Application of an Evolutionary Algorithm for Reconstructing the Dante Spectra

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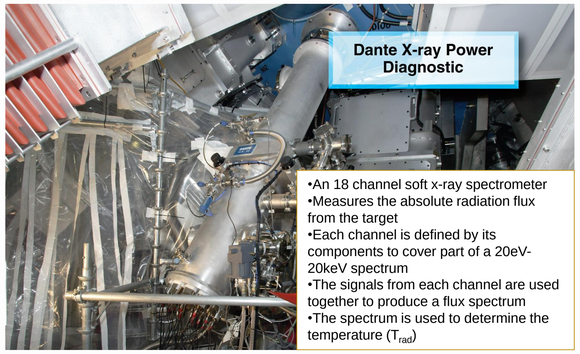
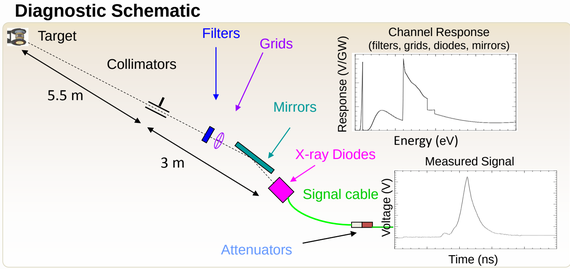


- An 18 channel soft x-ray spectrometer
- Measures the absolute radiation flux from the target
- Each channel is defined by its components to cover part of a 20eV-20keV spectrum
- The signals from each channel are used together to produce a flux spectrum
- The spectrum is used to determine the temperature (T_{rad})

Abstract

Dante is a key x-ray diagnostic on NIF that measures the spectrally and temporally resolved radiation flux from various targets. This diagnostic consists of a collection of 18 x-ray diodes located on an aperture of the Dante flange mounted to the target chamber wall. Each diode collects x-rays from a specific energy range after the x-rays have passed through a set of metal filters and mirrors. The measurements from each diode, filter, and mirror channel are designed to collect different overlapping energy ranges which can be combined to determine the spectra used for calculating the flux. The spectrum for each time step is determined by solving the Fredholm integral equation of the first kind. In this presentation, the unknown spectrum is expanded with a set of B-Spline basis functions. Because of the very large space of candidate solutions and the non uniform search space, we use a new method for identifying the B-spline node points that involves an evolutionary algorithm. Results of this algorithm are compared to the traditional UNSPEC algorithm, the NIF qualified algorithm for reconstructing the flux from Dante.

Diagnostic Schematic



Governing Equation

The governing equation for the Dante spectral unfold for each time step can be written in terms of the channel response functions as a set of Fredholm integral equations of the first kind

$$D_i = \int_0^\infty R_i(E)F(E)dE + \epsilon_i \cong \sum_{k=1}^{K} R_i(E_k)\,F(E_k)\,\Delta E + \epsilon_i$$

where D_i is the measured signal, R_i is the response function, and F is the spectrum. The response function will be assumed to be a linear approximation of basis functions

$$F(E) = \sum_{n=1}^{N} B_n(E)\,F_n$$

Writing as a matrix, the expression can be reduced to the following

$$\vec{D} = \overleftrightarrow{RI}\,\overleftrightarrow{BM}\,\vec{F} - \epsilon_i$$

which may be an ill-posed problems with potential ill-conditioned matrices without a unique solution. We consider the following expression instead

$$\overleftrightarrow{W}\,\vec{D} = \overleftrightarrow{W}\,(\overleftrightarrow{RI}\,\overleftrightarrow{BM}\,\vec{F} - \epsilon_i)$$

where $\overleftrightarrow{W}$ is a weighted matrix

Least Square Solution

The best solution in the least square sense occurs when with

$$\vec{F} = \overleftrightarrow{IO}\,\overleftrightarrow{W}\,\vec{D}$$

where $\overleftrightarrow{IO}$ is the Moore-Penrose pseudo inverse

$$\overleftrightarrow{IO} = PINV(\overleftrightarrow{W}\,\overleftrightarrow{RI}\,\overleftrightarrow{BM})$$

The weighting function is selected to be the covariance matrix which minimizes

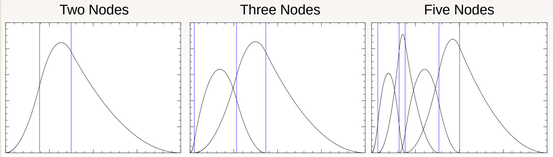
$$\chi_r^2 = \frac{1}{M-N}\sum_{i=1}^{M}\frac{\epsilon_i^2}{\sigma_i^2}$$

in the least square sense where σ_i is the standard deviation

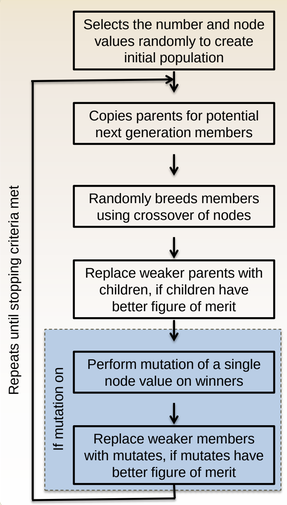
Basis Functions

- Selection of Basis Function
  - Minimal support with respect to a given degree
  - Smoothness and domain partition
- B-Spline node points are selected using an Evolutionary Algorithm
  - Allows searching very large spaces of candidate solutions
  - Allows for non uniform search spaces

