



# Approximate Error Conjugate Gradient Method for Accelerated Iterative CT Reconstruction

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# Motivation



- CT reconstruction and segmentation of objects of interest is affected by the environment in which these objects are found. The environment can contribute to artifacts such as:
  - Beam hardening
  - Photon starvation
  - Streaking
- Iterative reconstruction techniques have a number of knobs :
  - Regularization
  - Incorporation of *a priori* knowledge
  - Weighting of individual raysthat may alleviate some of the environmental effects
- Iterative techniques can take a great deal of memory and CPU time.

# Previous Iterative Reconstruction Work at LLNL



- Active and passive computed tomography (A&PCT) for radioactive waste drum assays\*.
  - Uses an energy discriminating detector.
  - Active CT source consisted of two radioactive isotopes.
  - Active CT for energy dependent attenuation profile of the waste drum reconstructed by filtered back projection.
  - Passive CT is SPECT and reconstructed using iterative techniques with attenuation profiles given by the active CT reconstruction. Iterative techniques included:
    - Maximum Likelihood Expectation Maximization.
    - Constrained Conjugate Gradient optimization.

\*D.C. Camp, H.E. Martz, G.P. Roberson, D.J. Decman, and R.T. Bernardi, "Nondestructive waste-drum assay for transuranic content by gamma-ray active and passive computed tomography," Nuc. Inst. & Methods in Phys. Res. A, Vol. 495, pp 69-83, 2002.

# Current Iterative Reconstruction Work at LLNL



- We are currently investigating a number of iterative reconstruction techniques at LLNL
  - Parametric model-based reconstruction
    - When we have a good idea of what we are imaging and want to extract a few parameters, e.g., object is a sphere and we want the radius and center
  - Reconstruction based on expectation maximization
    - For many situations, but at its best when dealing with photon starved data
    - We are examining variations on the Ordered Subset Expectation Maximization\* technique.
  - Reconstruction based on adjoint techniques with constrained conjugate gradient solvers
    - General purpose voxel or model based reconstruction technique
    - Adjoint technique yields directional derivative of error for conjugate gradient, which is constrained to positive solutions

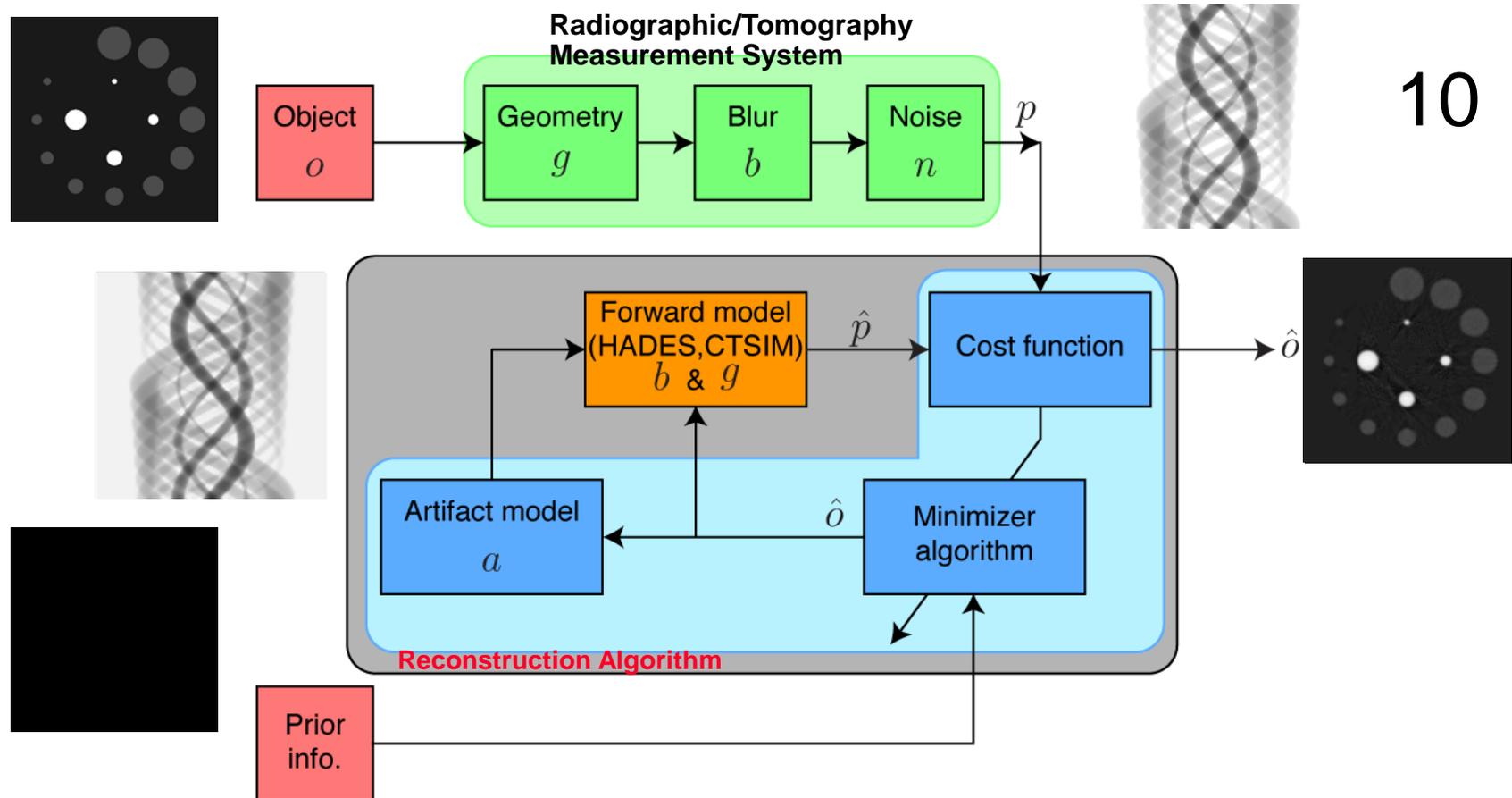
\* H. M. Hudson and R. S. Larkin, "Accelerated Image Reconstruction using Ordered Subsets of Projection Data," IEEE Trans. Medical Imaging, Vol. 13, No. 4, pp. 601-9.

