Two Planar Layer Foldy-Lax Simulator

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Sommerfeld Integral

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- If it were easy, it would be in *Gradshteyn & Ryzhik*

The Solution

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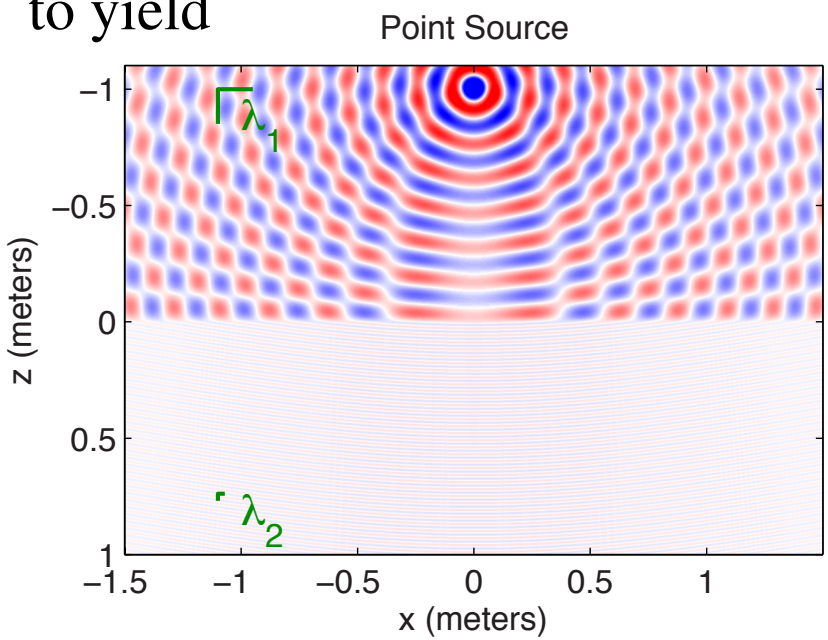
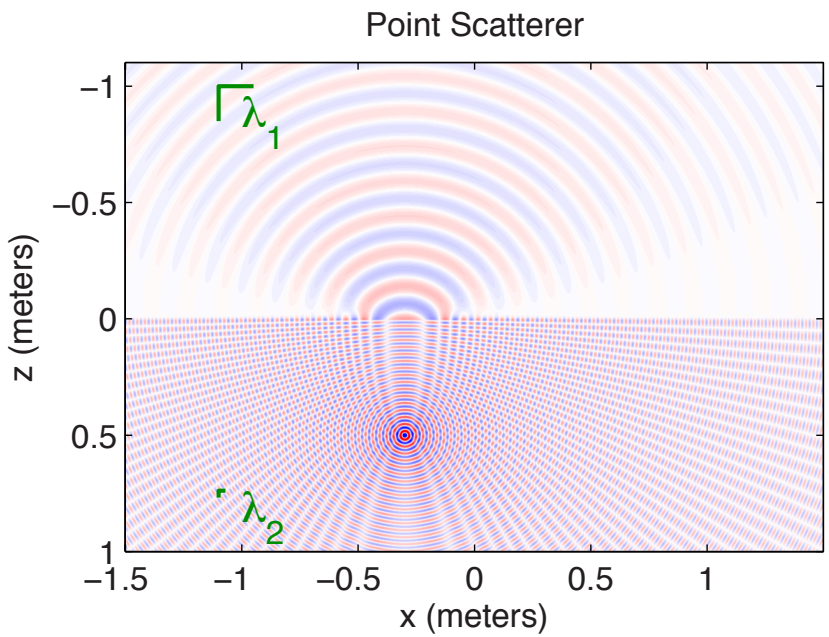
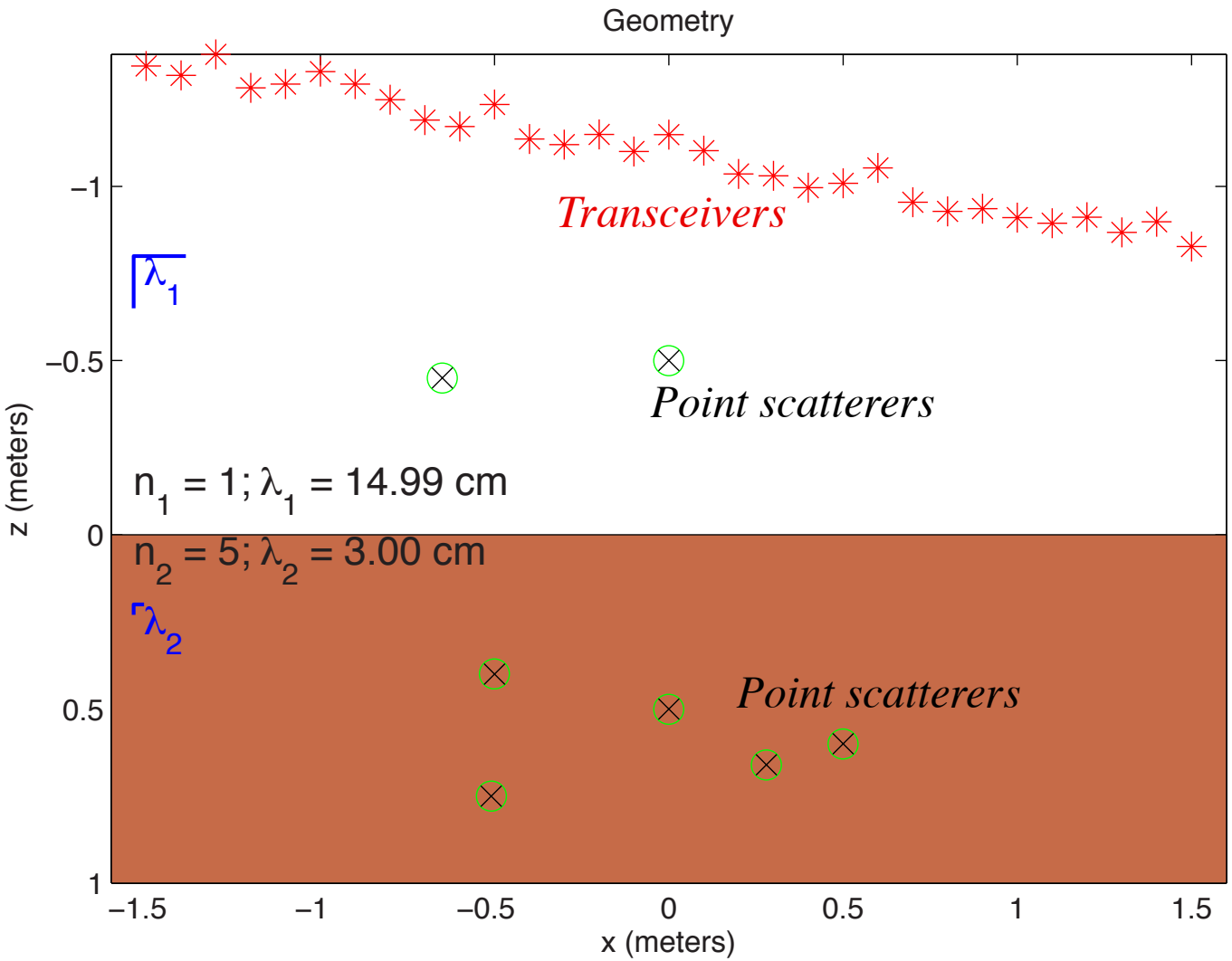
to yield



