Recent Advances in Gravity Sensor Array Signal Processing

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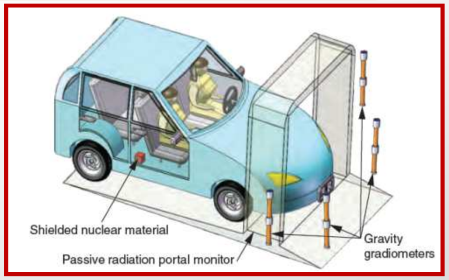
Gravity Sensor Project

Goal

Passively detect and "map" a shielded radioactive contraband in moving passenger vehicles.

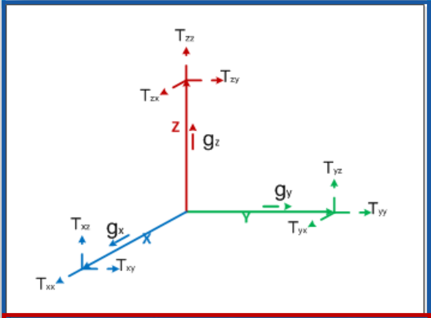


Instrument

Cold-atom interferometer based gravity gradiometer developed by AOSense, Sunnyvale

Measurement

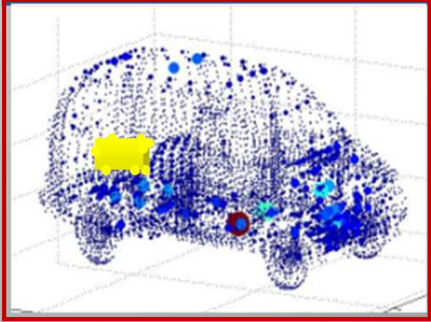
Changes in local gravitational acceleration **g** and its gradient (tensor) caused by the movement of massive objects across the sensor array



Signal Processing Problem

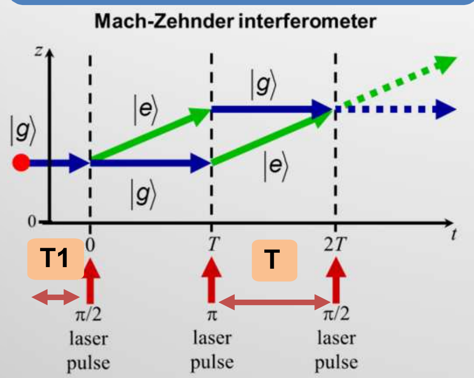
Detection: Detect whether an additional mass is present in vehicles. Ensure $P_D > 90\%$ and $P_{FA} < 10^{-3}$

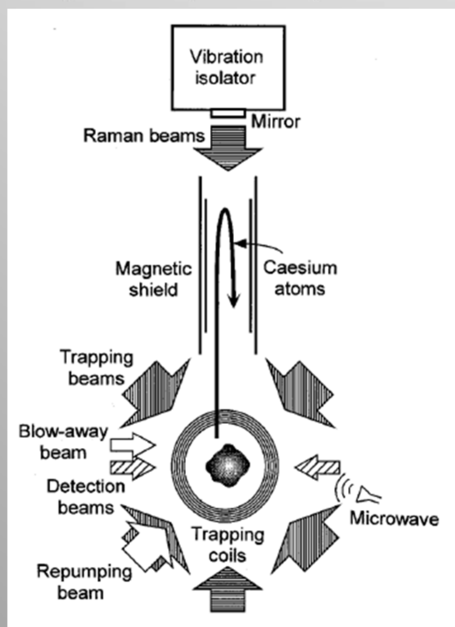
Estimation: Locate the mass within the vehicle and estimate its magnitude.



Instrument: Basic Gravimeter / Gradiometer Configuration

Gravimeter





Laser cool atoms to ~µK and load into a MOT

Launch atoms

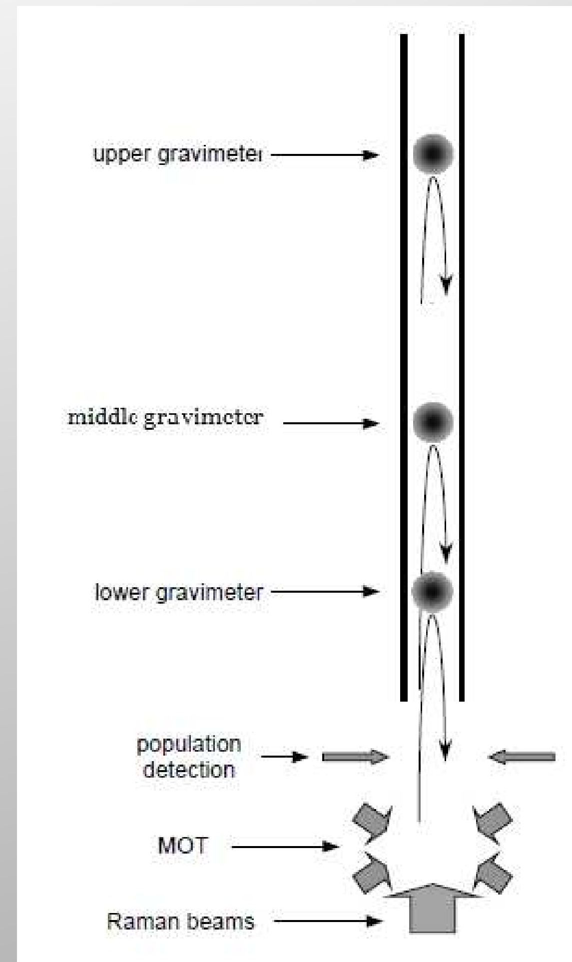
Put all atoms into the same initial quantum state

Start interferometer pulse sequence.

Measure interferometer phase shift

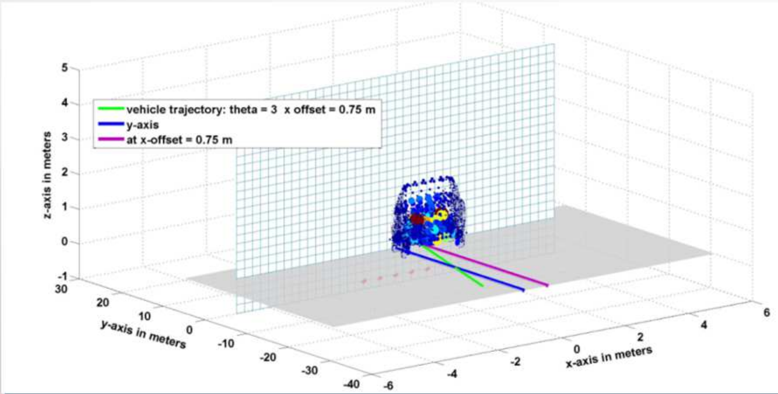
$$\Delta\phi_g = k_{eff} g T^2$$

Gradiometer



A. Peters, K. Y. Chung and S. Chu, *High-precision gravity measurements using atom interferometry*, Metrologia, 2001, 38, 25-61

Measurement: Gradiometer Response



Gradiometer response to a vehicle moving along a trajectory with parameters $\{\boldsymbol{v}_a, \theta, \delta x, t_0\}$ is modeled as a sum of responses to a set of point masses $\{m_i\}$ at positions $\{\boldsymbol{r}_i\}$ at time t

$$\mathbf{F}_a\left(\mathbf{r}-\mathbf{r}_a(t)+\Delta\mathbf{r}_c\right) = -\sum_{i=1}^{M} \frac{Gm_i\left(\mathbf{r}-\mathbf{r}_a(t)+\Delta\mathbf{r}_c\right)}{\left|\mathbf{r}-\mathbf{r}_a(t)+\Delta\mathbf{r}_c\right|^3}$$

$$\Delta\mathbf{r}_c = f\left(v_a, T_1, T, \mathrm{v}\right)$$

$\mathbf{r}_a(t)$ point mass location in sensor coordinates

M total number of point masses

$$\Delta\varphi(\mathbf{r},t) \propto \mathbf{F}_a$$

$$J_{\Delta\varphi}(\mathbf{r},t) = \Delta\varphi(\mathbf{r}+2\Delta\mathbf{r},t) - 2\Delta\varphi(\mathbf{r}+\Delta\mathbf{r},t) + \Delta\varphi(\mathbf{r},t)$$

3 gravimeter locations : $\mathbf{r}$, $\mathbf{r}+\Delta\mathbf{r}, \mathbf{r}+2\Delta\mathbf{r}$