# High Level Overview of CG Reconstruction



- Generate a model to fill space (blobs, regular voxels, pieces).
- Incorporate *a priori* information (or set everything to zero).
- Determine interactions between model and all rays.
- Perform an iteration of the conjugate gradient method:
  - Execute forward model for each ray
  - Determine mismatch between forward model and data for each ray.
  - Distribute error gradient to parts of model that interact with ray.
  - Generate appropriate direction given error gradient, regularization, and prior descent direction.
  - Perform line minimization to find minimum error in that direction.
  - If error is small enough, exit, otherwise repeat conjugate gradient iteration.
- Generate the output image.

# Overview of Iterative CT Reconstruction



The projection measurement ( $p$ ) is a function of the object ( $o$ ), geometry ( $g$ ), spatial blur ( $b$ ), and noise ( $n$ ) [ $p=f(o, g, b, n)$ ]. We want to recover the object; artifacts ( $a$ ) occur during the object recovery

# Major Tweak for Speed in Minimizer Algorithm



- One iteration of the Conjugate Gradient Algorithm in a nutshell:

$$\vec{x}_{k+1} = \vec{x}_k + \alpha_k \vec{d}_k, \quad \vec{x}_0 = 0$$

$$\vec{d}_{k+1} = -\vec{g}_{k+1} + \beta_k \vec{d}_k, \quad \vec{d}_0 = -\vec{g}_0$$

$$\vec{g}_k = \nabla_f E(\vec{x}_k), \quad \beta_k = \frac{(\vec{g}_{k+1} - \vec{g}_k)^T \vec{g}_{k+1}}{\vec{g}_k^T \vec{g}_k}$$

- **Major effort is the determination of  $\alpha_k$**  (the line search). It can require many evaluations of the error. Computing error for a system with 1E6 rays (1000 projections of 1000 data points each) on a 1E6 voxel grid can require  $> 2E9$  operations.
- **Approximating the error can reduce the effort** to less than that required for two full evaluations of the error (a speedup of approximately 30 times, although the result is non-deterministic). Our approach to approximation is to choose a random subset of the rays to approximate the error.
  - We use a different subset for each line search. We have been using a subset with  $\text{sqrt}(\text{original vector length})$  rays.
  - This technique is related to conjugate gradient methods with inexact searches \*.

\* D. F. Shanno, "On the convergence of a new conjugate gradient algorithm," SIAM J. Numer. Anal., Vol. 15, No. 6, pp. 1247-57, December 1978.