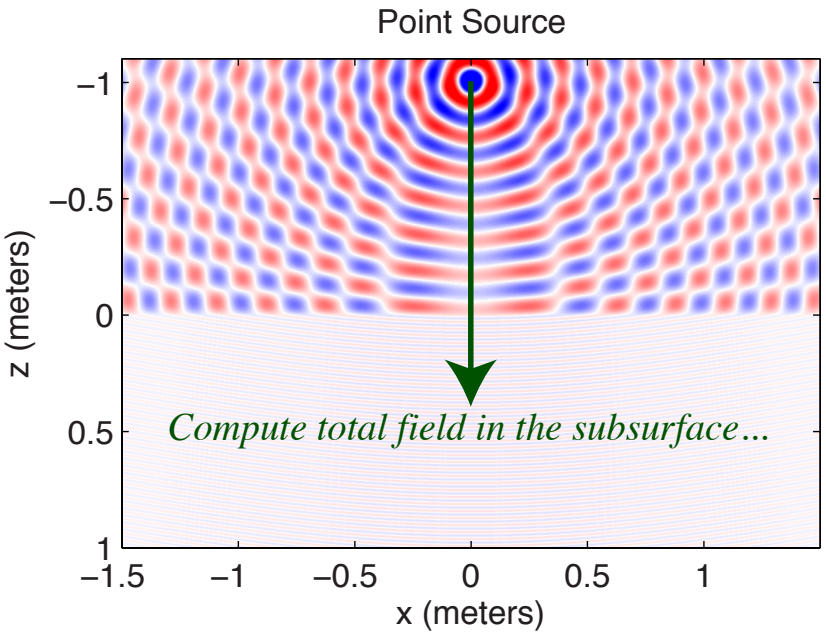


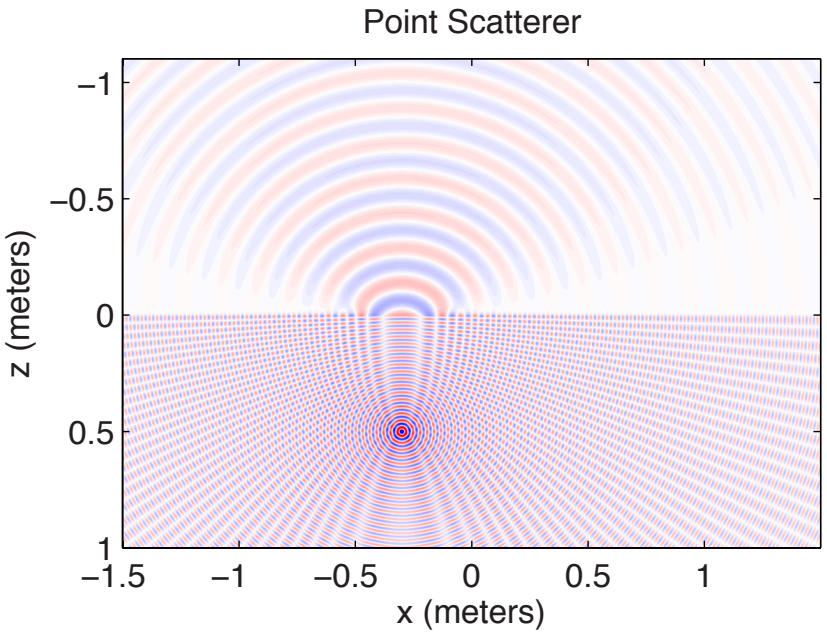
Proof-of-Concept Multistatic Geometry



Foldy-Lax

Foldy-Lax theory:





$$\psi^{tot}(\mathbf{r}, \mathbf{R}_n^t, \omega) = \psi^{inc}(\mathbf{r}, \mathbf{R}_n^t, \omega) + k_0^2 \sum_j \tau_j \, G(\mathbf{r}, \mathbf{r}_j, \omega) \, \psi^{tot}(\mathbf{r}_j, \mathbf{R}_n^t, \omega)$$

Foldy-Lax

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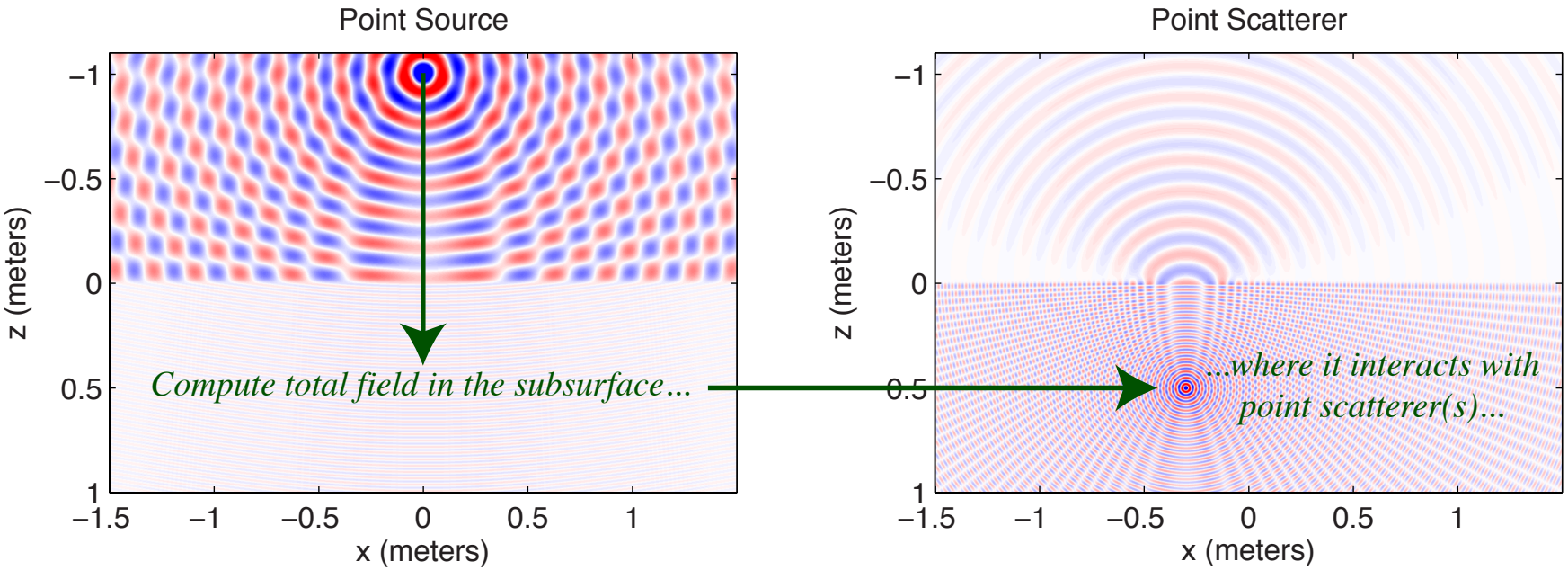

