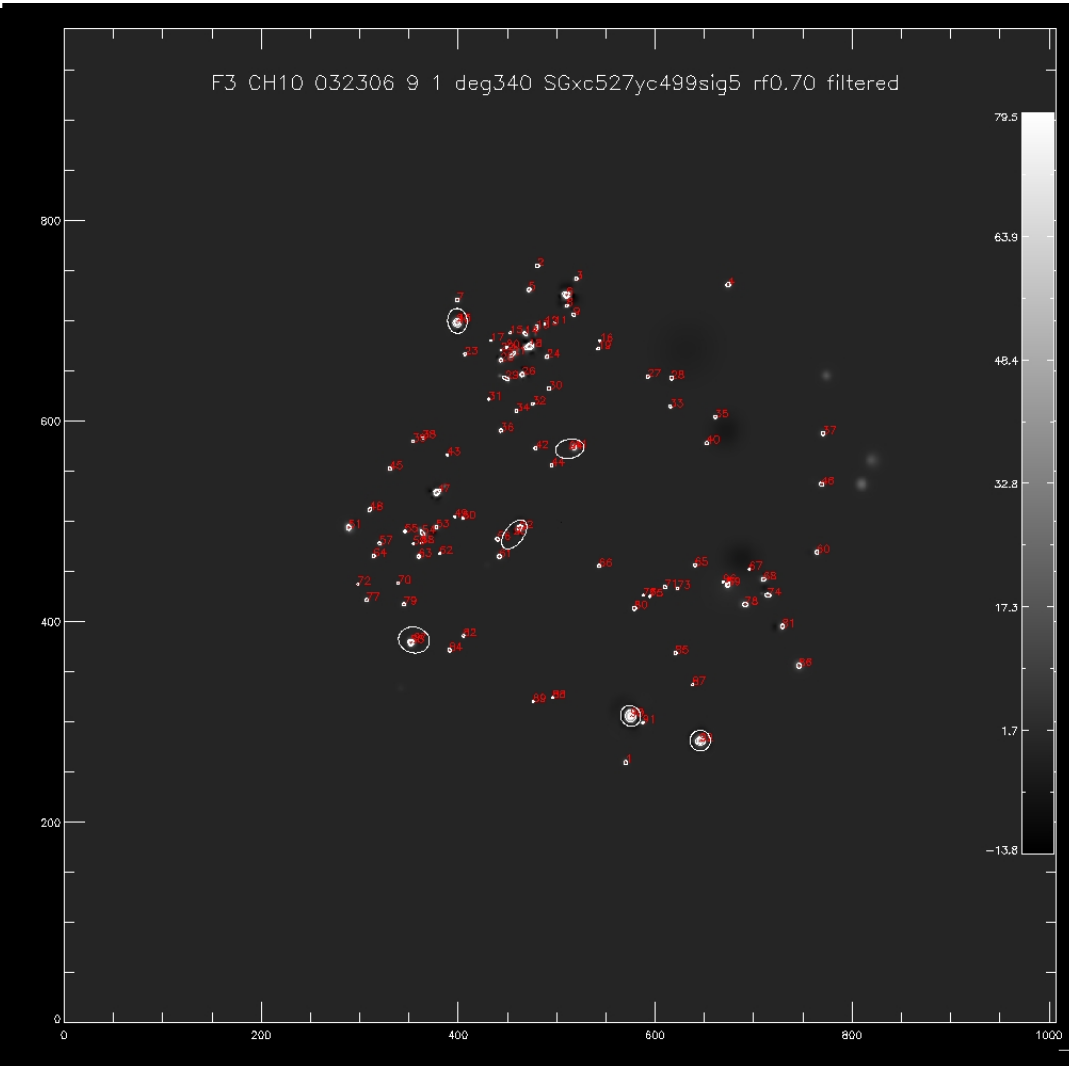