# Approximating the Error



- The error we are using for CT reconstruction is the squared projection difference error given by

$$E[\mu(\vec{r})] = \frac{1}{2} \sum_{m=1}^M w_m [I_m(s_{final}) - I_{m,observed}(s_{final})]^2$$

where  $E$  is the total error,  $\mu$  is attenuation (essentially the  $x$  vector of the conjugate gradient),  $r$  is position,  $M$  is the number of rays,  $m$  is the ray index,  $w$  is the ray weight,  $I$  is the current modeled ray intensity,  $s_{final}$  is the detector position, and  $I_{observed}$  is the data we are trying to match.

- The approximate error we use for CT reconstruction is the squared error of a random subset of the rays that have at least 0.1% mismatch between the modeled ray intensity and the detected ray intensity.
  - The value of the approximate error does not have to be close to the value of the true error. We need the minimum of the approximate error along the search direction to be near the minimum of the true error.
- One full error computation must be performed before the line search in order to generate the gradient for the entire problem.
- The majority of the implementation effort is in rewriting the error computation to deal with a set of selected rays and the selection of the rays themselves.

# Problems

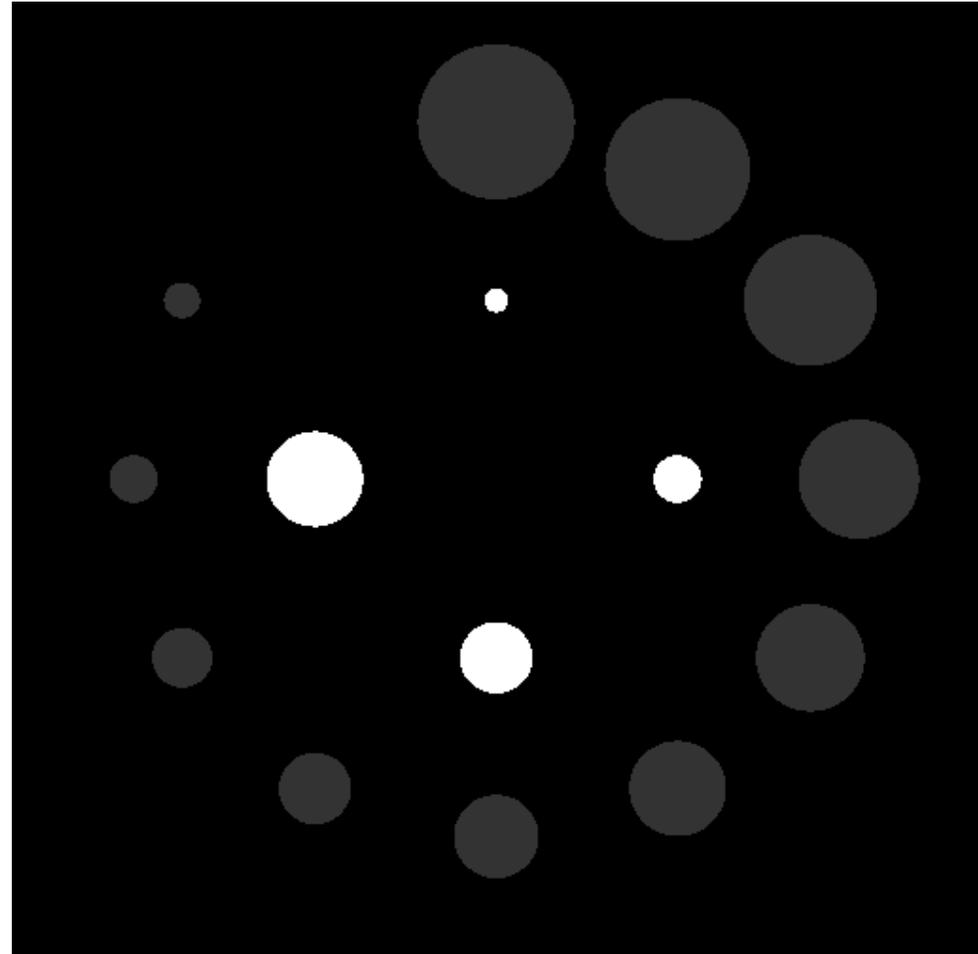


- Near the converged solution it becomes difficult to select an appropriate set of rays with which to approximate the error.
- At this point it is reasonable to switch to the full error conjugate gradient.
- Depending on when this switch occurs it may significantly reduce the time savings of this method.
- The same data will yield different results depending on which random sets of rays are used in the line search. This can be alleviated by switching to the full error conjugate gradient as the problem nears convergence.