Gravimeter Model Parameters

Atom cloud launch velocity v m/s upward

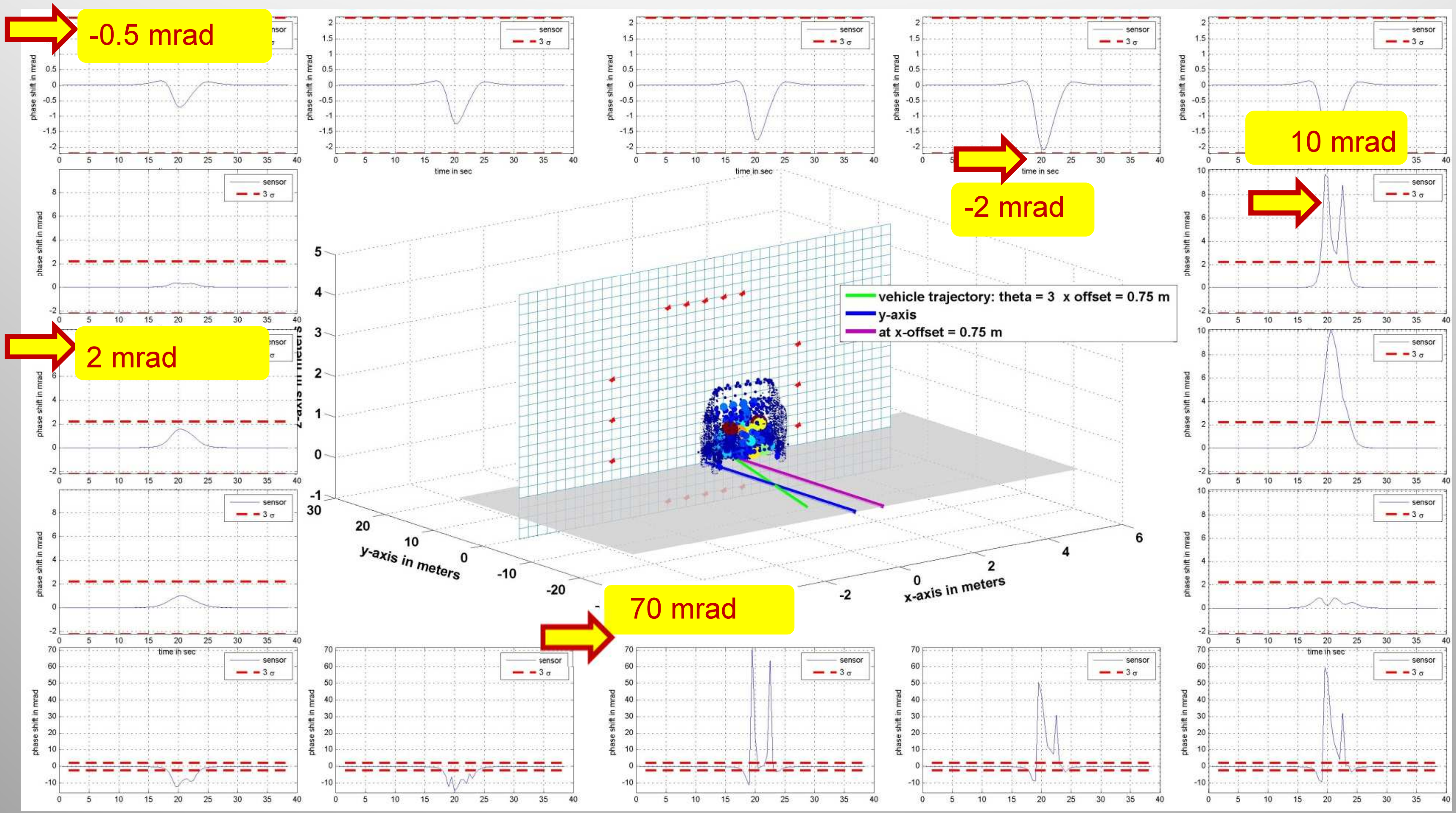
Sensor interrogation time T sec

Vehicle velocity: Known

Vertical sensors

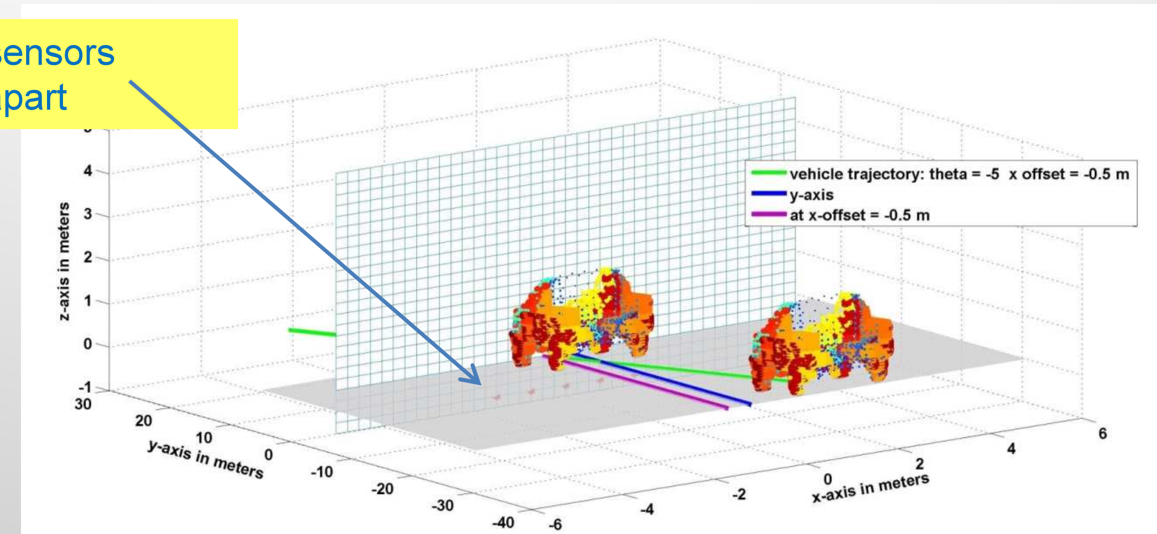
Measurement: Basic portal configuration

Work on portal configurations with sensors on the sides and top showed most significant signals come from lower (buried) sensors



Measurement: Portal Configuration and Vehicle Trajectory

Linear array of five sensors spaced 0.5 meters apart



Assume known vehicle moves past array at constant speed < 5 MPH

Parameters describing trajectory of vehicle are

- θ - Angle between vehicle path and array normal
- δX - Offset between vehicle path and array center
- t_0 - Time when origin of mass co-ordinates crosses the portal x-axis
- v – Constant velocity of vehicle

In principle, these parameters could be estimated from sensor data

*Gravity Gradiometer Signal Processing: **Earlier Work***

$$\mathbf{F}_a(\mathbf{r}-\mathbf{r}_a(t)+\Delta\mathbf{r}_c) = -\sum_{i=1}^{M} \frac{Gm_i(\mathbf{r}-\mathbf{r}_a(t)+\Delta\mathbf{r}_c)}{|\mathbf{r}-\mathbf{r}_a(t)+\Delta\mathbf{r}_c|^3}$$

$\Delta\mathbf{r}_c = f(v_a, T_1, T, \mathrm{v})$

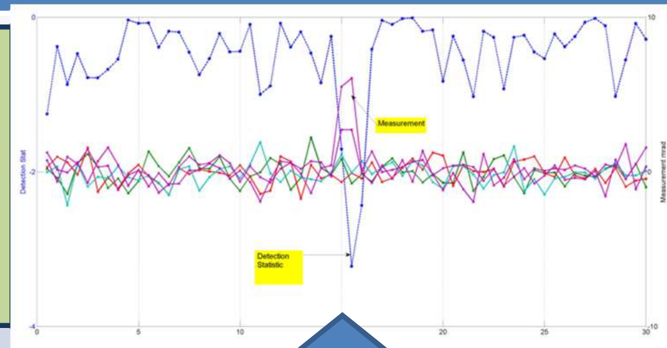
$\mathbf{r}_a(t)$ point mass location in sensor coordinates

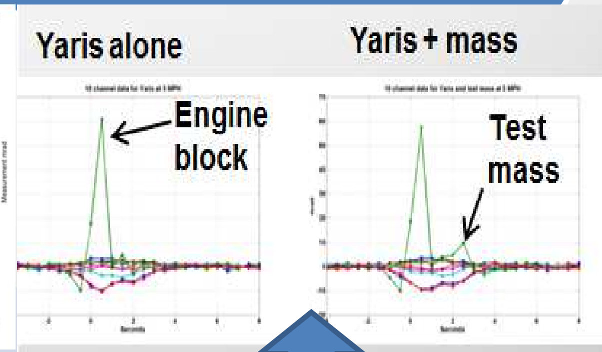
M total number of point masses

$\Delta\varphi(\mathbf{r},t) \propto \mathbf{F}_a$

$J_{\Delta\varphi}(\mathbf{r},t) = \Delta\varphi(\mathbf{r}+2\Delta\mathbf{r},t) - 2\Delta\varphi(\mathbf{r}+\Delta\mathbf{r},t) + \Delta\varphi(\mathbf{r},t)$

