Quantum Sensing and Information Processing

Lecture 5: Quantum Computing Algorithms

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Los Alamos National Laboratory

July 31st, 2019

LLNL-PRES-xxxxxx
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Lecture Schedule

Quantum Computing Algorithms

Wednesday, July 31st at 2:00 *and* Thursday, August 1st at 2:00

B543 Auditorium, R1001

Quantum Radar

Thursday, August 22nd at 2:00, B543 Auditorium, R1001

Schedule posted to Lab calendar – subscribe to receive updates

https://casis.llnl.gov/seminars/quantum_information

Guest Lecturer Dr. Andreas Baertschi

- Postdoctoral Research Associate, Los Alamos National Laboratory
- Research interests
  - Quantum Alternating Operator Ansatz (QAOA),
    including initial state preparation and the design, compilation and analysis of mixing operators and phase-separation operators;
  - Quantum Annealing with D-Wave,
    including classical preprocessing and graph embedding tasks.
- PhD Computer Science, ETH Zurich

Dr. Baertschi's lectures are co-sponsored by the Advanced Simulation and Computing Program (LANL) and the Center for Applied Scientific Computing (LLNL).

https://cnls.lanl.gov/External/people/Andreas_Baertschi.php

Quantum Computing Algorithms
Tricks and Tools

LLNL CASIS Quantum Sensing and Information Processing Series

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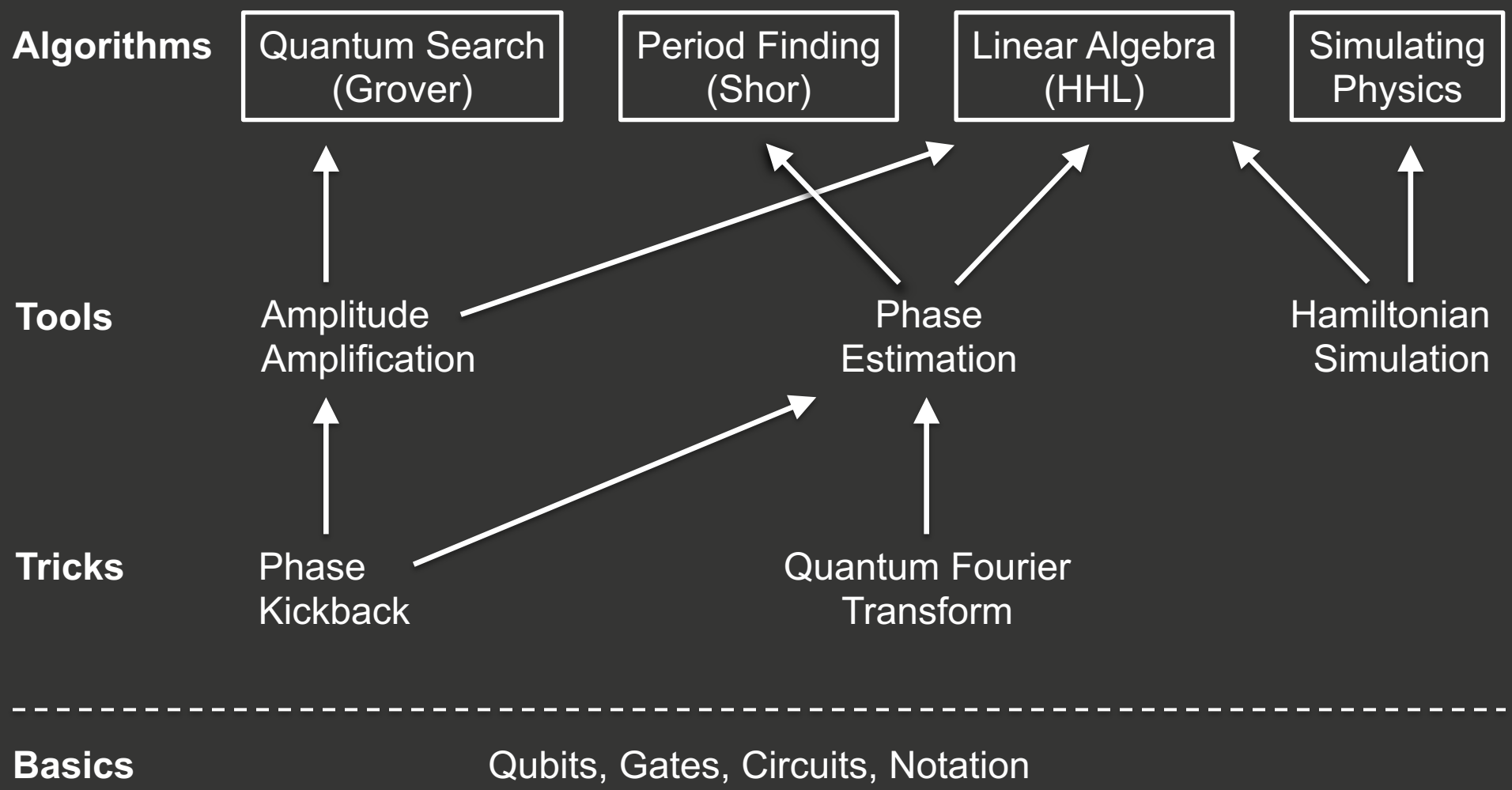
July 31 / August 1, 2019