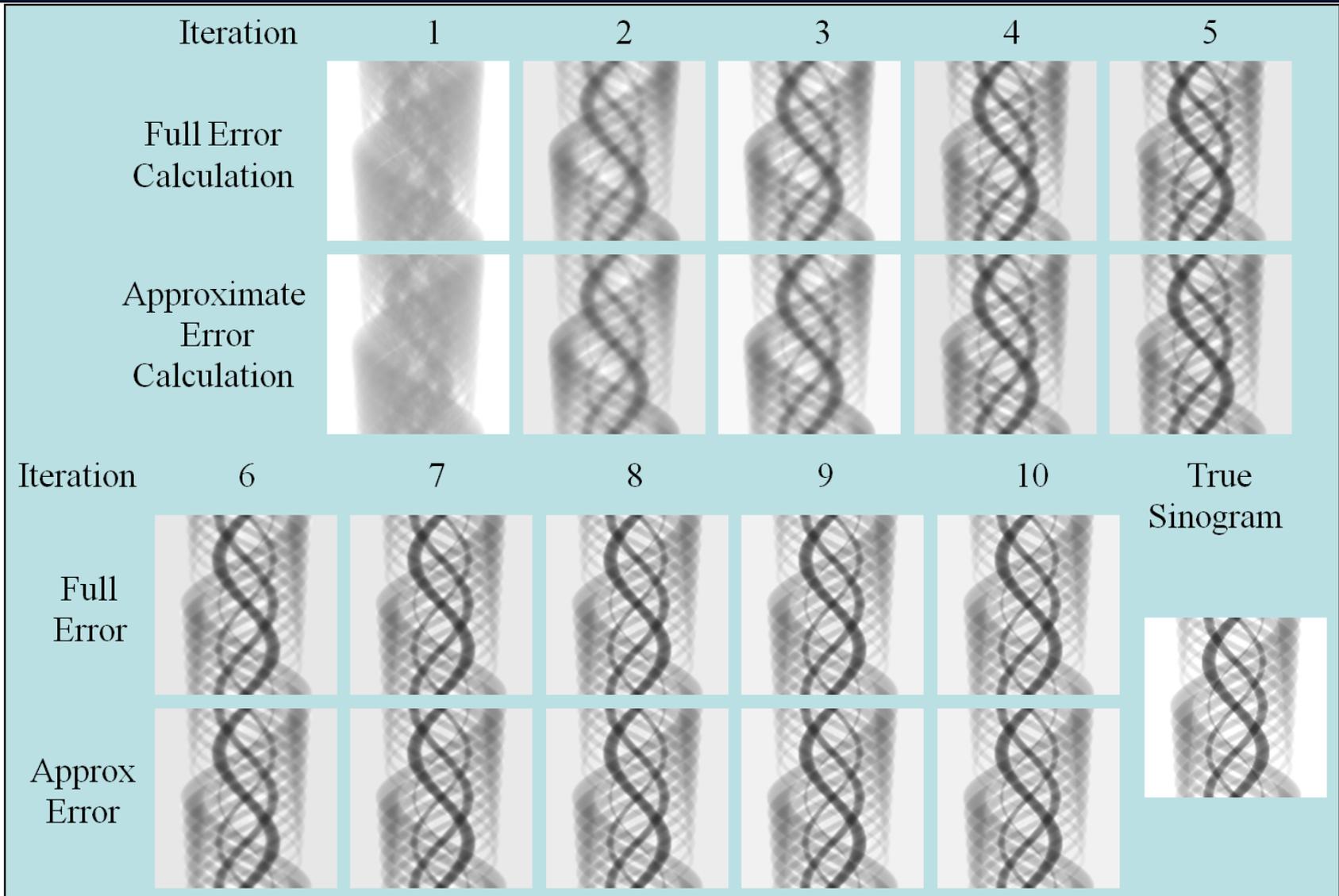
# Example: Clock Phantom



- Clock Type Phantom
- Dim circles have attenuation value 0.2
- Bright circles have attenuation value 1.0
- Region is 4 units on a side.