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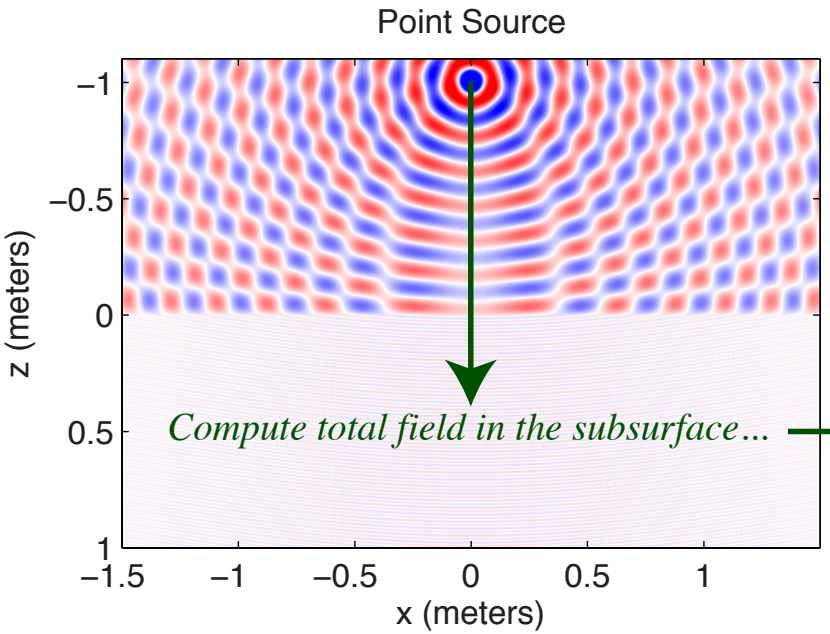