Managed by Triad National Security, LLC for the U.S. Department of Energy's NNSA


Algorithms
Quantum Search (Grover)
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Amplitude Amplification
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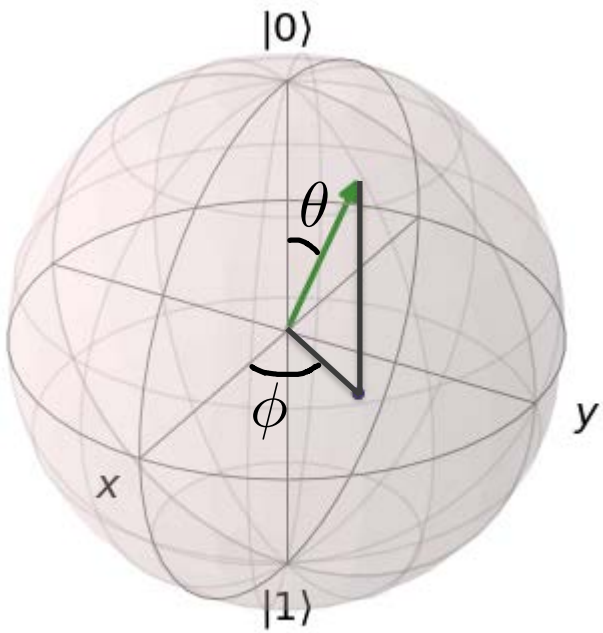
Phase Kickback
Quantum Fourier Transform
Basics
Qubits, Gates, Circuits, Notation

Theory (Slides) and Practice (Quirk)

https://cnls.lanl.gov/~baertschi/QCA/

From Bits to Qubits

Bits: Either 0 or 1



Qubits: $\alpha|0\rangle + \beta|1\rangle$ with $\alpha, \beta \in \mathbb{C}$

$$|\alpha|^2 + |\beta|^2 = 1$$

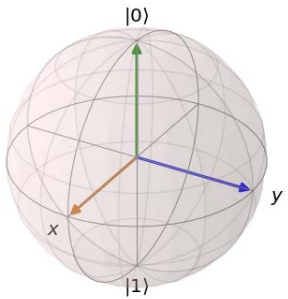
$$\Rightarrow e^{i\delta}\left(\cos(\theta/2)|0\rangle + e^{i\phi}\sin(\theta/2)|1\rangle\right)$$

global phase relative phase

$$|0\rangle = \begin{pmatrix}1\\0\end{pmatrix} \qquad |1\rangle = \begin{pmatrix}0\\1\end{pmatrix}$$

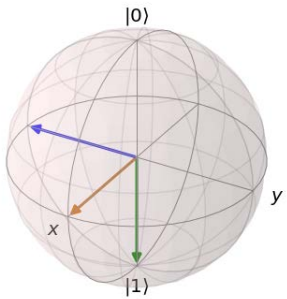
$$|+\rangle := \frac{|0\rangle + |1\rangle}{\sqrt{2}} \qquad |-\rangle := \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

Single Qubit Gates

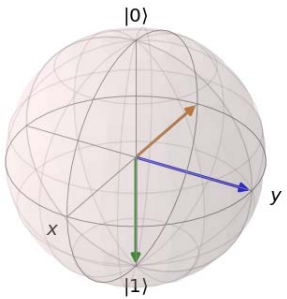


Pauli

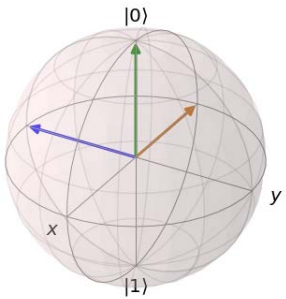
Clifford



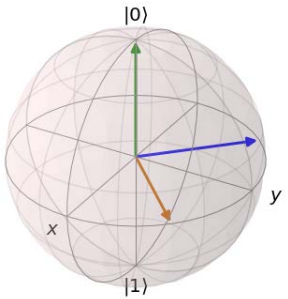
$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$



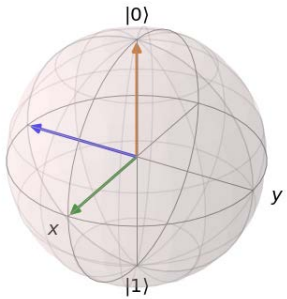
$$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$



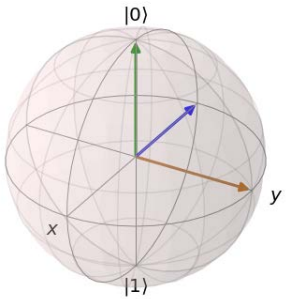
$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$



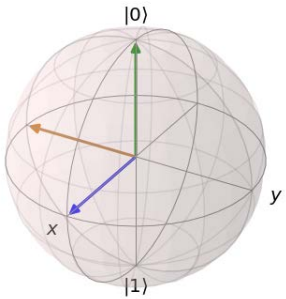
$$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$$