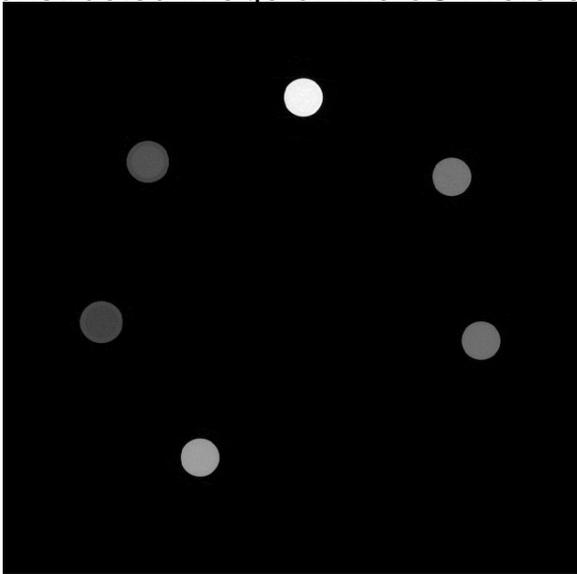
# Approximate Error Line Search



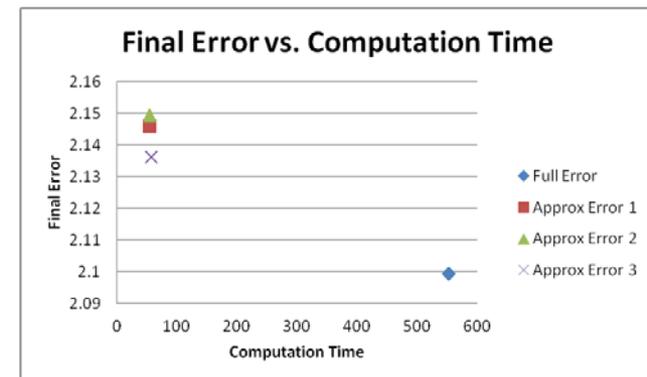
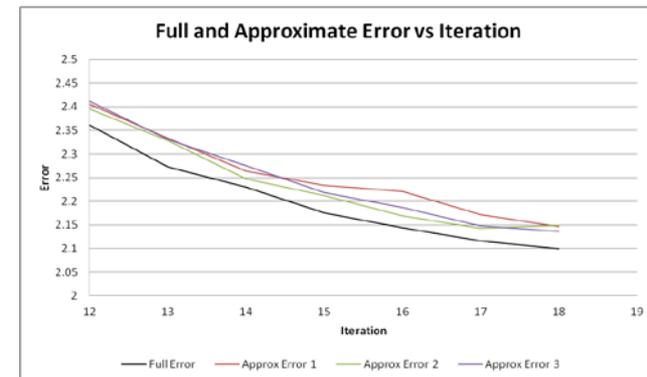
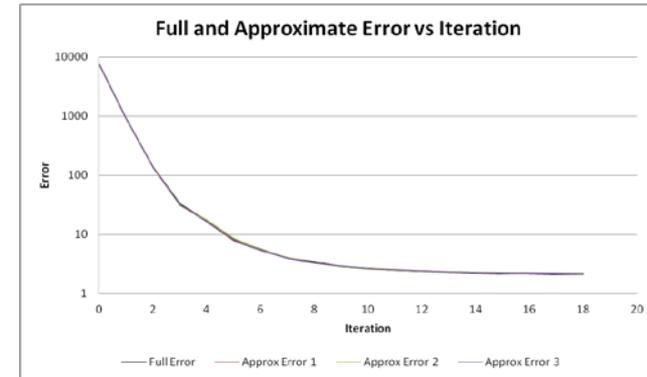
# MicroCT References Example



Reconstructed image of MicroCT references



- Reconstruction of a MicroCT slice with reference materials.
- 18 CCG iterations
  - Once with full error
  - Three times with approx. error (each with slight differences)
  - Approximate error results took 1/10<sup>th</sup> the time to generate.
    - Factor of 10 difference is actually smaller than usually observed because of the large amounts of empty space in the image.