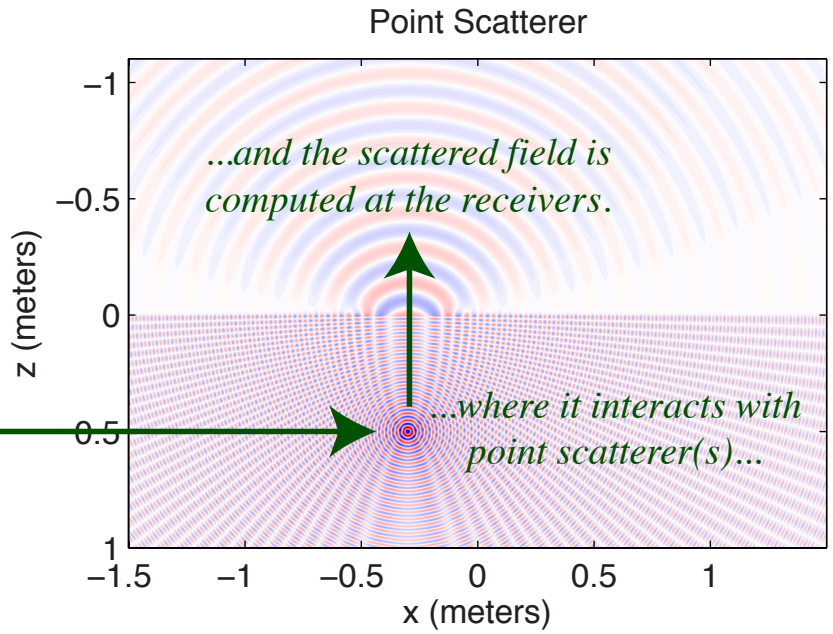

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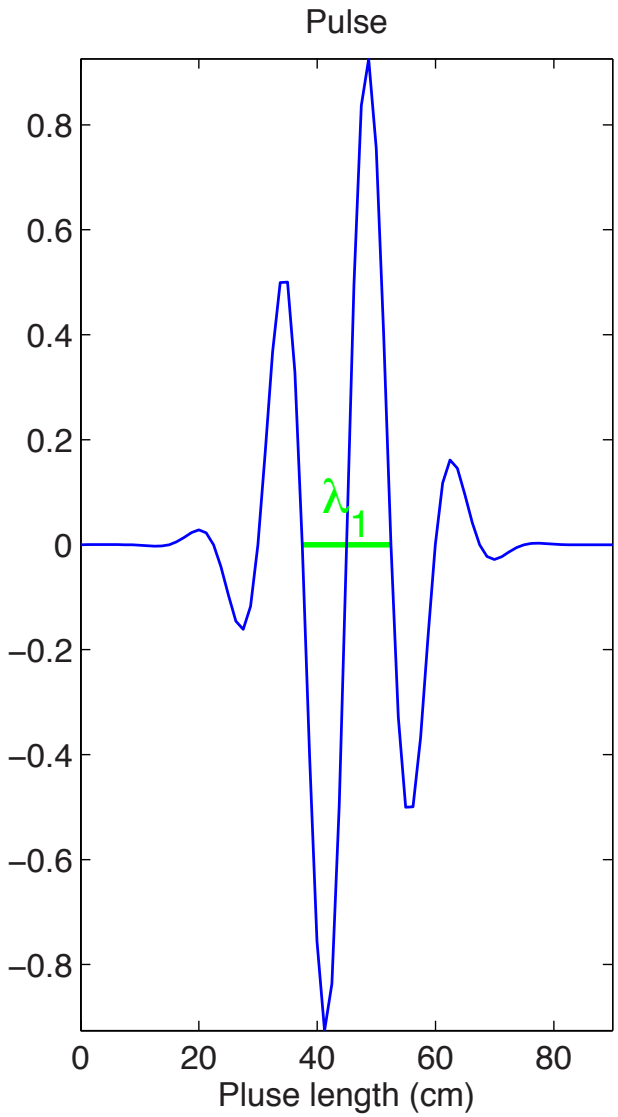
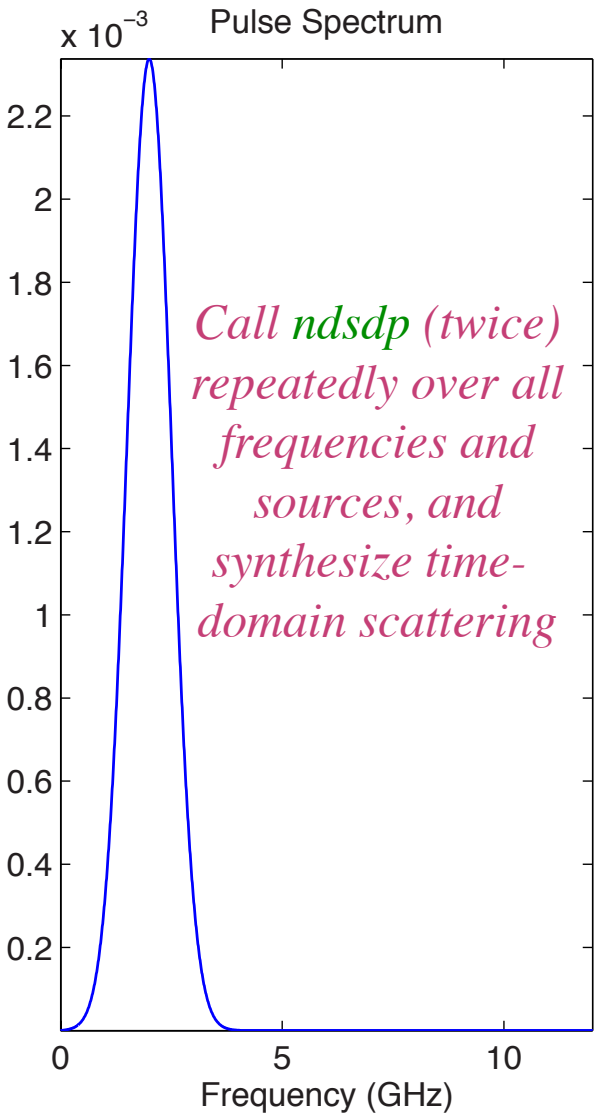
$$\psi^{scat}(\mathbf{r},\mathbf{R}_n^t,\omega) = k_0^2 \sum_j \tau_j \, G(\mathbf{r},\mathbf{r}_j,\omega) \, \psi^{tot}(\mathbf{r}_j,\mathbf{R}_n^t,\omega)$$