$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$



$$S = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/2} \end{pmatrix}$$



$$S^\dagger = \begin{pmatrix} 1 & 0 \\ 0 & e^{-i\pi/2} \end{pmatrix}$$

Multiple Qubits

$$\left.\begin{array}{l}|\Phi\rangle \\ |\Psi\rangle\end{array}\right\} |\Psi\rangle \otimes |\Phi\rangle = \begin{pmatrix}\psi_1 \\ \psi_2\end{pmatrix} \otimes \begin{pmatrix}\phi_1 \\ \phi_2\end{pmatrix} = \begin{pmatrix}\psi_1 \begin{pmatrix}\phi_1 \\ \phi_2\end{pmatrix} \\ \psi_2 \begin{pmatrix}\phi_1 \\ \phi_2\end{pmatrix}\end{pmatrix} = \begin{pmatrix}\psi_1 \cdot \phi_1 \\ \psi_1 \cdot \phi_2 \\ \psi_2 \cdot \phi_1 \\ \psi_2 \cdot \phi_2\end{pmatrix}$$

$$|\Psi\rangle = \begin{pmatrix}\psi_1 \\ \psi_2\end{pmatrix} \quad |\Phi\rangle = \begin{pmatrix}\phi_1 \\ \phi_2\end{pmatrix}$$

Example:

$$|1\rangle \otimes |0\rangle = \begin{pmatrix}0 \\ 1\end{pmatrix} \otimes \begin{pmatrix}1 \\ 0\end{pmatrix} = \begin{pmatrix}0 \\ 0 \\ 1 \\ 0\end{pmatrix}$$

3rd Entry: $|3\rangle$ or $|10\rangle$

***n* Qubits live in a *N* = 2^n-dimensional Hilbert Space:**

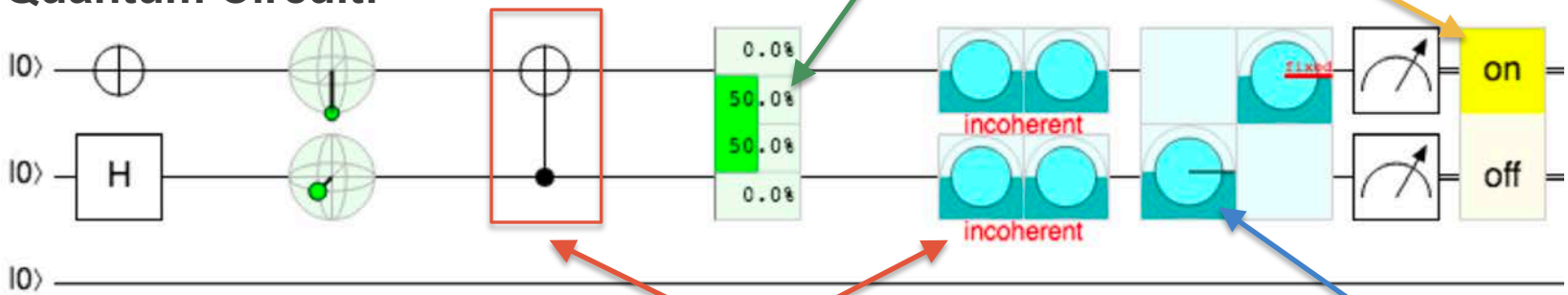
$$\alpha_0|0..00\rangle + \alpha_1|0..01\rangle + \alpha_2|0..10\rangle + \alpha_3|0..11\rangle + \ldots + \alpha_{2^n-1}|1..11\rangle$$

2-Qubit Gates and Entanglement

Probability to measure $|01\rangle$

Measurement: $|01\rangle$

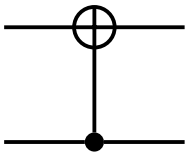
Quantum Circuit:



Entanglement

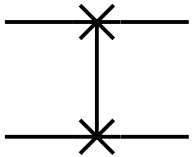
Amplitude $\alpha_3|10\rangle$ with Mag^2 = 0.5, Phase 1

Notable 2-Qubit Gates:



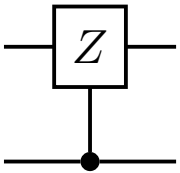
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \quad \begin{array}{lcl} |00\rangle & \rightarrow & |00\rangle \\ |01\rangle & \rightarrow & |01\rangle \\ |10\rangle & \rightarrow & |11\rangle \\ |11\rangle & \rightarrow & |10\rangle \end{array}$$

CNOT



$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

SWAP



$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Controlled-Z

Algorithms

given by quantum circuits with 1- and 2-qubit gates

Qubits:

- Total Number of Qubits
- Number of Ancillas (“extra qubits”)

Gates:

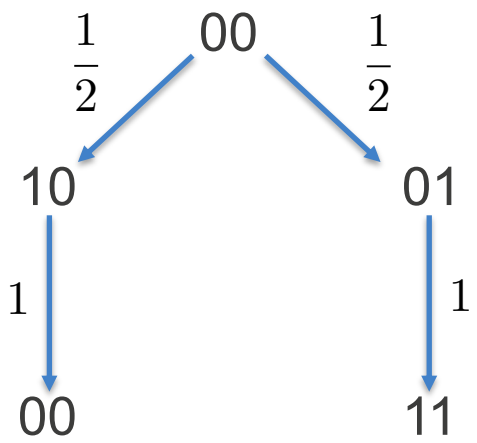
- Total Number of Gates
- Depth of Circuit
- Number of T-Gates

