Curvature Wavefront Sensing

Using an Extra-focal Image and an Intra-focal Image of a Bright Star:

Reconstruction Algorithm and Wave Optics Simulations

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2006 Signal and Imaging Sciences Workshop

Center for Advanced Signal and Image Sciences

Acknowledgements to Stacie Hvisc and Kevin Baker

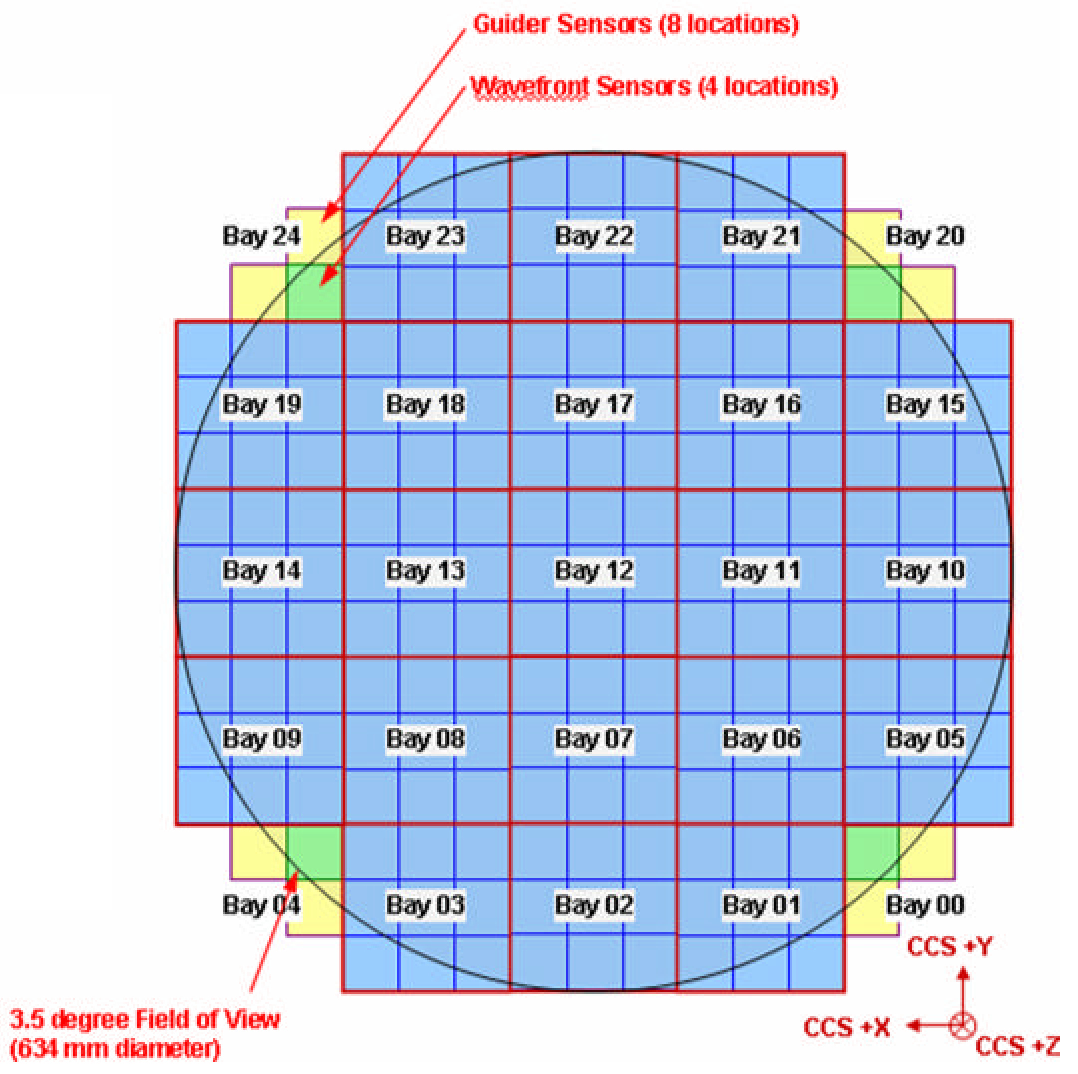
This work was performed under the auspices of the U.S. Department of Energy by University of California, Lawrence Livermore National Laboratory under Contract W-7405-Eng-48.