3 gravimeter locations: $\mathbf{r},\ \mathbf{r}+\Delta\mathbf{r}, \mathbf{r}+2\Delta\mathbf{r}$





Built physics model of the cold-atom gravity gradiometer

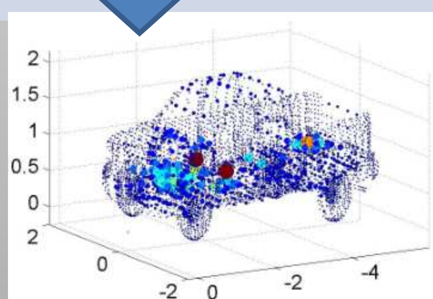
Constructed FEM for several passenger vehicles

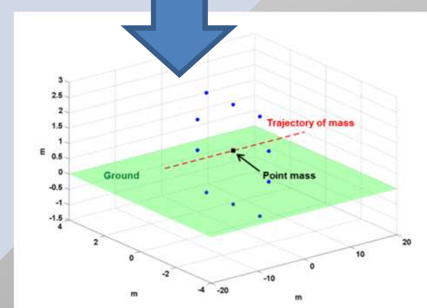
Formulated detection statistics

(sliding window ANOVA)

Demo' d the ability to detect 50Kg point mass moving at < 5MPH

Tested point masses combined Toyota Yaris





*Gravity Gradiometer Signal Processing: **Recent Advances***

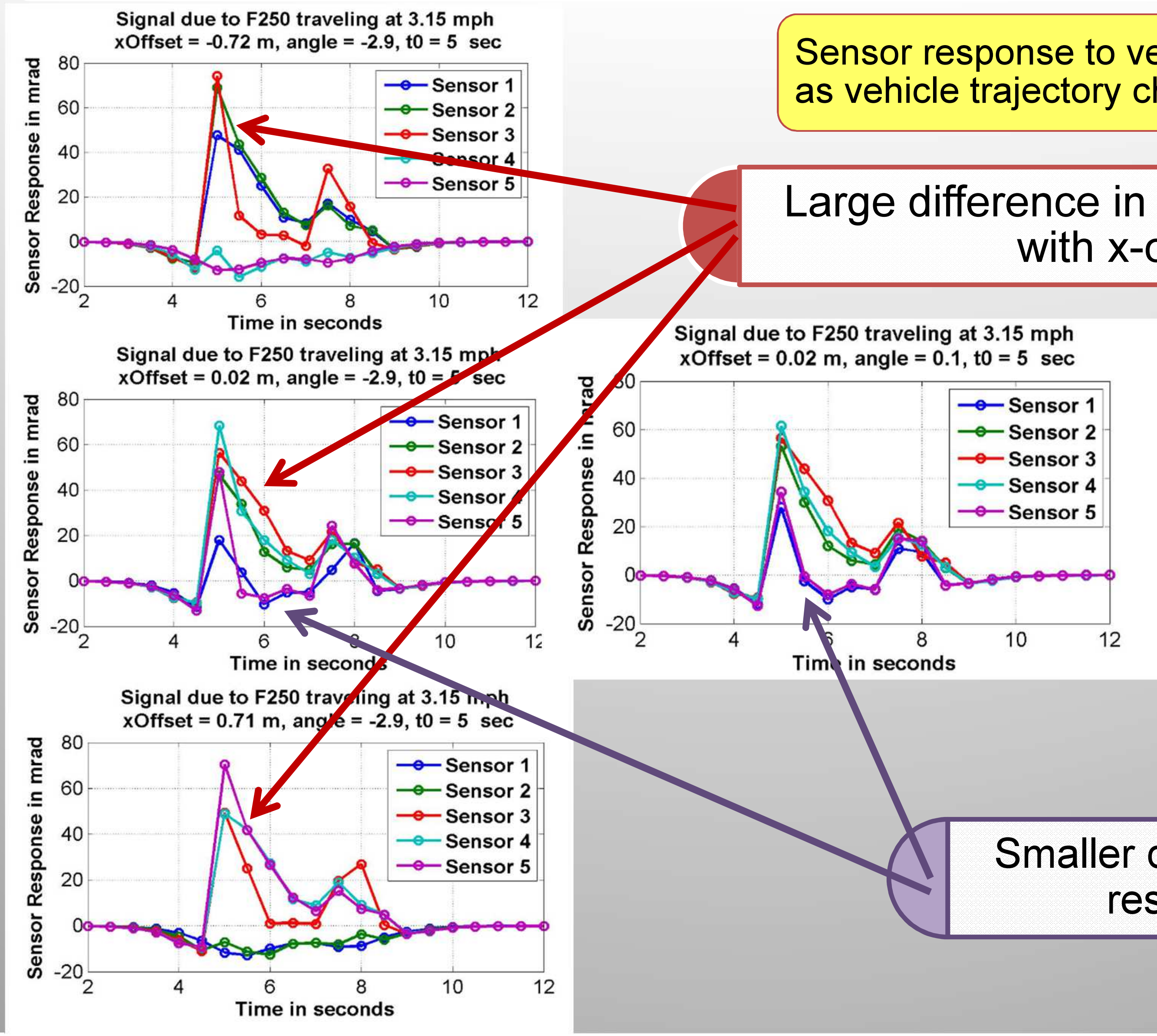
Extract threat mass signal by removing sensor response to vehicle

Detect 50 kg mass with P_D>95% and P_{FA}<10^{-3} for all 8 vehicle models tested

Locate mass to within 5% of vehicle length and width, and ~10% of vehicle height

Estimate mass to ~20%

Estimating Vehicle Trajectory: Motivation



Sensor response to vehicle changes as vehicle trajectory changes

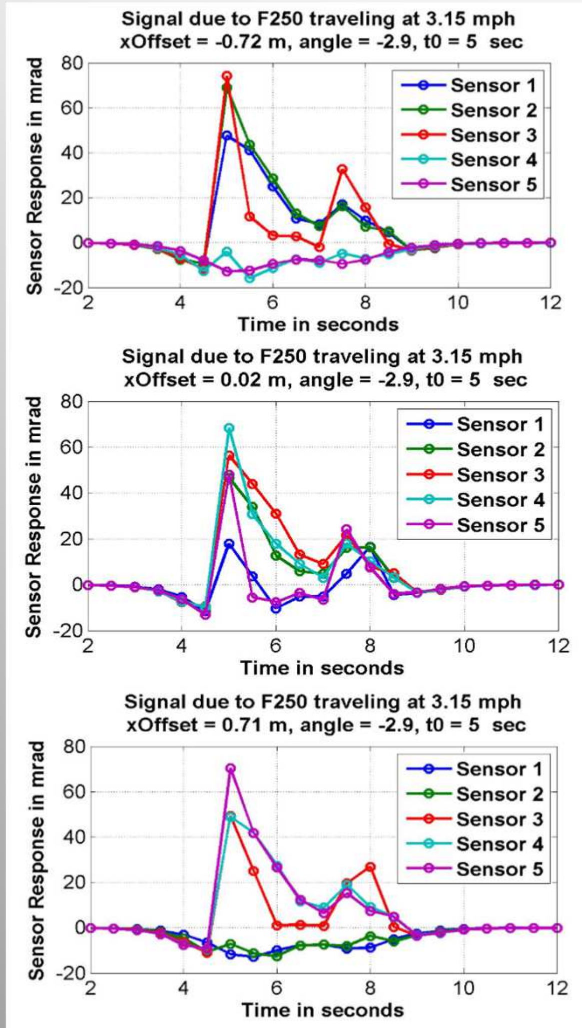
Large difference in sensor response with x-offset

Smaller difference in sensor response with θ

Estimating Vehicle Trajectory:
How well do we need to estimate the vehicle trajectory? (1/2)

Vehicle (F250)