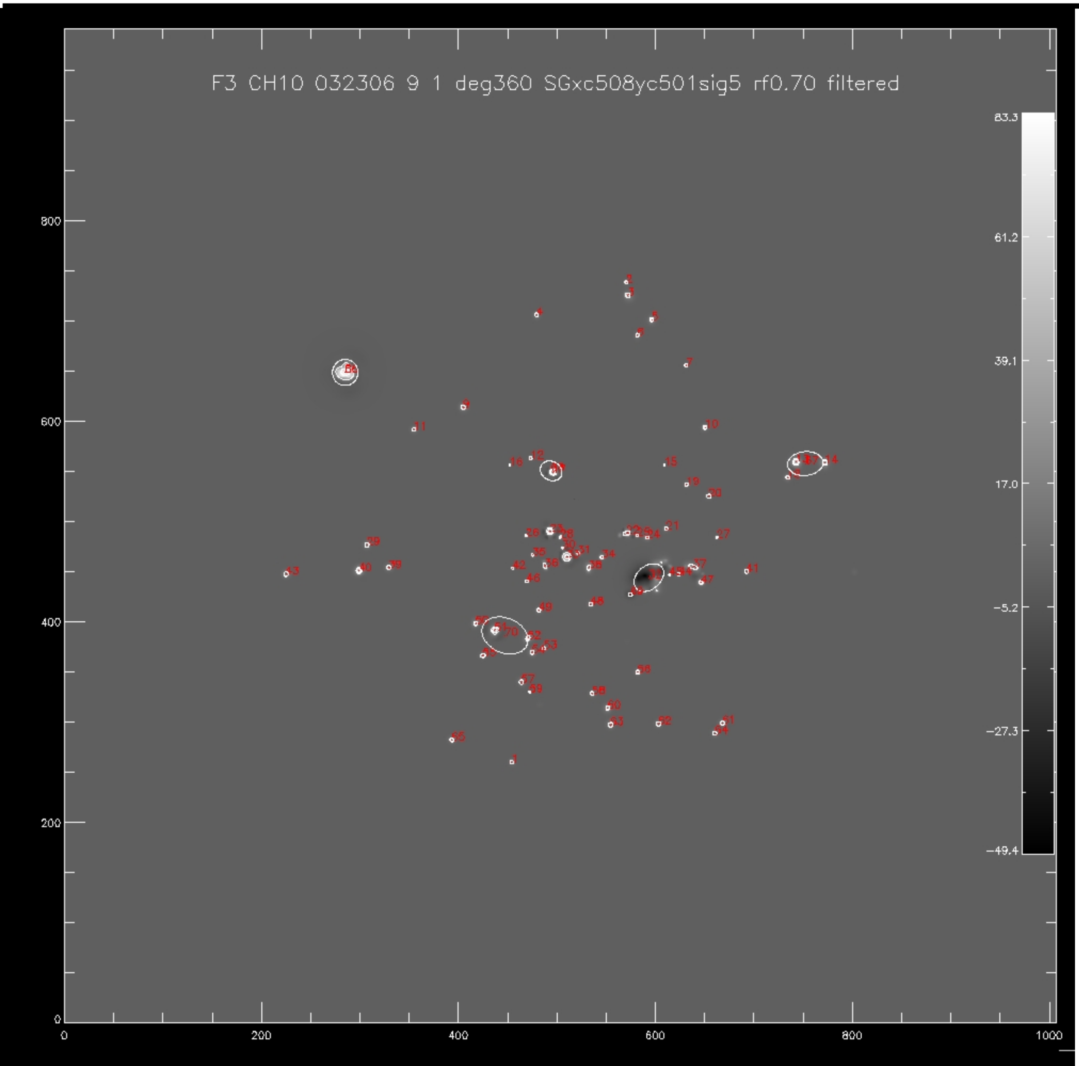