The curvature wavefront sensor work was motivated by requirements for the tomographic wavefront reconstruction for the Large Synoptic Survey Telescope

- Collection of phase information from multiple field angles allows the tomographic reconstruction of the mirror aberrations and rigid body misalignments
- Each field angle corresponds to a wavefront sensor located at a different position in the focal plane
- Simulation software was developed to determine the effect of the number and configuration of the wavefront sensors on reconstructing the mirror aberrations and rigid body misalignments
  - Ray tracing was used to determine the phase at the wavefront sensor
  - This work led to the LSST project making the decision to use four wavefront sensors located in the corners of the focal plane
  - The LSST project also made the decision that the baseline wavefront sensors to be used are curvature wavefront sensors

Focal Plane Array Layout for the Large Synoptic Survey Telescope

- The green squares are the location of the curvature sensors



- Curvature wavefront sensing (CWFS) was developed as an alternate method of wavefront sensing by François Roddier in 1988. Gureyev and Nugent developed a matrix method for solving the TIE equation in their 1996 K. Opt. Soc. Am. A paper. Our method is a generalization of Gureyev and Nugent's matrix method.

- A curvature wavefront sensor measures the spatial intensity distribution at two positions, one on either side of focus. These are called the intra- and extra-focal intensity images. These are relayed to object space where the light is collimated. The relaying to object space changes only the coordinates since the relaying is done to the conjugate images in object space. The reconstruction is done in object space so as not to be dominated by a huge focus term.

- The difference in the intensity is proportional to the Laplacian of the phase and is given by the transport of intensity equation (TIE):

$$k\frac{\partial \mathrm{I}(\mathbf{r})}{\partial z} = -\nabla_{\perp} \cdot \left[\mathrm{I}(\mathbf{r}) \nabla_{\perp} j(\mathbf{r}) \right]$$

Derivation of the TIE equation

Start with the Helmholtz equation $\nabla^2 \tilde{\mathrm{E}}(x,y,z) + k^2 \tilde{\mathrm{E}}(x,y,z) = 0$

Substitute $\tilde{\mathrm{E}}(x,y,z) = \exp(i\,k\,z)\,\tilde{E}(x,y,z)$ to obtain

$$\frac{d^2\tilde{E}}{dz^2} + 2ik\frac{d\tilde{E}}{dz} - k^2\tilde{E} + \nabla_\perp{}^2\,\tilde{E} + k^2\tilde{E} = 0$$

Make the paraxial approximation to obtain

$$2ik\frac{d\tilde{E}}{dz} + \nabla_\perp{}^2\,\tilde{E} = 0$$

Substitute $\tilde{E} = \sqrt{I}\exp(i\,k\,W)$ The phase W is a distance.

Setting the real and imaginary parts to zero gives the two equations:

$$\frac{dW}{dz} = \frac{1}{2}\left(k^{-2} I^{-1/2} \nabla_\perp{}^2 I^{1/2} - \nabla_\perp W \bullet \nabla_\perp W\right)$$ real part

$$\frac{dI}{dz} = -\nabla_\perp \bullet \left(I\,\nabla_\perp W\right)$$ imaginary part

Solving the TIE equation

The TIE equation can be solved by expanding W in some set of basis functions and expanding dI/dz in another set of basis functions. These two sets of functions do not have to be the same. Orthogonality and normalization are not essential to the method.

Let $W\left(x,y\right)=\sum_i a_i\ f_i\left(x,y\right)$ Let another set of basis functions be $g_j\left(x,y\right)$

We obtain the integral equations:

$$\int \frac{dI}{dz} g_j\, d^2S = \int -\nabla_\perp \bullet \left(I\, \nabla_\perp \sum_i a_i\ f_i\left(x,y\right) \right) g_j\, d^2S$$

Integrating the right side by parts and using Stoke's theorem gives:

$$\int \frac{dI}{dz} g_j\, d^2S = \int \left(-I\ g_j \nabla_\perp \left(\sum_i a_i\ f_i\left(x,y\right) \right) \right) \bullet \mathbf{n}\, dl + \int I\ \nabla_\perp \left(\sum_i a_i\ f_i\left(x,y\right) \right) \bullet \nabla_\perp g_j\, d^2S$$

In order for the first term to go to zero as r goes to infinity, we must have

$$\left| I\ g_j \nabla_\perp \left(\sum_i a_i\ f_i\left(x,y\right) \right) \right| = o\left(r^{-1}\right)$$