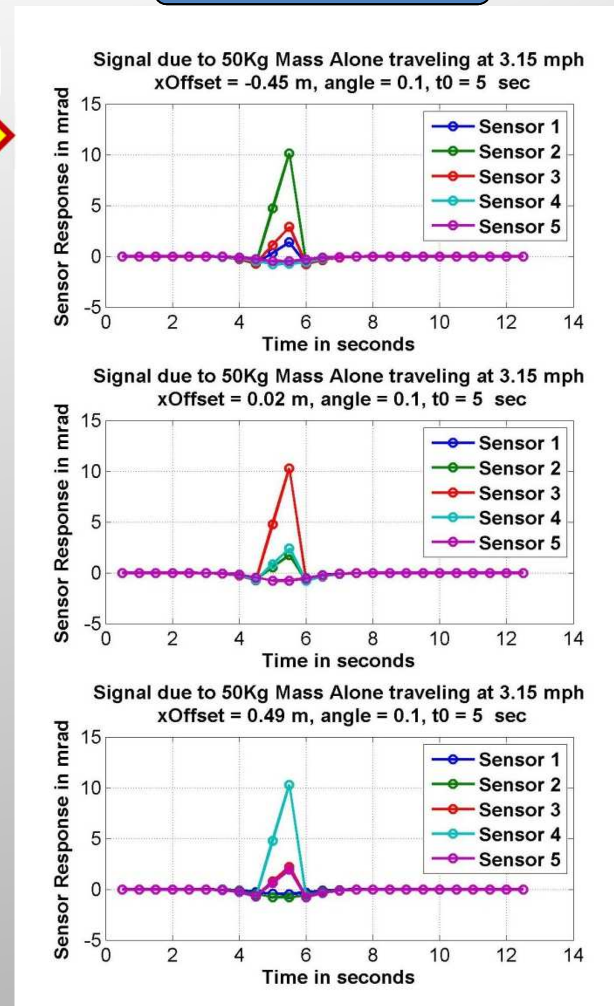
50 kg mass



80mrad

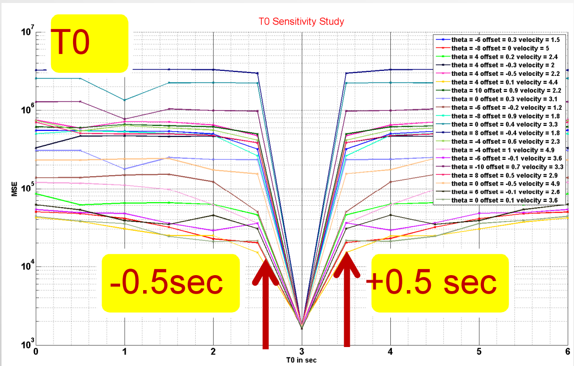
10 mrad

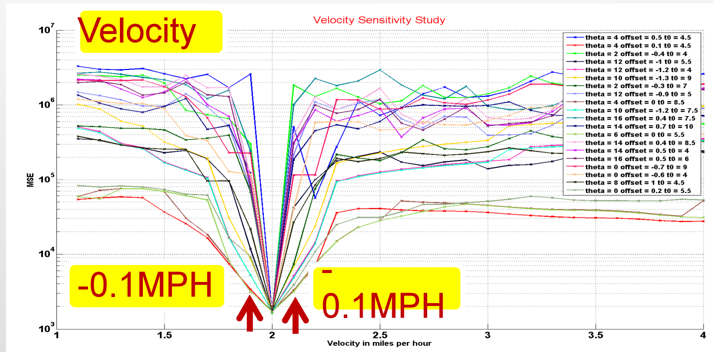
Response to vehicle is ~8X more than response to a 50 kg mass

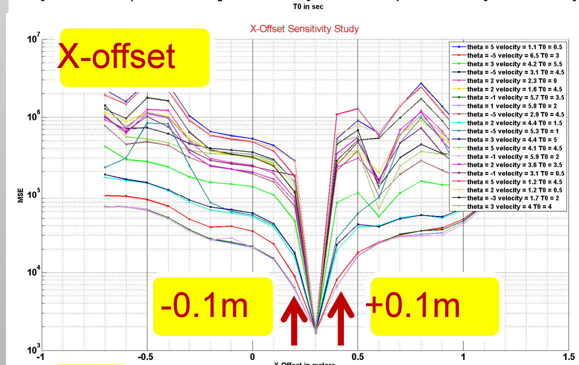


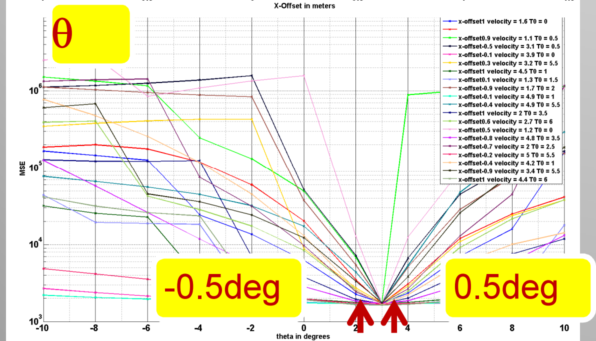
Need to calculate response to vehicle to better than 10%

Estimating Vehicle Trajectory:
How well do we need to estimate the vehicle trajectory? (2/2)









MSE is very sensitive to velocity and T0 errors

- 4D search space
- Multiple Local Minima

Estimating all 4 trajectory parameters is computationally challenging

- Optimization algorithms do not guarantee the best solution
- Template matching requires either ~250MB of storage for each vehicle type or impractical CPU requirements

Estimation of Vehicle Trajectory: 4D to 2D

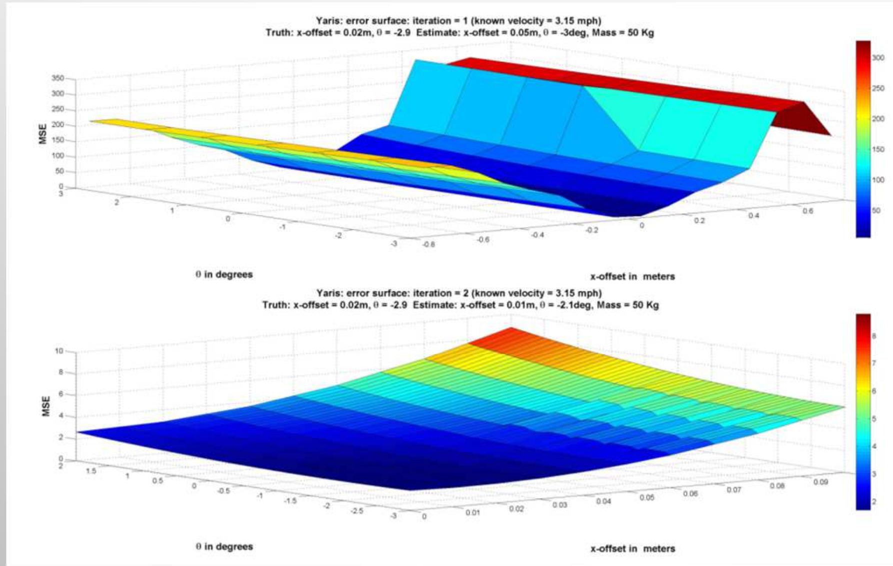
Simplify 4D Trajectory Parameter Estimation Problem to 2D

- Shift sensor data to known time origin (when the signal exceeds 5 σ of instrument noise RMS)
- Obtain velocity from an alternate sensor to within 0.1MPH
- Estimate θ and X-offset using MMSE with $-3° < \theta < 3°$, $-0.75m < x < 0.75m$

What does it does it take solve 2D parameter estimation problem?

- Template matching to generate all possible trajectories and sensor responses
- Typical number of templates for each vehicle = 105 ($-3° < \theta < 3°$, $-0.75m < x < 0.75m$, $\Delta\theta = 1°$, $\Delta x = 0.05m$)
- Typical time taken to estimate θ and X-offset using MMSE is ~2 minutes in Matlab

Estimate Vehicle Trajectory via Template Matching: Quadratic Error Surface of Solution Neighborhood



Vehicle entering at θ= -2.9° and x-offset=0.02m.

Coarse template matching leads to solution neighborhood θ= -3° and x-offset=0.05m.

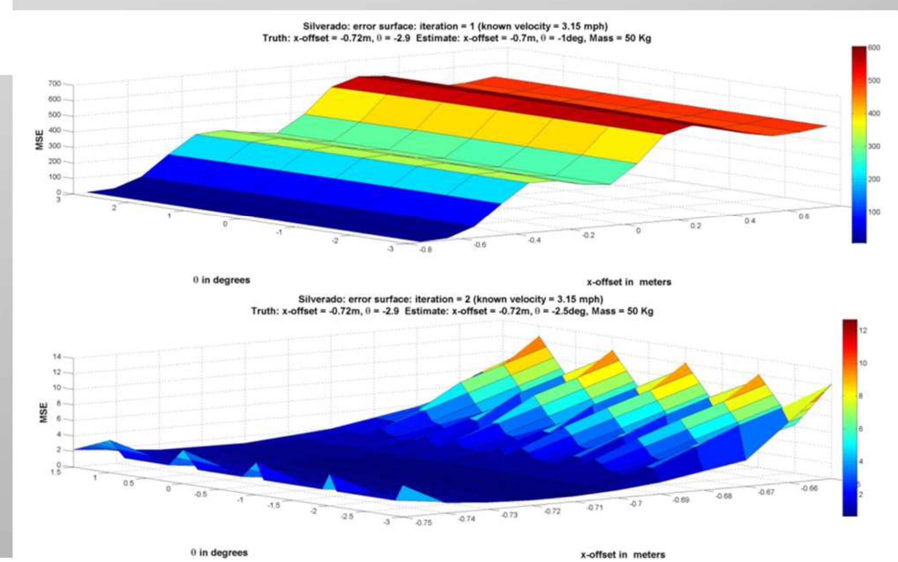
Fine matching within the solution neighborhood leads to the closest match.