# MicroCT References Example

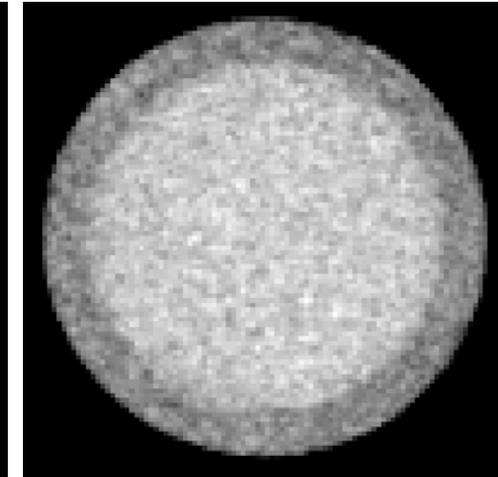
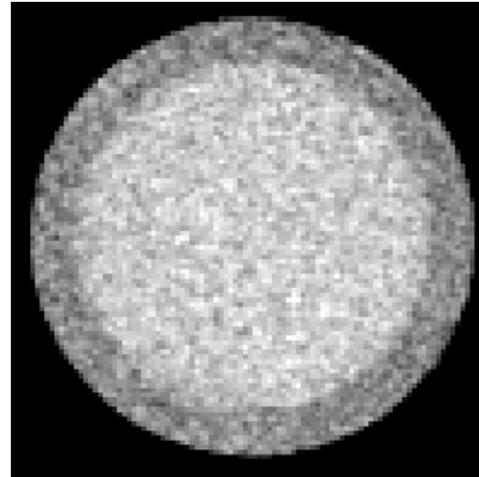


There are subtle differences between the reconstructions that are evident when viewed carefully. An example is the very slightly sharper edge on the Delrin sample seen in the graph.

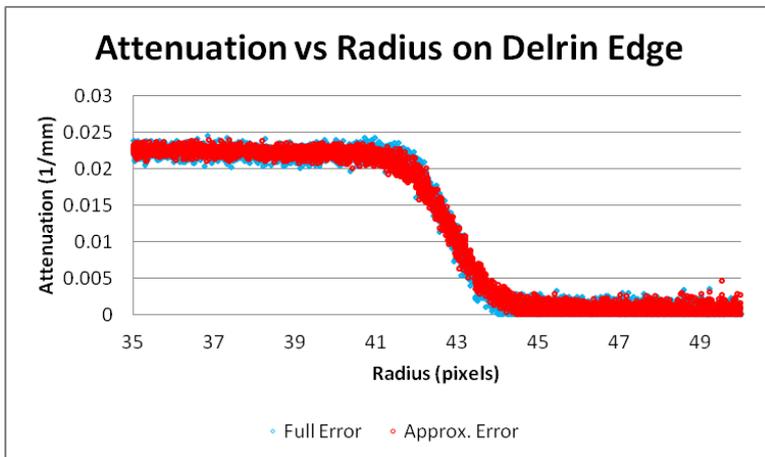
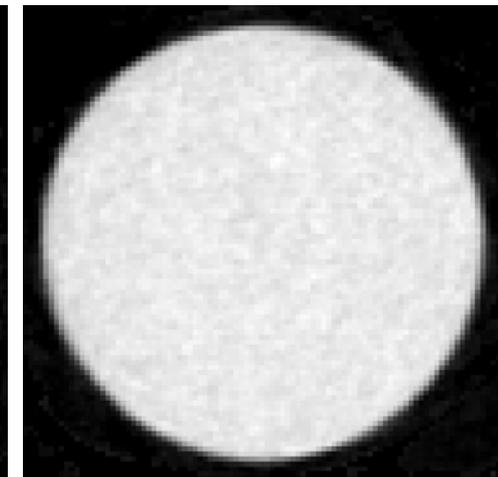
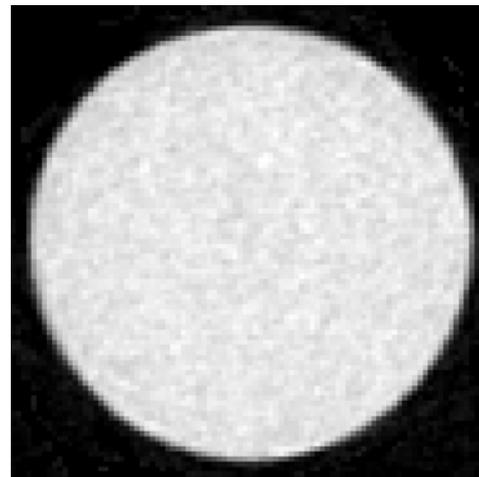
**Full Error**

**Approximate Error**

Water in plastic container



Delrin



# Summary



- We have found the line search to be the most computationally costly part of the constrained conjugate gradient CT reconstruction algorithm.
- We have found that using a randomly chosen subset of the data to approximate the error can reduce the computation time required for one iteration of the conjugate gradient algorithm to less than that of two full error computations.
  - One full error computation is required to compute the gradient.
  - Subsequent approximate error computations for the line search are much faster.



# Questions?