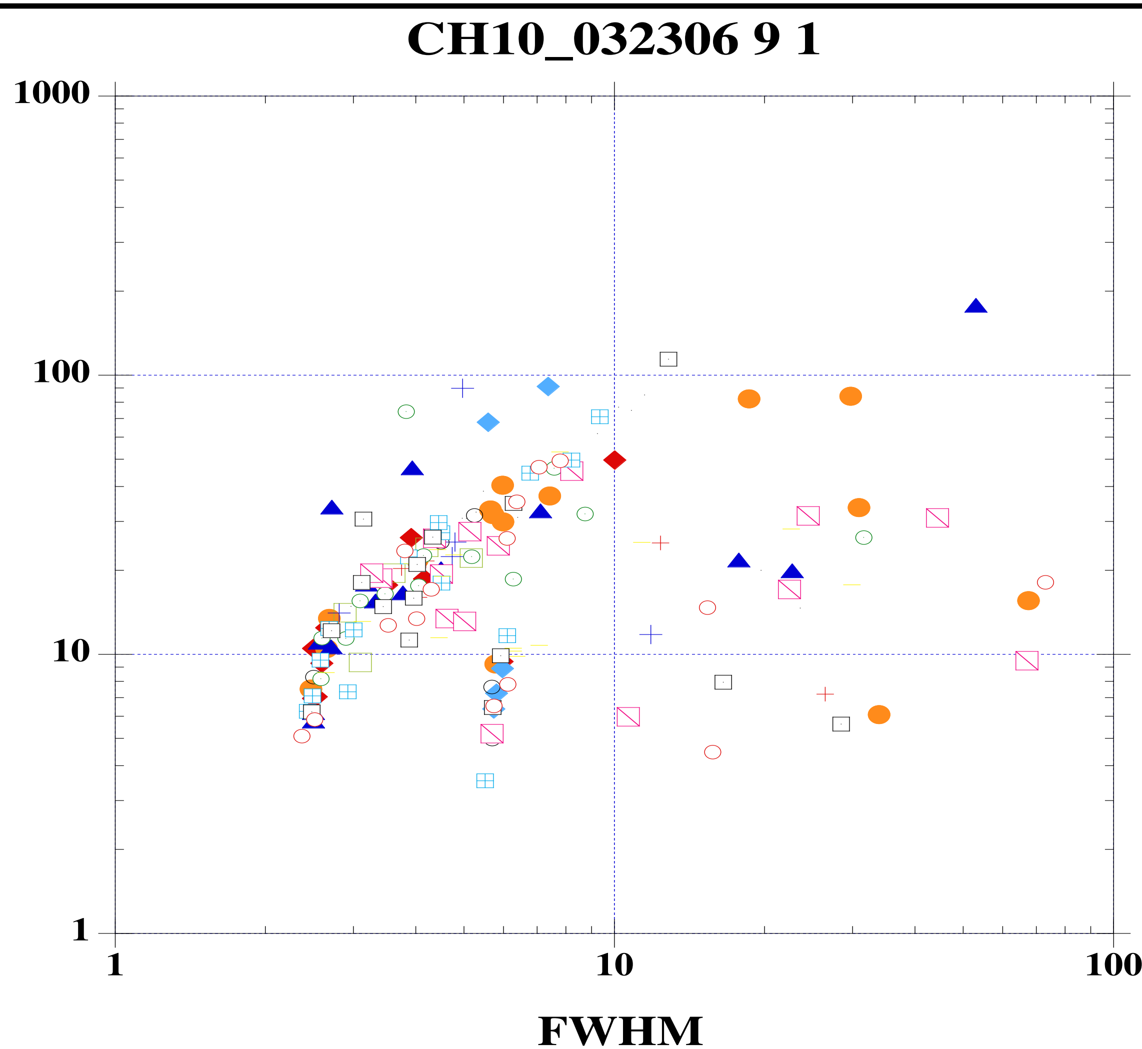