Vehicle entering at θ= -2.9° and

x-offset = -0.72m.

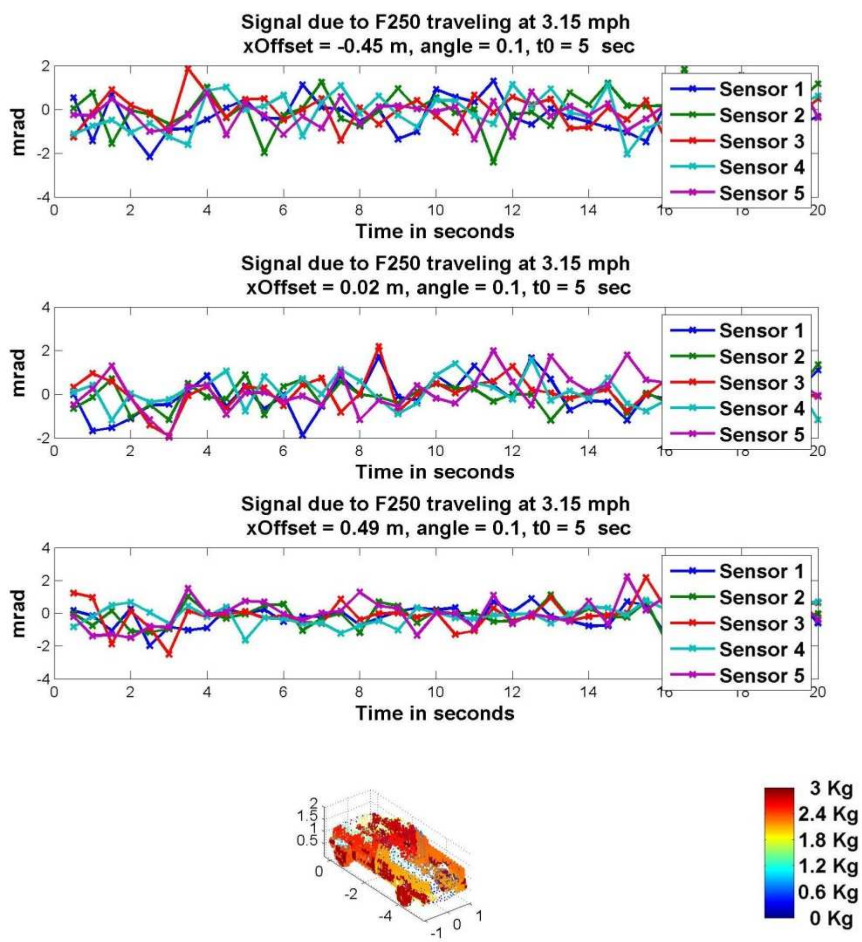
Coarse template matching leads to solution neighborhood θ= -1° and x-offset= -0.7m.

Fine matching within the solution neighborhood leads to the best solution.

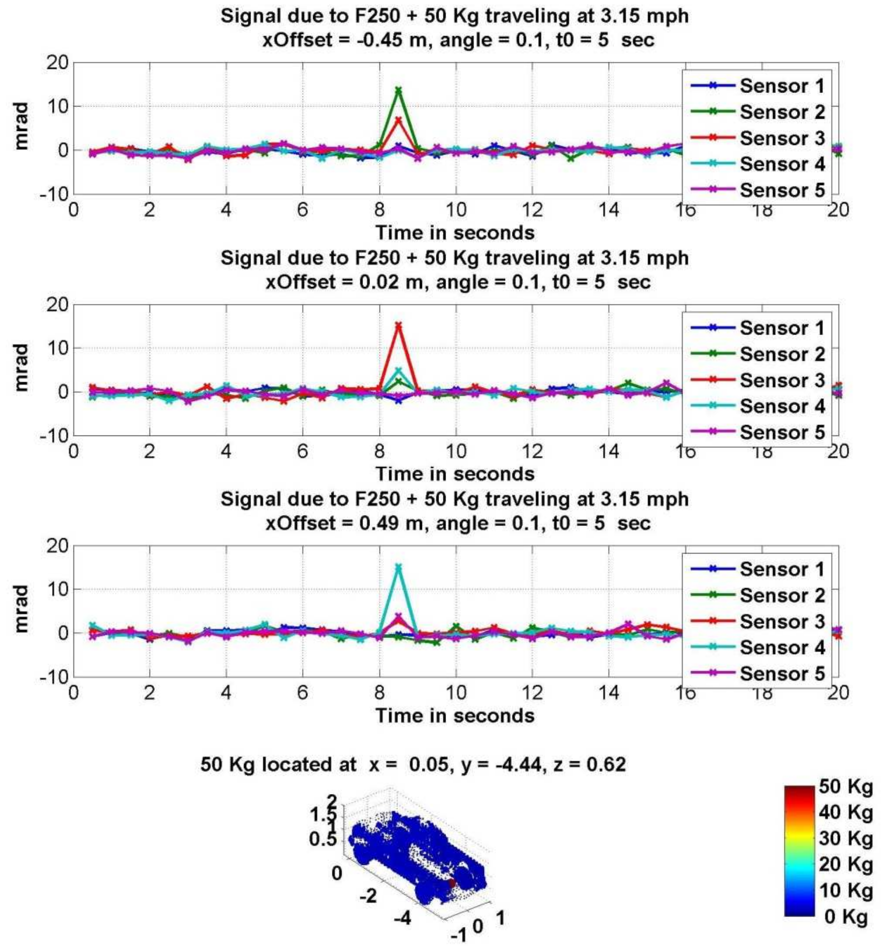


Extract Point Mass Signal from Residual

Residual when no mass is present



Residual when a 50 Kg mass is present



Detect Mass: Detection Statistic Computation

Residual Signal Characteristics

- Exact form of threat mass signal is unknown
- Signal duration is very short (2 samples or less for 50 Kg Mass)