Solving the TIE equation (continued)

Define

$$F_j = \int \frac{dI}{dz} g_j \, d^2S$$

$$M_{ji} = \int I \, \nabla_\perp g_j \bullet \nabla_\perp f_i(x,y) \, d^2S$$

Notice that the integrals are over all x y space. We then have the matrix equation:

$$F_j = \sum_i M_{ji} \, a_i$$

We would usually choose the functions f_i to be the circular Zernikes for a circular aperture and the annular Zernikes for an annular aperture. If the intensity I did not go to zero sufficiently rapidly so as to make the integration by parts valid when choosing the f and g functions to be the same, the functions g_j could instead be chosen to be something else so as to make the integration by parts valid. For instance, the functions g_j could be chosen to be the Laguerre-Gaussian functions for a circular aperture.

Example: TIE solved for astigmatism

Astigmatism has the property that it only changes the shape of the boundary in the near field where the TIE equation is valid. The intensity is uniform and does not change with z within the boundary. This is because the Laplacian of either the 0°-90° astigmatism function or of the 45°-135° astigmatism function vanishes:

$$\nabla_{\perp}^{2}\left(x^{2}-y^{2}\right)=\nabla_{\perp}^{2}\left(x\,y\right)=0$$

Define $Z_A = x^2 - y^2$ and let $W = e\,Z_A$

The ray direction **n** is normal to the wavefront:

$$\mathbf{n}=\nabla\left(W+z\right)=e\left(2\,x\,\mathbf{a_x}-2\,y\,\mathbf{a_y}\right)+\mathbf{a_z}$$

If the intensity is a uniform circle of radius r=1 at z=0, then the intensity change from z=-Δz to z=+Δz can be regarded as being concentrated on the radial boundary r=1 in the integral for F_j:

$$\Delta I = 2e\cos\left(2f\right)d\left(r-1\right)\Delta z$$

We find $M=\int I\,\nabla_{\perp}Z_A\bullet\nabla_{\perp}Z_A\,dA=\frac{p}{2}$ $\quad F=\int\partial_z I\;Z_A\;dA=\frac{p}{2}e$

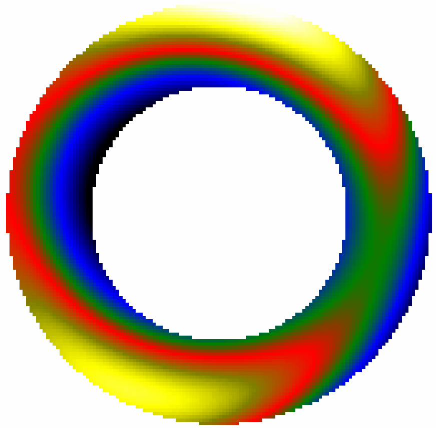
Solving $M\;a=F$ gives $a=e$

Goals for our study of CWFS

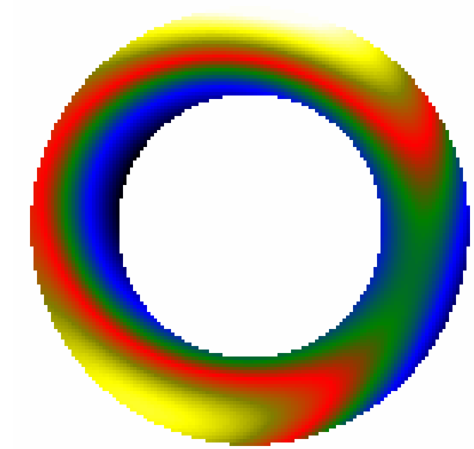
- **Find an algorithm which works for any pupil shape and does not need any special treatment of the boundaries and does not even have to know where the boundaries are! We have found such an algorithm in an extension of the work of Gureyev and Nugent. The integrals are over all space so that the only role of the boundary is in guiding our choice of basis functions.**

- **Answer the questions:**
  - **Does the CWFS reconstruction algorithm work for the least demanding case where the focal plane is moved between the intra- and extra-focal positions so that the vignetting is the same and the registration is known? Yes**
  - **Can the CWFS reconstruction algorithm be made to work for the case where the focal plane is split so that the intra- and extra-focal images are of different stars? In this case the vignetting will be different for the two images and the registration will be unknown. The vignetting produces large errors if uncorrected but can be accurately corrected if the vignetting is accurately known for both images. The registration is done iteratively so as to make the tilts vanish in the reconstruction.**
  - **What should the Δz should be for the intra- and extra-focal planes? Does this Δz depend upon the spectral band? We have found that Δz should be between 1 and 2 millimeters independent of the spectral band. This is supported by complete wave optics simulations for generating the intensity images which include turbulence and vignetting.**