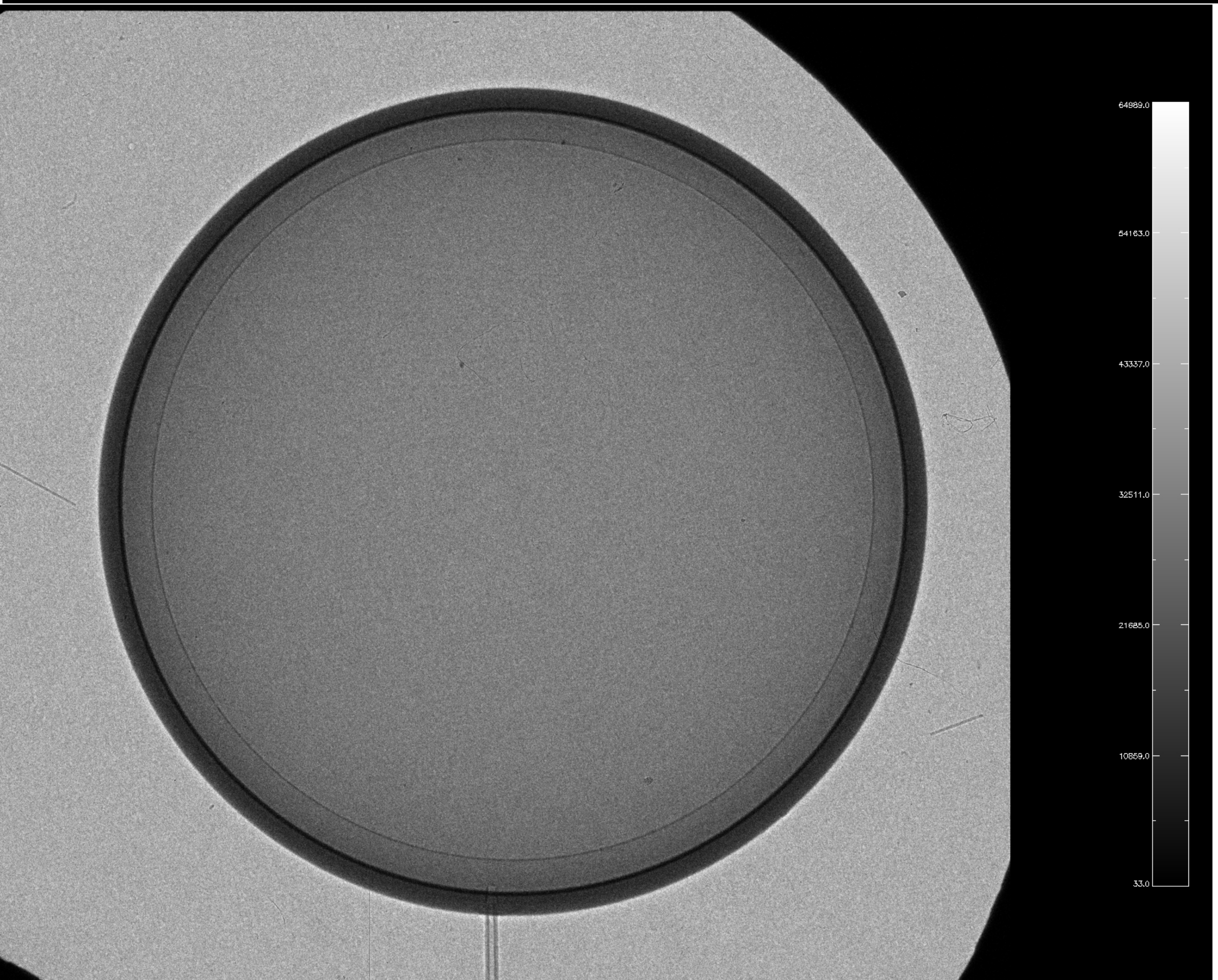
A CH Sequence of 18 Medallions around a Great Circle ~ 20% Area of the Surface of a Sphere (360 Deg)