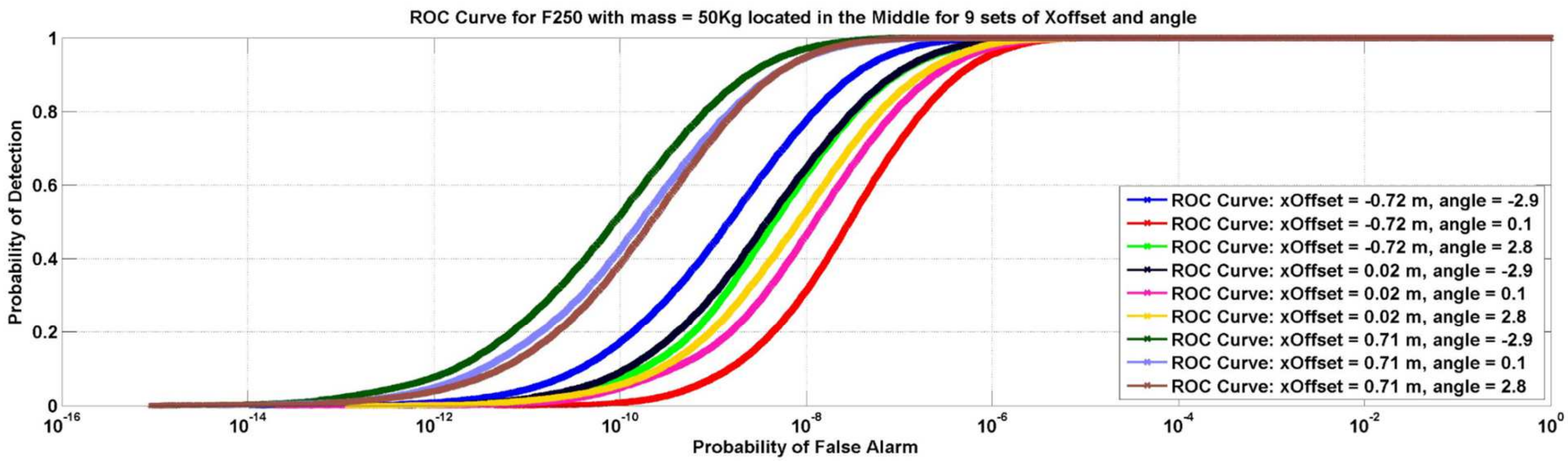
Noise Characteristics

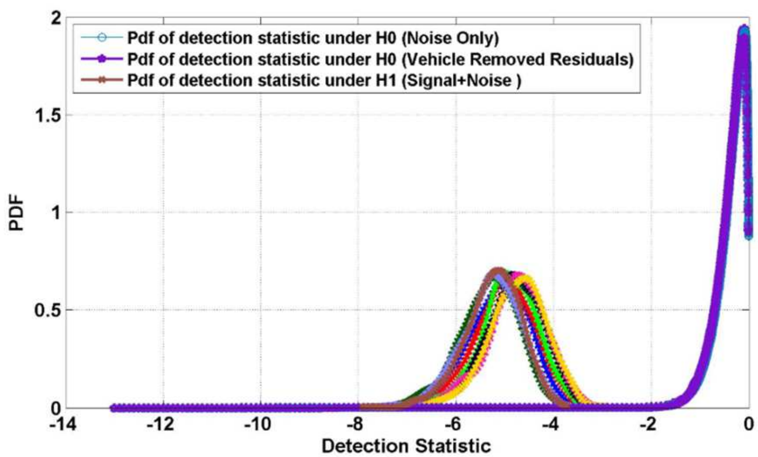
- All channel measurements are independent and have almost equal variance
- Noise from all channels is approximately Gaussian

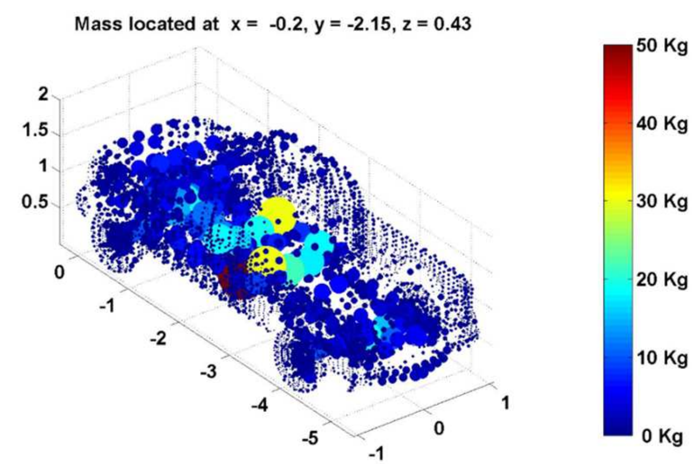
Detection Statistic Computation

- Use a sliding window one-way ANOVA to compute the F-ratio. F ratio and signal-to-noise power ratio are related by $F = 1 + P_{SNR}$
- Apply a monotonic transformation to computed F-ratio followed by a log transform.

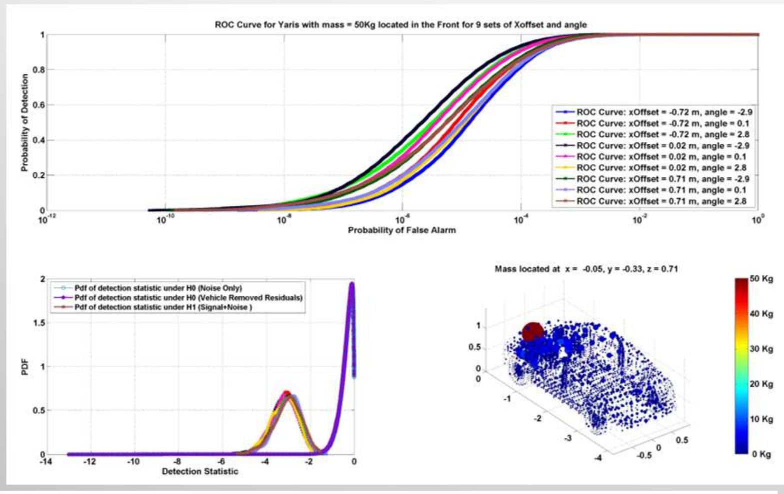
Mass Detection Performance: ROC Curves (1/2)

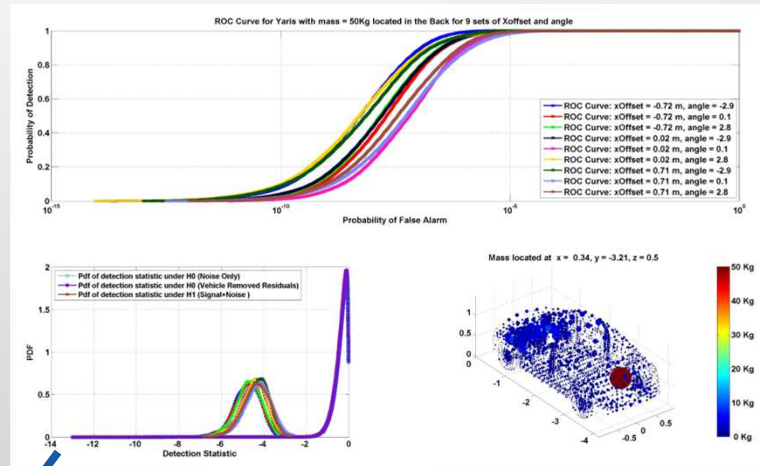


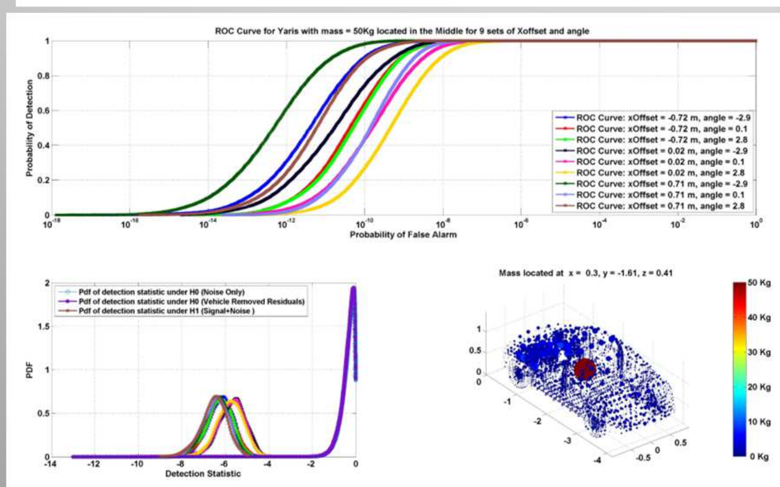




Mass Detection Performance: ROC Curves (2/2)