Measurements:

- Total Number of Measurements
- Number of Consecutive Measurements (Error Correction)

Quantum Computing in One Slide

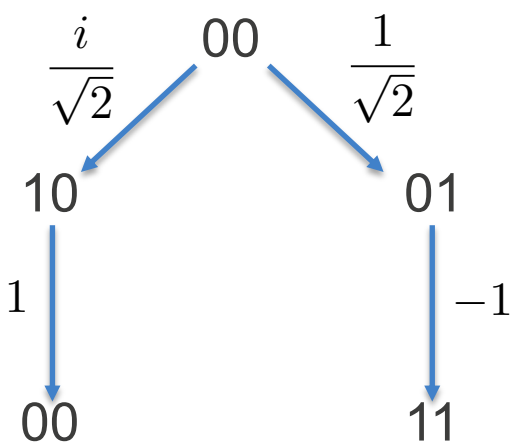
Classical Randomized Algorithm:



$$\Pr[x] = \sum_{\text{paths } p \text{ to } x} \prod_{\text{edges } e \text{ in } p} p_e$$

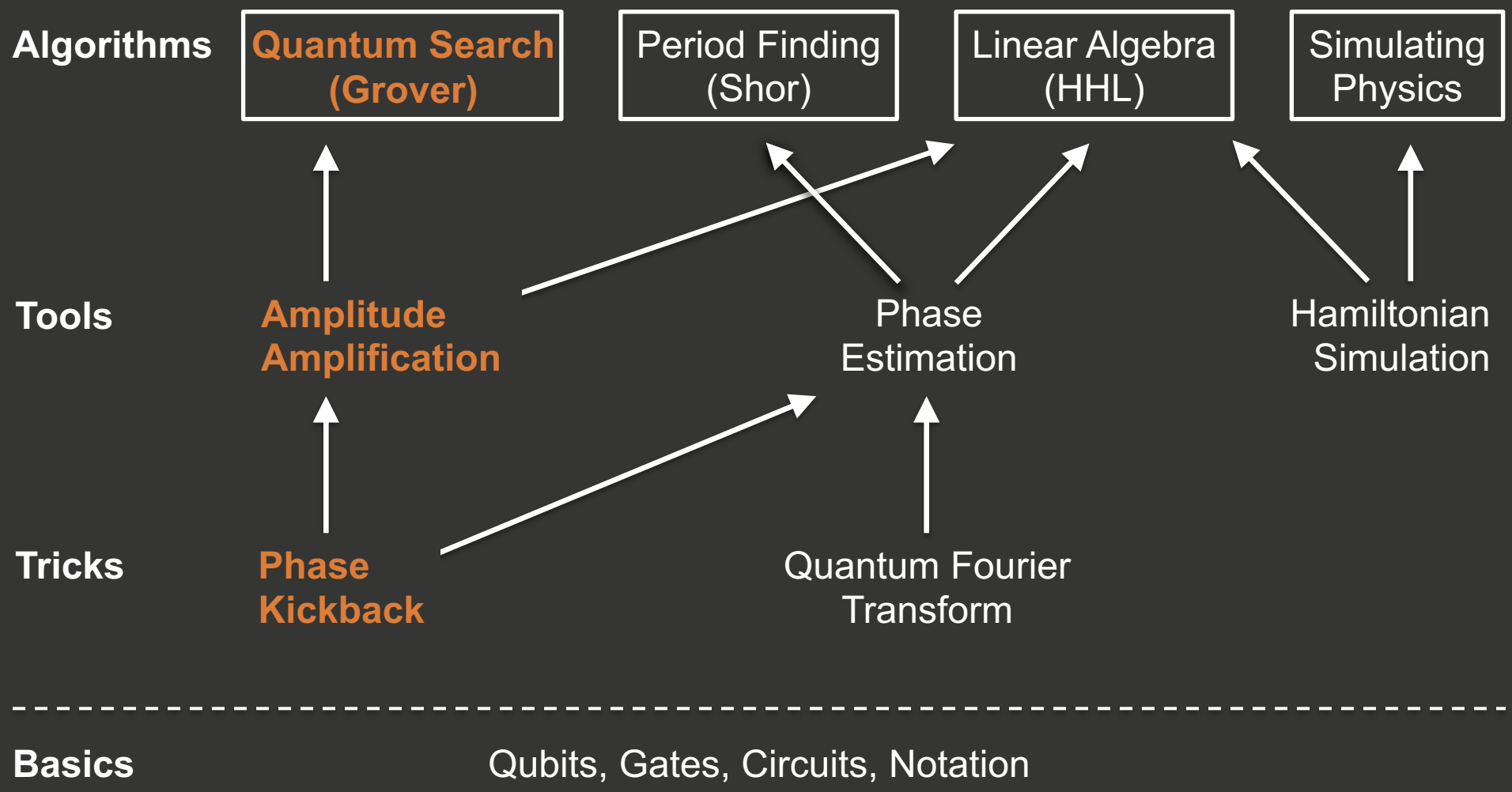
Stochastic Matrices

Quantum Algorithm:



$$\Pr[x] = \left| \sum_{\text{paths } p \text{ to } x} \prod_{\text{edges } e \text{ in } p} \alpha_e \right|^2$$