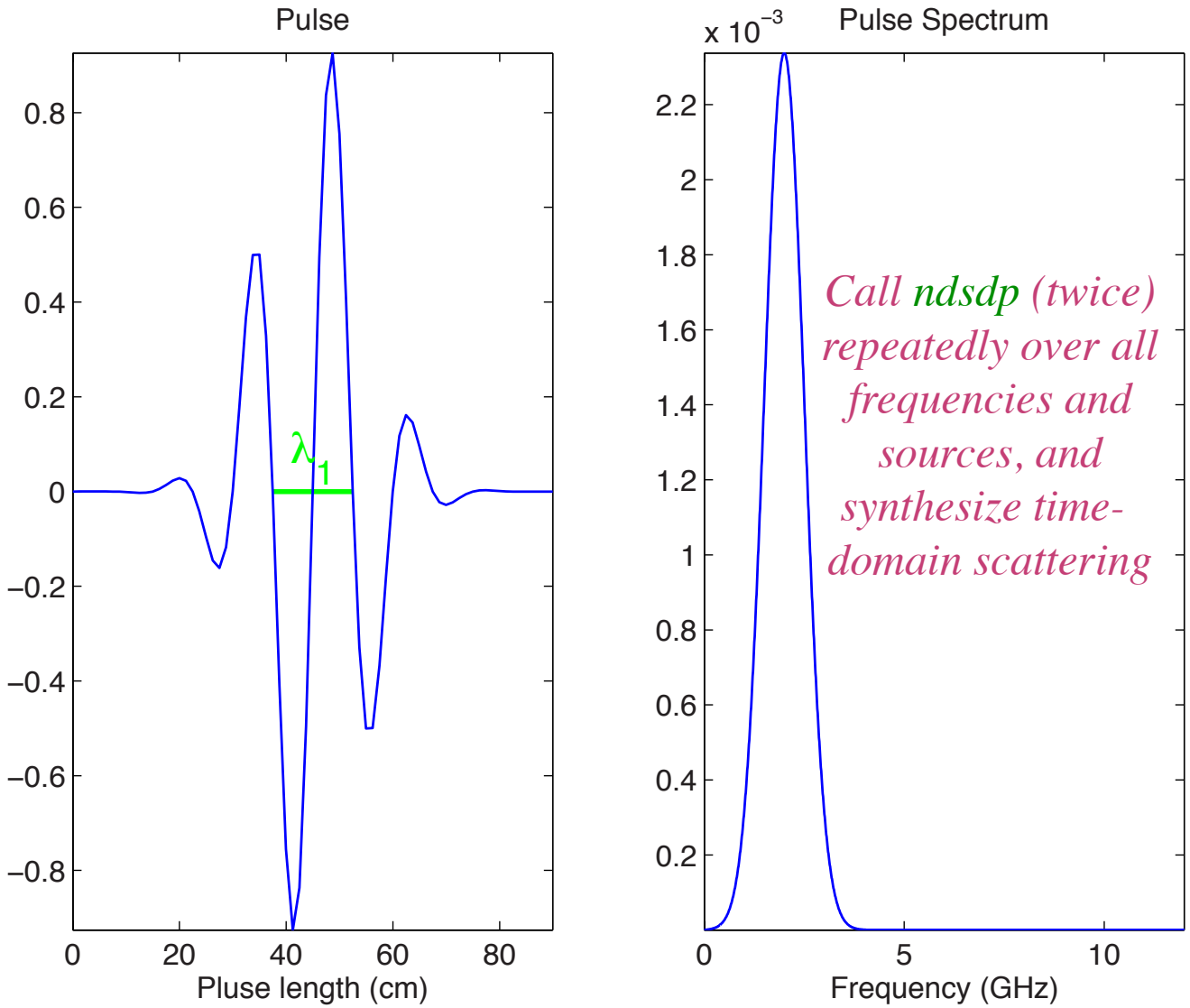
Proof-of-Concept Incident Pulse





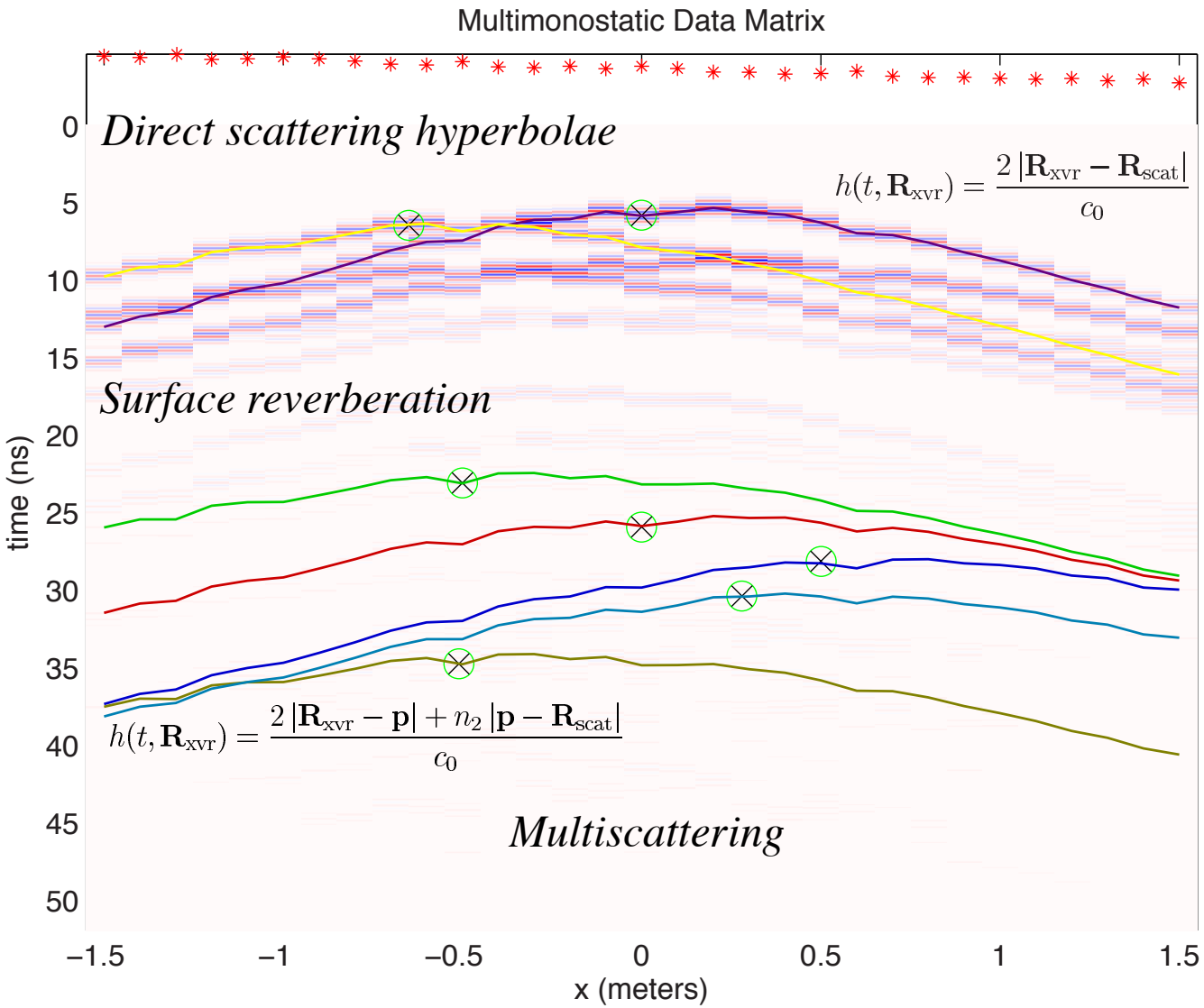
Call ndsdp (twice) repeatedly over all frequencies and sources, and synthesize time-domain scattering