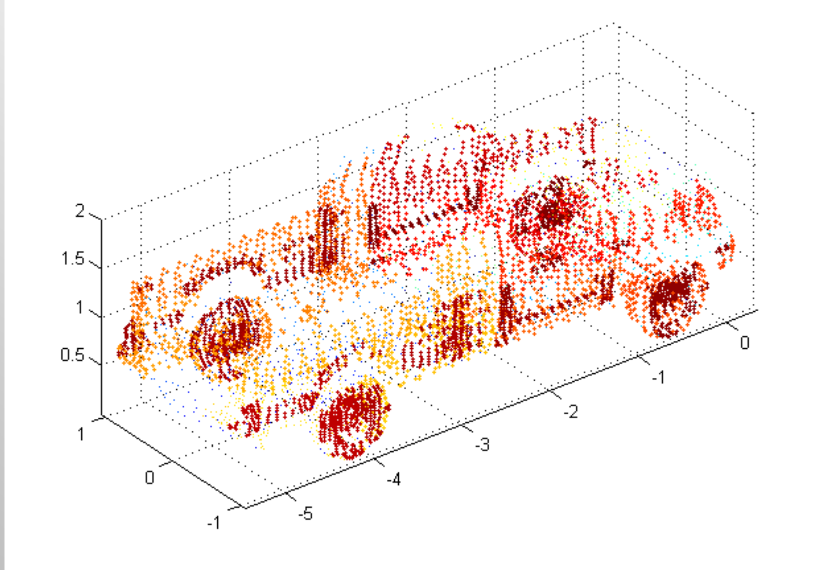


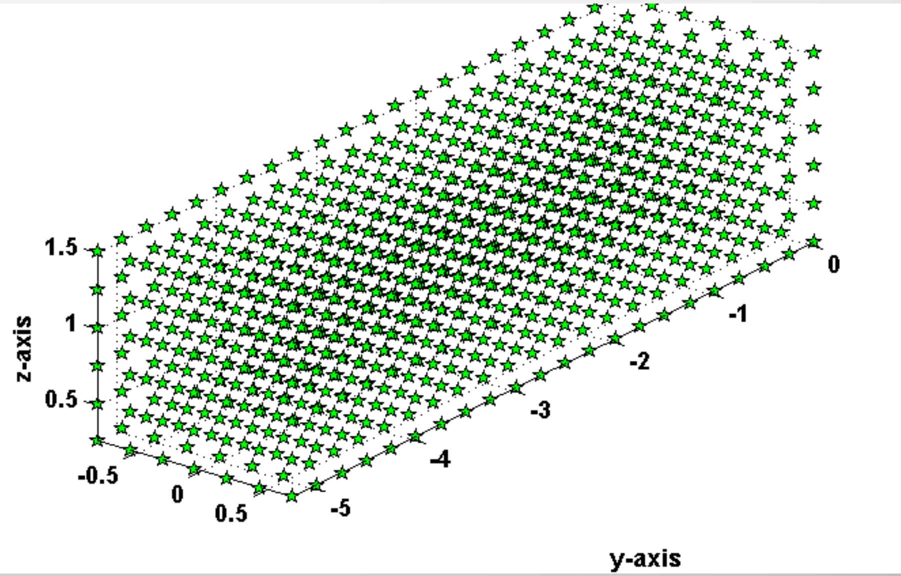
ROC curves for Yaris with 50 Kg located front, back, and middle of vehicle at different heights

ROC curves for 8 different vehicles (Silverado, F250, Rav, Caravan, Taurus, Neon, Metro, and Yaris) for 9 different trajectories and 3 different positions (front, middle, and back) of a 50 Kg mass (a total of 216 ROC curves (each requiring 1000 Monte Carlo runs) were generated.

ROC curves show PD > 0.95 for a PFA < 10^{-3}.

Estimate Mass Location, Magnitude from Residual Signal (1/4)



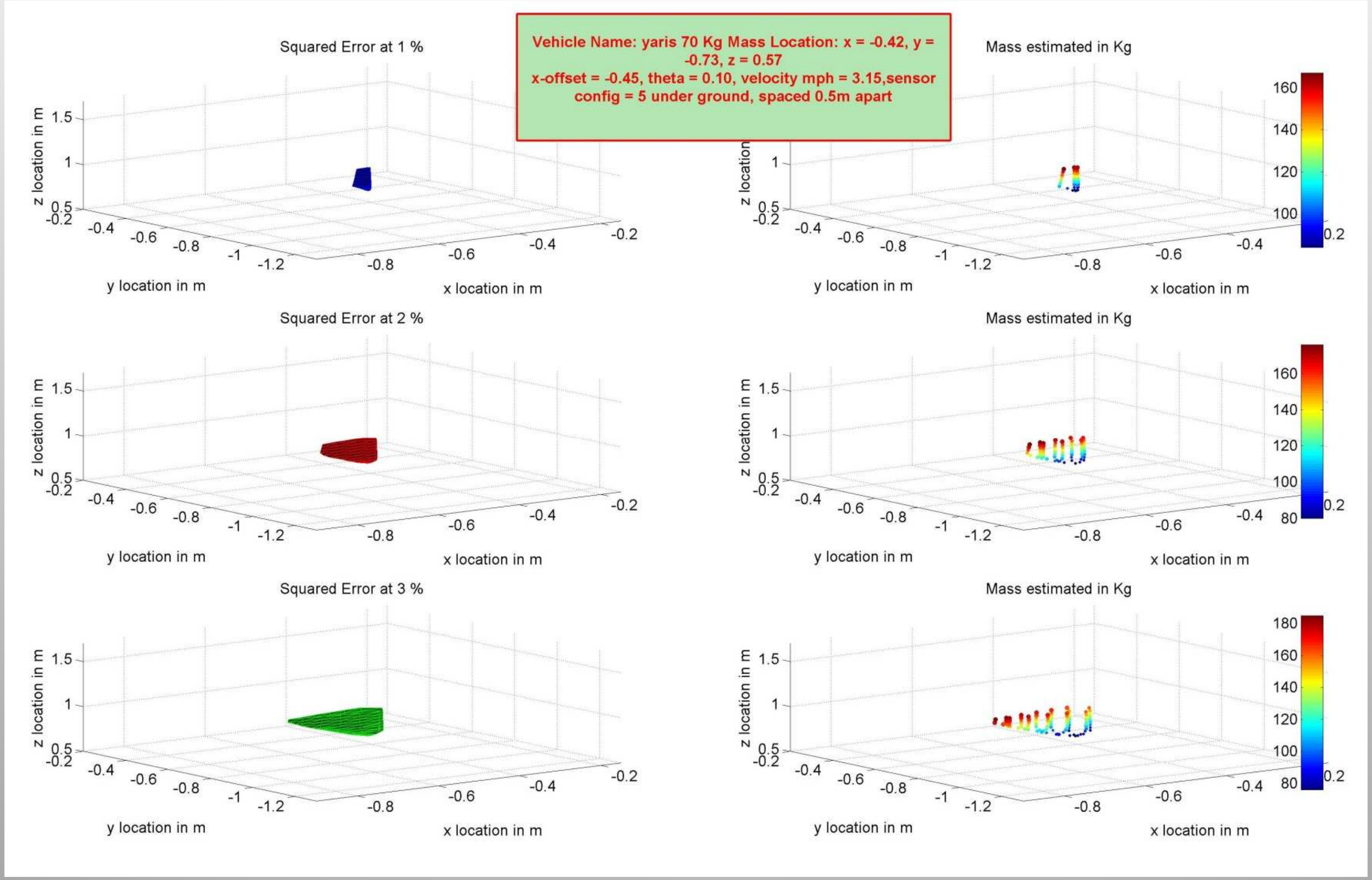


Generate gravity sensor response templates for a 50 Kg mass at grid points

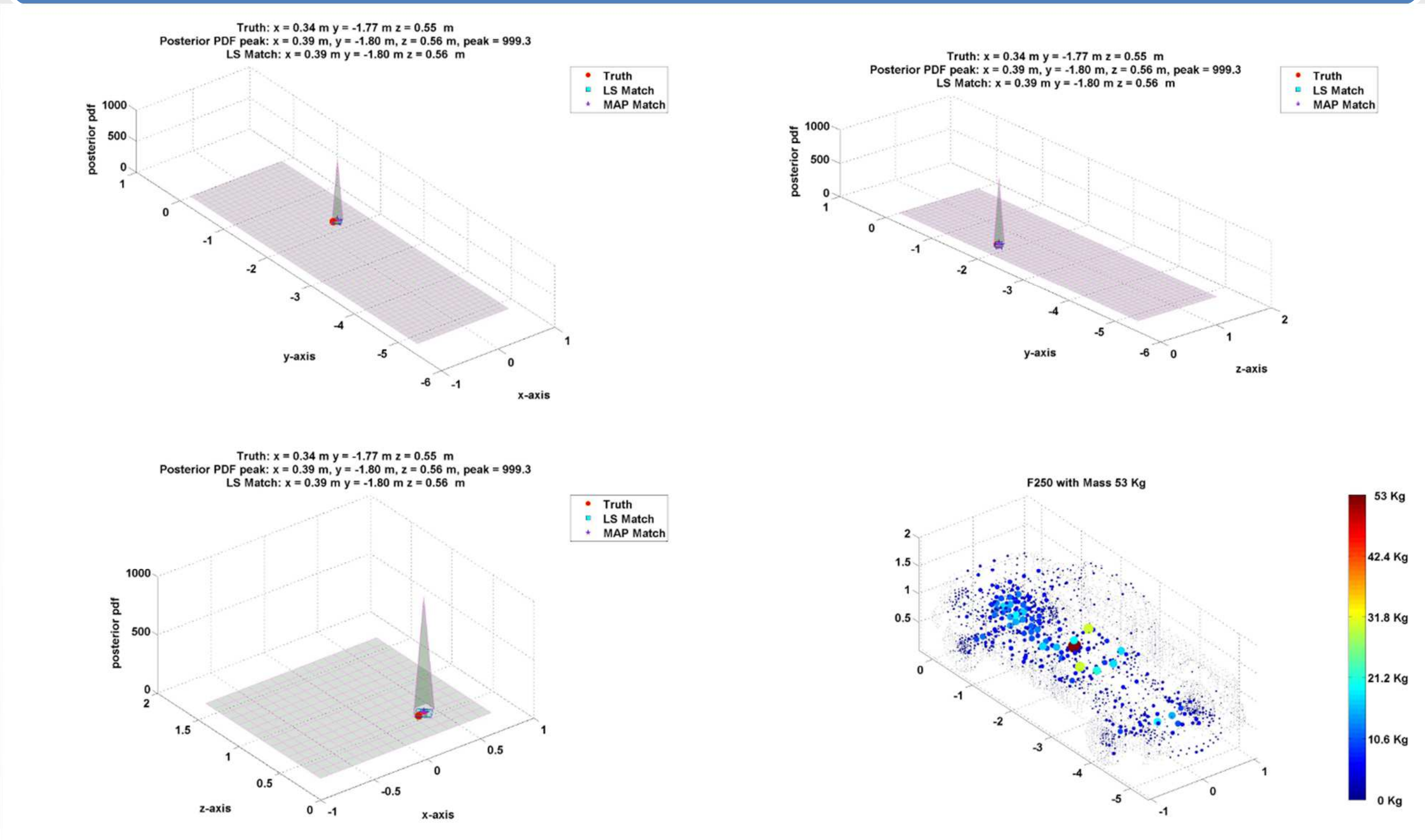
Estimate mass that would have generated measured residual by LS as $\hat{M} = M_0 \dfrac{(\mathbf{t}_{x,y,z} \bullet \mathbf{d})}{(\mathbf{t}_{x,y,z} \bullet \mathbf{t}_{x,y,z})}$