Unitary Matrices $U^{-1} = U^\dagger$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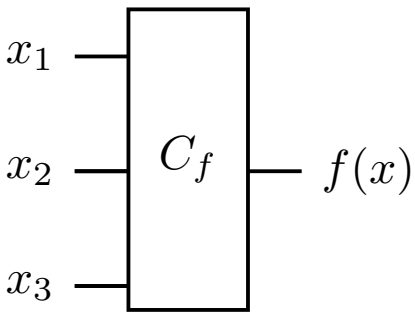
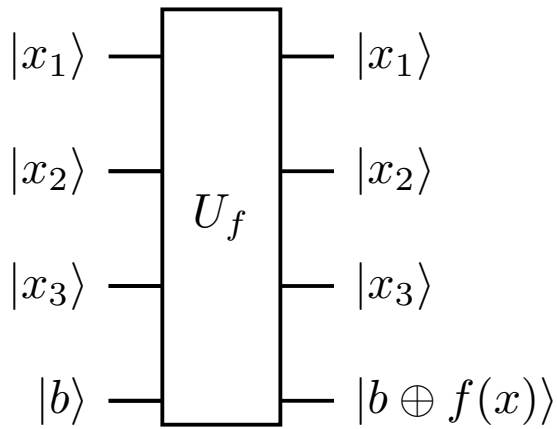
3SAT: hard to solve, easy to verify

$$f(x) = f(x_1, x_2, x_3) = (\overline{x_1}) \wedge (x_1 \vee \overline{x_2} \vee x_3) \wedge (x_1 \vee x_2)$$

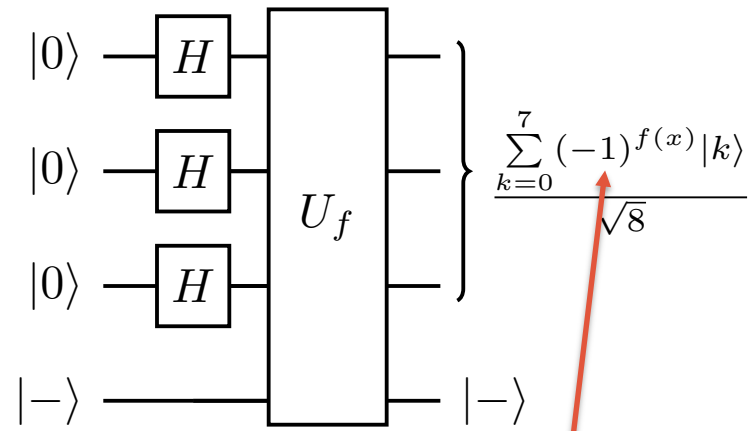
Only one solution: $(x_1, x_2, x_3) = (0, 1, 1)$



Classical Circuit (irreversible)



Quantum Circuit (reversible)



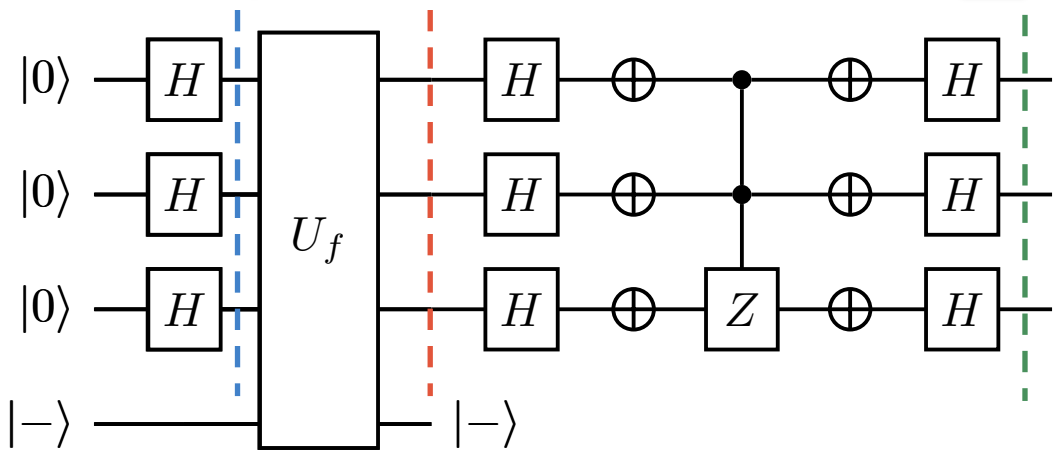
Phase Kickback

$$\frac{|0 \oplus 1\rangle - |1 \oplus 1\rangle}{\sqrt{2}} = \frac{|1\rangle - |0\rangle}{\sqrt{2}} = -\frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