Accumulated Statistical Distribution of Heights and Sizes of Localized Structures Around the 20% Belt

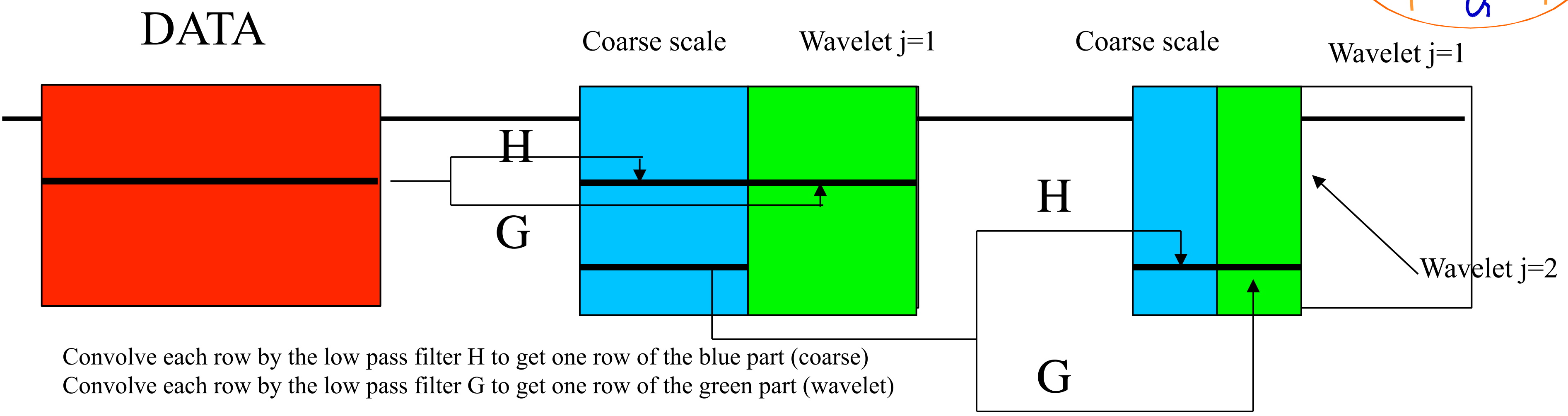


ICE Roughness: X Ray Phase Contrast Imaging Generated Single (not 200 averaged) Image



Annulet Transform: (1+1)D (bi-) orthogonal WLTs, eg. 3 scales along x, two scales along y

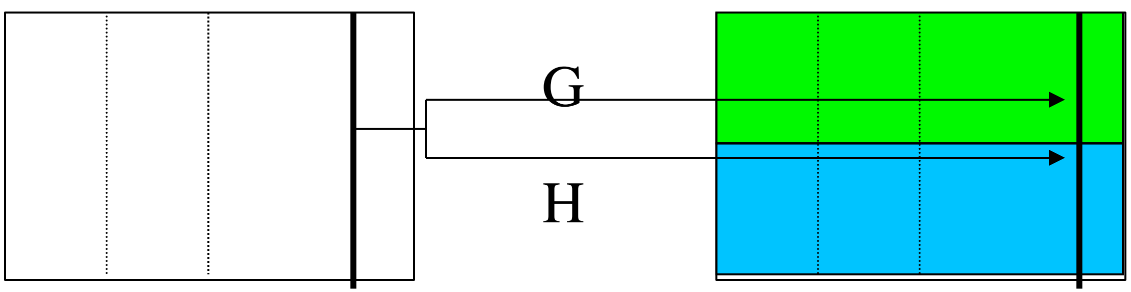
I. 1D WLT TRANSFORM ALONG ROWS



Convolve each row by the low pass filter H to get one row of the blue part (coarse)
Convolve each row by the low pass filter G to get one row of the green part (wavelet)

Repeat the same process for the next scale

II. 1D WAVELET TRANSFORM ALONG COLUMNS



OUTPUT

3,1	2,1	1,1
3,2	2,2	1,2

Convolve each column by the low pass filter H to get one column of the blue part (coarse)
Convolve each column by the low pass filter G to get one column of the green part (wavelet)

The (1+1)D WLT or Annulet Transform for Anisotropic Data

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$$\phi_{j,l}(k)=\phi\left(\frac{k-l}{2^j}\right)$$