Find the best match template and report its x, y, z and scaled mass

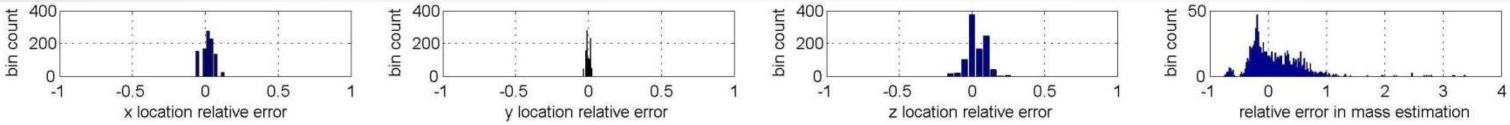
Mass estimation: Uncertainty in Height / Mass Estimation (2/4)



Mass estimation: Uncertainty in Height / Mass Estimation (3/4)



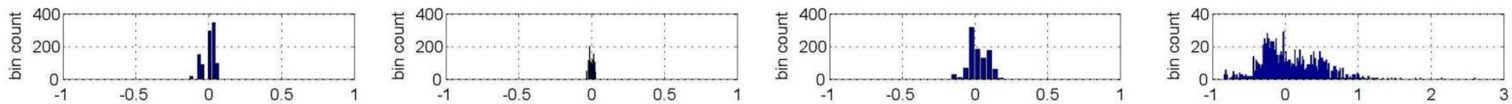
Mass estimation from residual signal (4/4): Performance



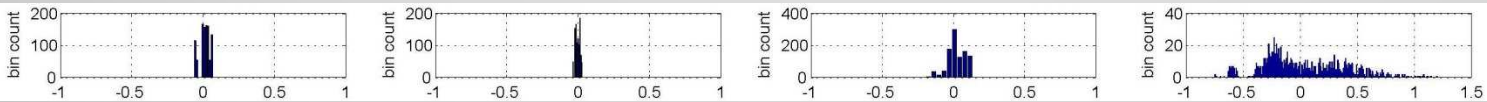
x-offset = 0.49, theta = 1.80,
velocity mph = 3.15,
sensor config = 5 under ground,
spaced 0.5m apart

x, (or y,z) position error is calculated as [(estimated x (or y,z) - true x (or y,z)) / vehicle length in x (or y,z)] so that postive error indicates overestimation and negative error implies underestimation.
Relative error in mass is calculated as (estimated mass-true mass)/true mass so relative error > 0 indicates overestimation and relative error < 0 implies underestimation.

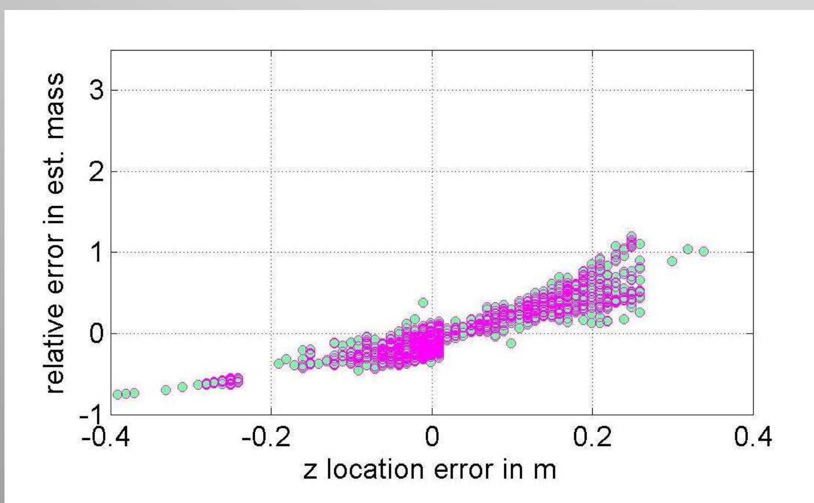
Mass can be located to within 5% of vehicle length and width, and ~10% of vehicle height



x-offset = -0.45, theta = -1.90,
velocity mph = 3.15,
sensor config = 5 under ground,
spaced 0.5m apart



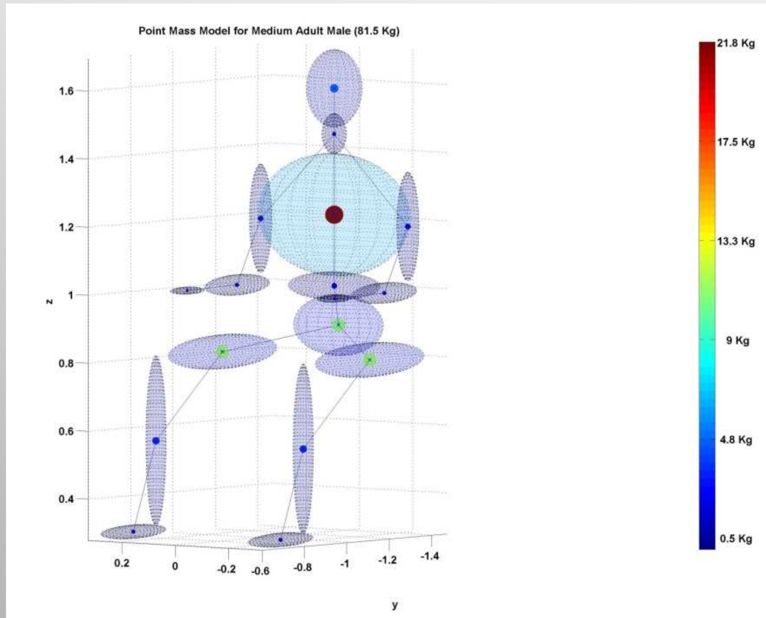
x-offset = 0.02, theta = 0.10,
velocity mph = 3.15,
sensor config = 5 under ground,
spaced 0.5m apart



Mass estimate is more uncertain, ~20%

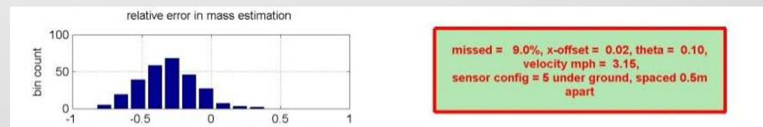
Correlation between height and mass errors are consistent with physics ambiguity between distance and mass

Preliminary work on effect of "clutter" mass

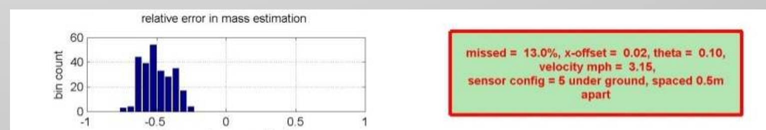


Add "mass man" in driver's seat: based on 1988 anthropometry report* on male aviators

Distribution of mass estimates for 50 kg point mass in F250



Distribution of mass estimates for "mass man" in F250



Mass man looks like point mass with ~50% of true mass

Both 50 kg point mass and mass man can be located to within 10% of vehicle dimensions

*ANTHROPOMETRY AND MASS DISTRIBUTION FOR HUMAN ANALOGUES Volume I: Military Male Aviators, March 1988, Naval Biodynamics Laboratory P.O. Box 29407,New Orleans, UI 70189-0407

In summary, we can …

- **Extract** threat mass signal by removing sensor response to vehicle
- **Detect** 50 kg mass with P_D>95% and P_{FA}<10^{-3} for all 8 vehicle models tested
- **Locate m**ass to within 5% of vehicle length and width, and ~10% of vehicle height
- **Estimate** mass to ~20%

with the caveat…

No gravity gradiometer prototype has yet been built to verify the results reported here.