Amplitude Amplification

Repetitions:

$$\sim \frac{\pi}{4}\sqrt{N}$$

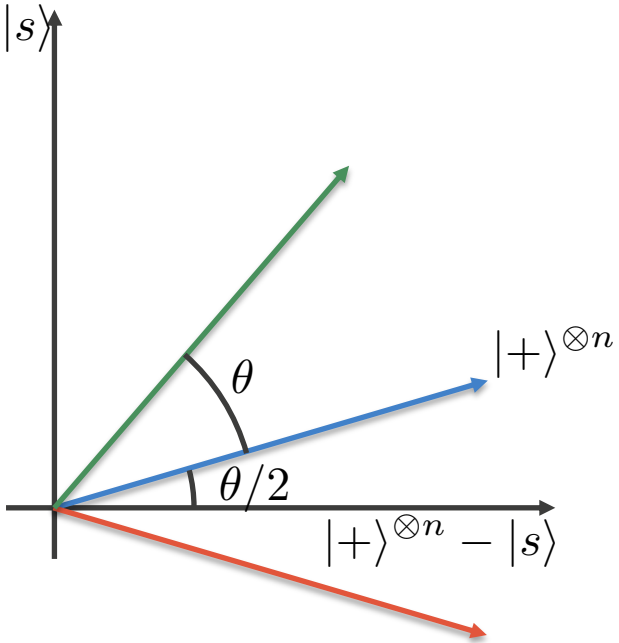


Reflection at $|+\rangle^{\otimes n} - |s\rangle$

Reflection at $|+\rangle^{\otimes n}$

Rotation Angle: $\sin(\theta/2) = \frac{1}{\sqrt{N}}$

$$\Rightarrow \theta \approx \frac{2}{\sqrt{N}}$$



Quantum Search (Grover)