Proof-of-Concept Incident Pulse

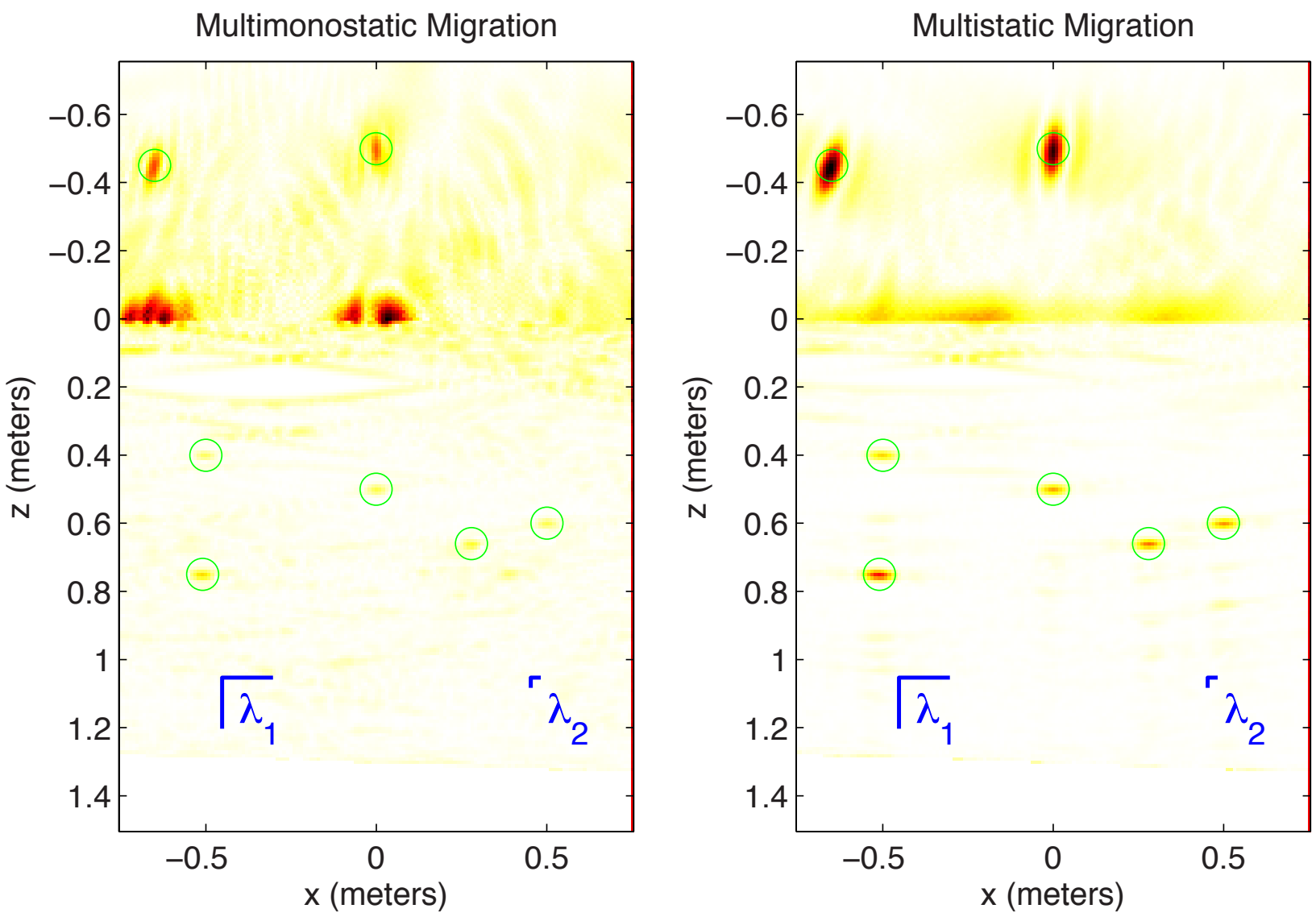


`ndsdp` is fast

Multimonostatic Data Extracted from Multistatic Example



Migration


Multimonostatic Migration
Multistatic Migration
z (meters)
x (meters)
−0.6
−0.4
−0.2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
−0.5
0
0.5
λ1
λ2

Comments

- Have an accurate working two-layer Foldy-Lax simulator
- Need to incorporate near field