Three Randomly Selected Basis Function for a second order B-Spline



Evolutionary Algorithm



Breeding
-Select number of nodes randomly
-Select each nodes randomly

Example
Parent 1 $\{a_1, a_2, a_3\}$
Parent 2 $\{b_1, b_2, b_3, b_4, b_5\}$
Child $\{b_1, b_2, a_3, b_4\}$

Mutations
-Select one node in member randomly
-Select mutant node randomly

Example
Member $\{a_1, a_2, a_3\}$
Child $\{a_1, a_2, m_3\}$

Stopping Criteria

- The algorithm has two stopping criterion
  - Diversity Measure – The population has not changed significantly over a specified number of generations
  - Number of Iterations – Maximum number of generations have occurred

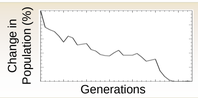
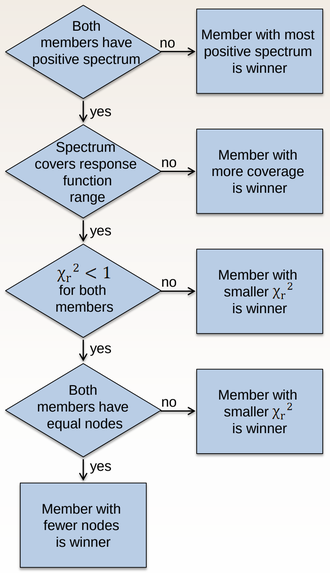


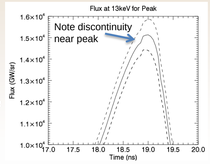
Figure of Merit

- The selection of the figure of merit dramatically effects the results for evolutionary problems
- For our problem we will consider four main factors in constructing the figure of merit
  - Chi-square (want $\chi_r^2 < 1$)
  - Simplest solution (least number of nodes)
  - Spectrum Constraints
    - Positive spectrum
    - Covers support of the response function energies



Solutions from the Evolutionary Algorithm

- The Evolutionary Algorithm produces a family of solutions for each time step
  - Family of solutions within the error
  - The member with the best "score" is selected to represent family as the solution
- The solutions of the Fredholm integral equation is solved for each time step independently
  - The number of nodes varies depending on the time
  - Discontinuities occur when the number of nodes changes
  - The discontinuity size is within the expected uncertainty calculated using the matrix formulation



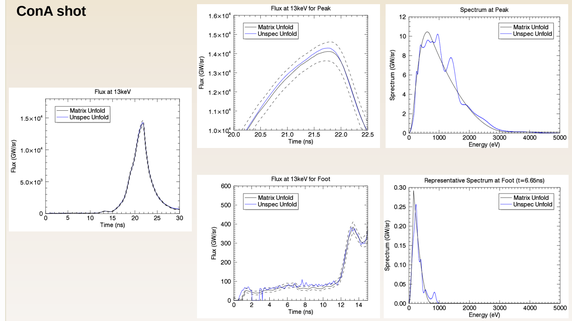
Comparison to the NIF approved Dante Unfold Algorithm

- The Unspec algorithm identifies the flux from the Dante diagnostic
  - Algorithm was qualified against several unfold algorithms from Sandia and Livermore in 2008
- Typically, the spectrum produced from UNSPEC is not smooth
  - Significant peaks/valleys occur at the channel response function means energies
  - Can vary significantly if the energy ranges are missing

Algorithm
- Estimates the spectrum using the black body spectrum for low energy channels (channel energy less than 2 keV)
- Iteratively refines the spectrum
  - Calculates the difference in flux between measured and calculated voltages
  - Modifies spectrum with Gaussian bumps based on channel response

Experimental Comparison to UNSPEC

N120408-001-999
ConA shot



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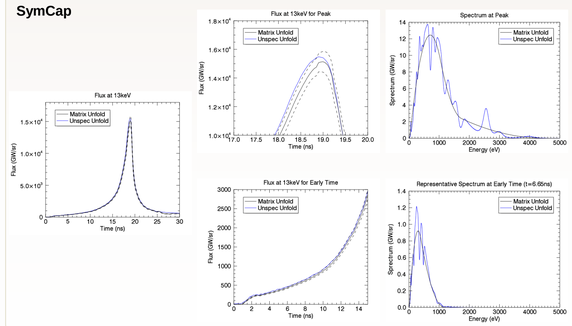

Summary

- Results of the evolutionary algorithm are consistent with UNSPEC, the NIF peer reviewed Dante spectral reconstruction algorithm
  - Flux
    - Both peaks agree to within the expected unfold error bars
    - Small discontinuities within the error bars occur
    - Less noise in the foot
  - Spectra
    - Spectra are much smoother
    - Spectral bumps at channel energies do not occur

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