$$\psi_{j,l}(k)=\psi\left(\frac{k-l}{2^j}\right)$$

$$P^{(1)}_{l,m}(k_x,k_y)=\phi_{J_x,l}(k_x)\phi_{J_y,m}(k_y)$$

$$P^{(2)}_{j_y,l,m}(k_x,k_y)=\phi_{J_x,l}(k_x)\psi_{j_y,m}(k_y)$$

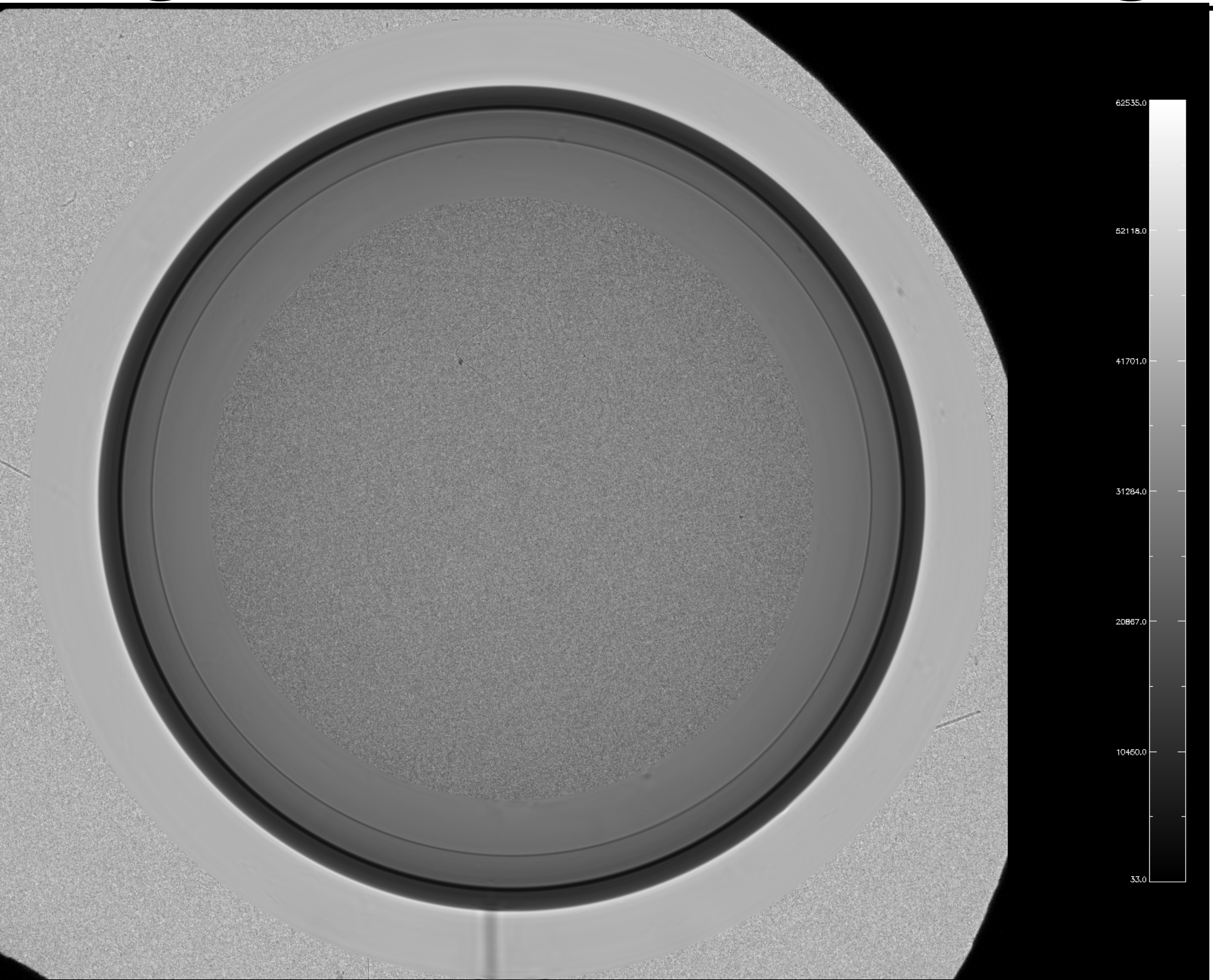
$$P^{(3)}_{j_x,l,m}(k_x,k_y)=\psi_{j_x,l}(k_x)\phi_{J_y,m}(k_y)$$