Searching over all Function Inputs with $\mathcal{O}(\sqrt{N})$ Iterations

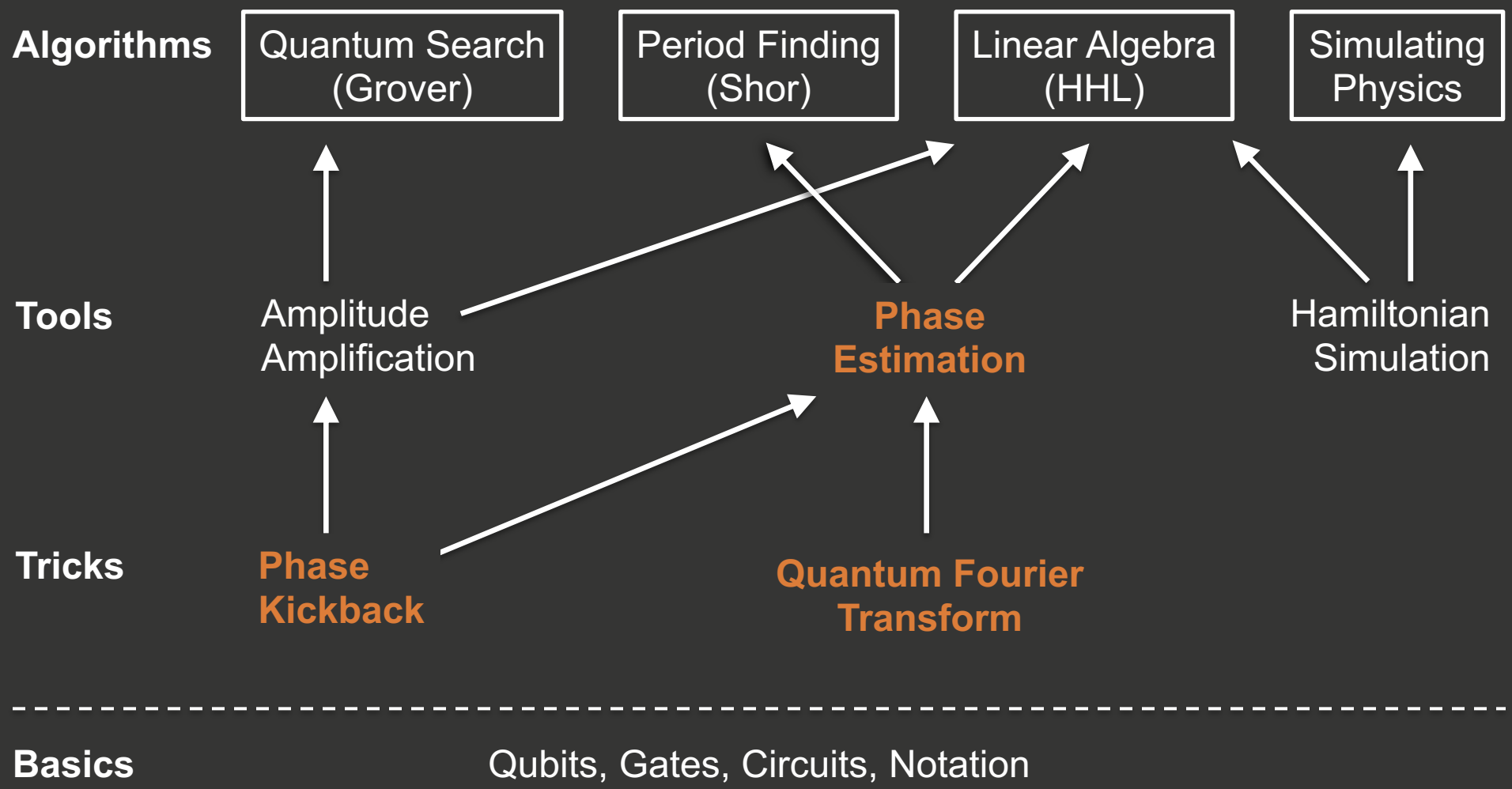
Details

- What when there are M items? $\Rightarrow \mathcal{O}(\sqrt{N/M})$ Iterations
- Grover is known to be optimal (in black-box sense).
- 3SAT Grover $\widetilde{\mathcal{O}}(\sqrt{2^n}) \approx \widetilde{\mathcal{O}}(1.41^n)$
  - vs. Schöning Randomized Alg. with $\approx \widetilde{\mathcal{O}}(1.34^n)$ rounds of success probability $\approx \frac{1}{1.34^n}$
  - Use Amplification $\Rightarrow \widetilde{\mathcal{O}}(\sqrt{1.34^n})$!

Applications

- Database Search? needs Quantum Memory…
- Quantum Speedups for Dynamic Programming
- Distributed QC
  - Sublinear Diameter Computation in CONGEST model

Quantum Speedup: Polynomial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Quantum Fourier Transform

$$QFT: |x\rangle = |x_4 x_3 x_2 x_1\rangle \rightarrow \frac{1}{\sqrt{2^4}} \sum_{k=0}^{2^4-1} e^{2\pi i k x/2^4} |k\rangle$$

$$= \frac{x_2}{2} + \frac{x_1}{4}$$

$$= \frac{1}{\sqrt{2^4}} (|0\rangle + e^{2\pi i 0.x_1}|1\rangle) \otimes (|0\rangle + e^{2\pi i 0.x_2 x_1}|1\rangle)$$

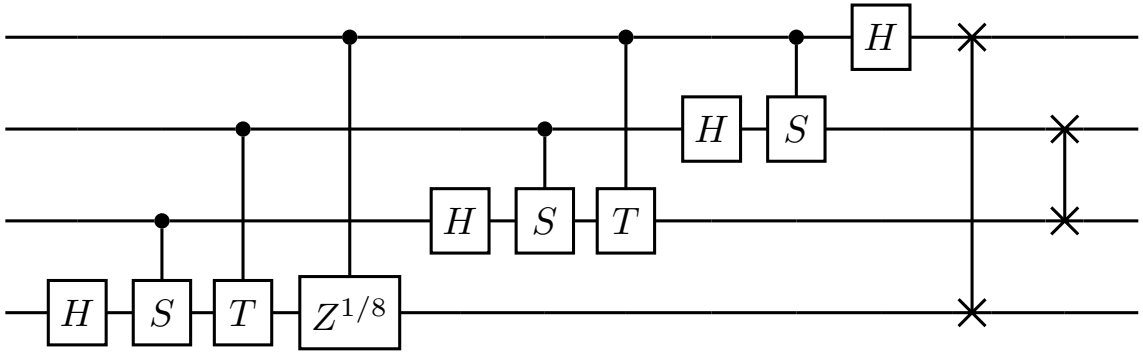
$$\otimes (|0\rangle + e^{2\pi i 0.x_3 x_2 x_1}|1\rangle) \otimes (|0\rangle + e^{2\pi i 0.x_4 x_3 x_2 x_1}|1\rangle)$$

$$\frac{1}{\sqrt{2}} \left(|0\rangle \pm e^{\pi i x_3/2} \cdot e^{\pi i x_2/4} \cdot e^{\pi i x_1/8} |1\rangle \right)$$

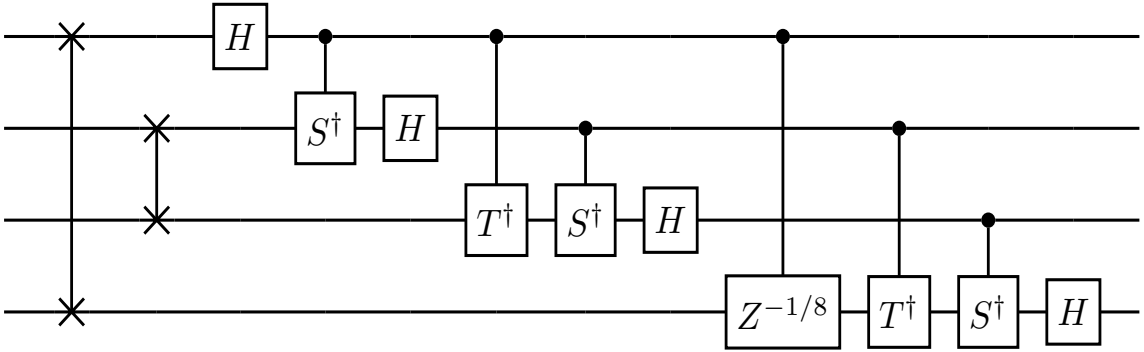
H $\quad S = Z^{1/2}$ $\quad T = Z^{1/4}$ $\quad Z^{1/8}$

QFT Circuits

$QFT:$



$QFT^\dagger:$



Quantum Speedup: Exponential

Quantum Phase Estimation

Finding the phase of a Unitary and its Eigenstate

For a Unitary U and an Eigenstate $|u\rangle$ we always have:

Phase $\phi \in [0,1)$

$$U|u\rangle = e^{2\pi i \phi}|u\rangle = e^{2\pi i \, \underline{\underline{0.\phi_n \phi_{n-1} \ldots \phi_2 \phi_1}}}|u\rangle$$

Terms from QFT!