$$P^{(4)}_{j_x,j_y,l,m}(k_x,k_y)=\psi_{j_x,l}(k_x)\psi_{j_y,m}(k_y)$$

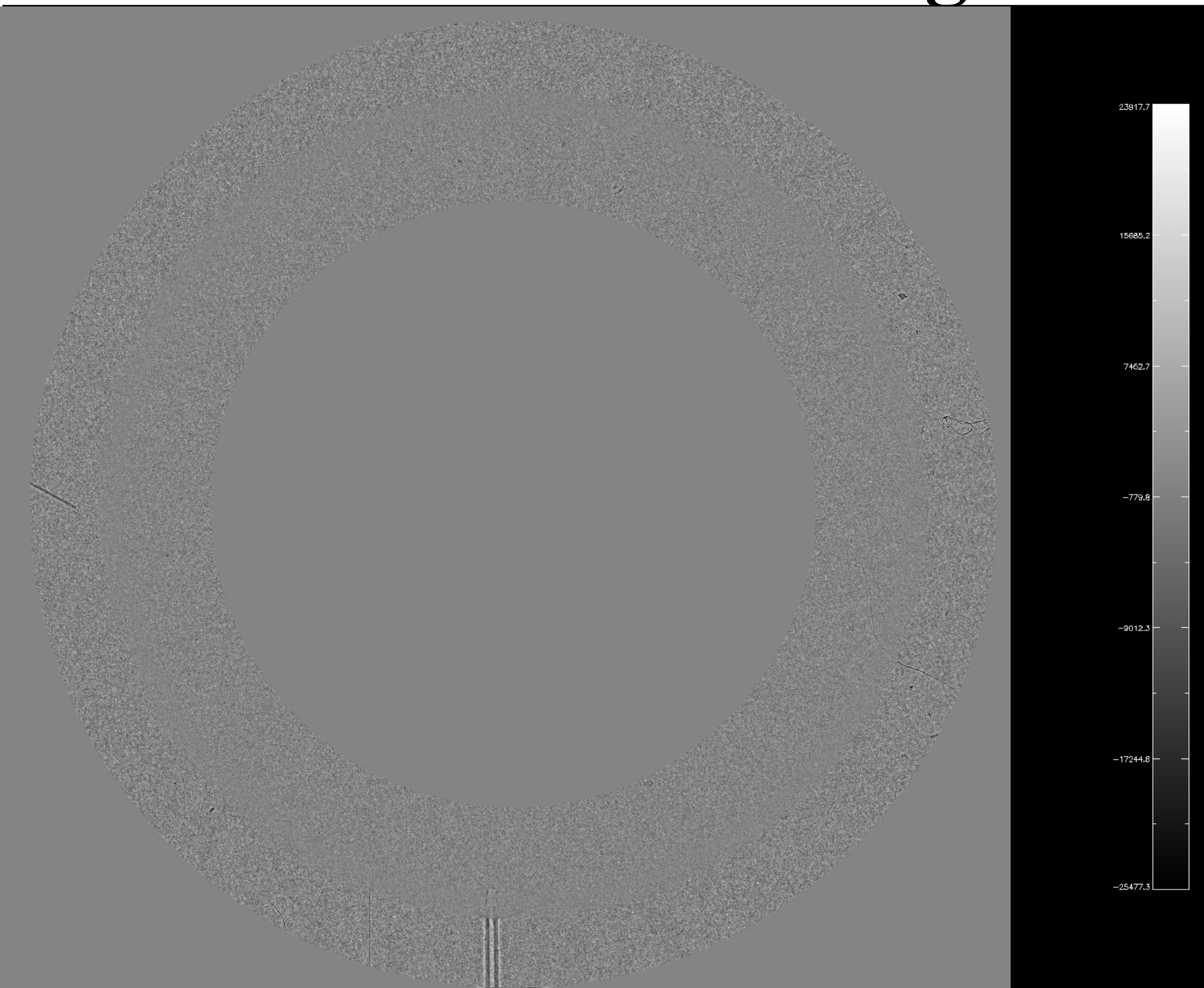
s = data, $s_{l,m}$=pixel value at position (l,m)
J_x=Number of scales in the x directions,
J_y= Number of scales in the y directions
j_x,j_y= scale index in the x and y directions,
k_x,k_y = pixel index in the x and y directions,
c = coarse scale coefficients,
w = wavelet coefficients,
ϕ = 1D scaling function,
ψ= 1D wavelet function

$$s_{l,m}=\sum_{k_y}\sum_{k_x}c_{J_x,J_y,k_x,k_y}P^{(1)}_{l,m}(k_x,k_y)+\sum_{k_y}\sum_{k_x}\sum_{j_y=1}^{J_y}w_{J_x,j_y,k_x,k_y}P^{(2)}_{j_y,l,m}(k_x,k_y)+$$
$$\sum_{k_y}\sum_{k_x}\sum_{j_x=1}^{J_x}w_{j_x,J_y,k_x,k_y}P^{(3)}_{j_x,l,m}(k_x,k_y)+\sum_{k_y}\sum_{k_x}\sum_{j_y}^{J_y}\sum_{j_x=1}^{J_x}w_{j_x,j_y,k_x,k_y}P^{(4)}_{j_x,j_y,l,m}(k_x,k_y)$$

ICE Roughness: X Ray Phase Contrast Imaging Generated Single Annulet Denoised Image



ICE Roughness: X Ray Phase Contrast Imaging Generated Single Image's Residual After Denoising



The ICE Layer (Single Image) Unwrapped, Denoised, Residual

Registration & Averaging of 1, 2, 5, 10, 20 & 80 ICE X ray PCI Images Followed By Annulet Denoising and $r_{min}(\theta)$ and $r_{max}(\theta)$ Extraction (Differences Are Sub Pixel w/o Fitting (yet))

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