Note that:

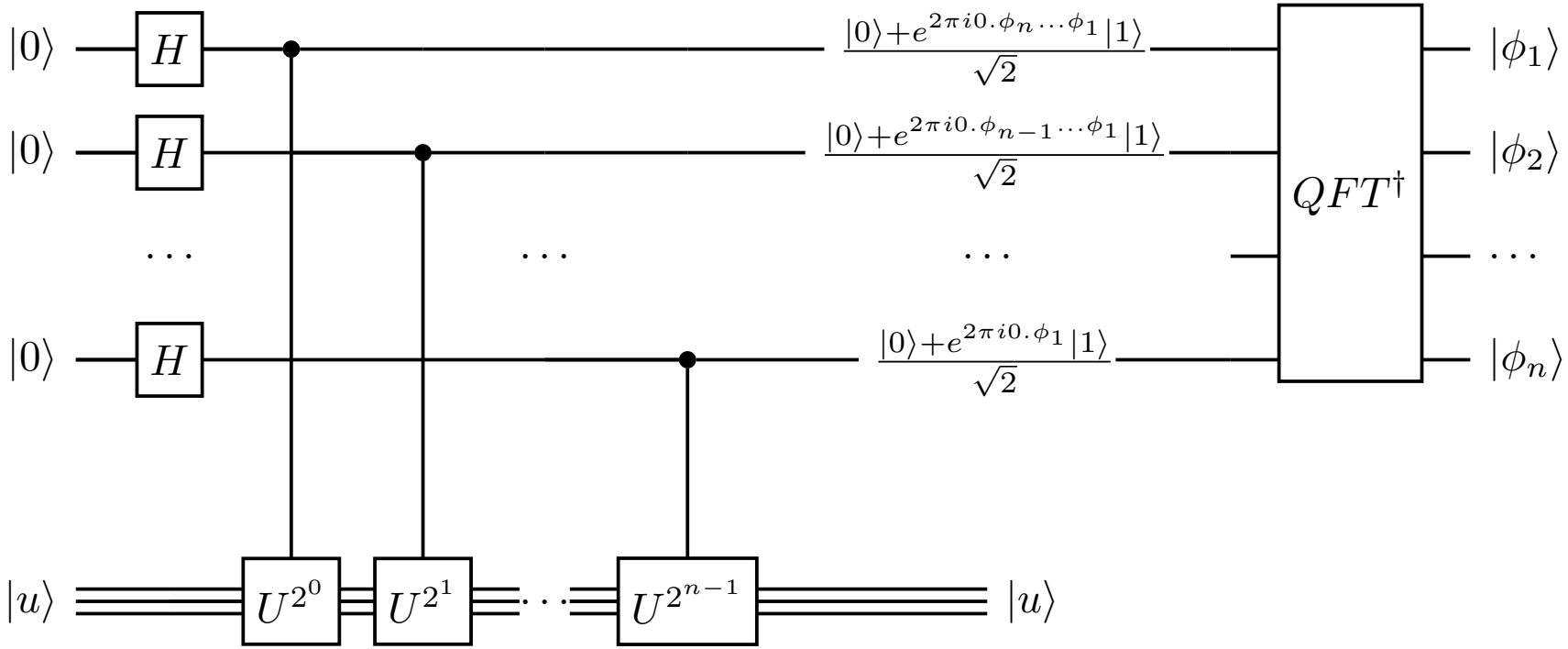
$$U^2|u\rangle = e^{2\pi i \, \phi_n . \phi_{n-1} \ldots \phi_2 \phi_1}|u\rangle = e^{2\pi i \, \underline{0.\phi_{n-1} \ldots \phi_2 \phi_1}}|u\rangle$$

$$\ldots$$

$$U^{2^{n-1}}|u\rangle = e^{2\pi i \, \phi_n \phi_{n-1} \ldots \phi_2 . \phi_1}|u\rangle = e^{2\pi i \, \underline{0.\phi_1}}|u\rangle$$

If only one could kick back the phases into an (ancilla) register and do inverse Quantum Fourier Transform on it…

QPE Circuit


$|0\rangle$
$|0\rangle$
$|0\rangle$
H
H
H
$\frac{|0\rangle+e^{2\pi i 0.\phi_n\cdots\phi_1}|1\rangle}{\sqrt{2}}$
$\frac{|0\rangle+e^{2\pi i 0.\phi_{n-1}\cdots\phi_1}|1\rangle}{\sqrt{2}}$
$\frac{|0\rangle+e^{2\pi i 0.\phi_1}|1\rangle}{\sqrt{2}}$
$QFT^\dagger$
$|\phi_1\rangle$
$|\phi_2\rangle$
$|\phi_n\rangle$
$|u\rangle$
U^{2^0}
U^{2^1}
$U^{2^{n-1}}$
$|u\rangle$