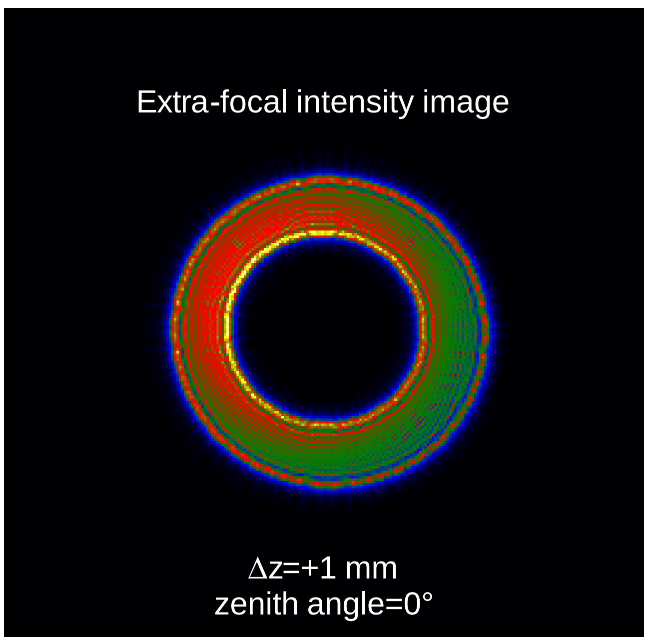
CWFS reconstruction example with no turbulence and |Dz|=1 mm

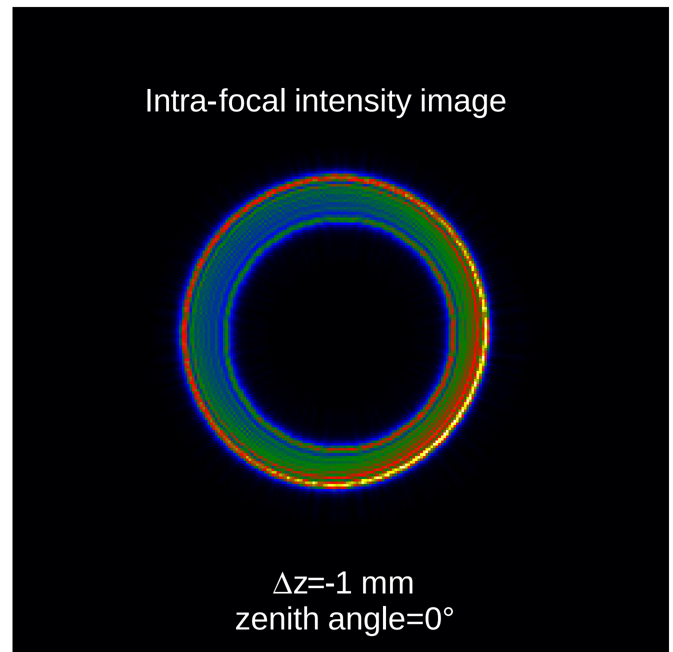


ACTUAL PUPIL PHASE
c_4 to c_9 = +400, -400, +300, -300, +200, -200 nm

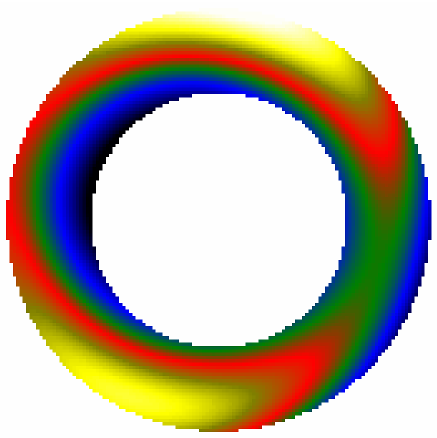


RECONSTRUCTED PUPIL PHASE
c_4 to c_9 = +398, -388, +290, -272, +185, -169 nm (z=0°)
c_4 to c_9 = +400, -365, +290, -260, +183, -154 nm (z=80°)

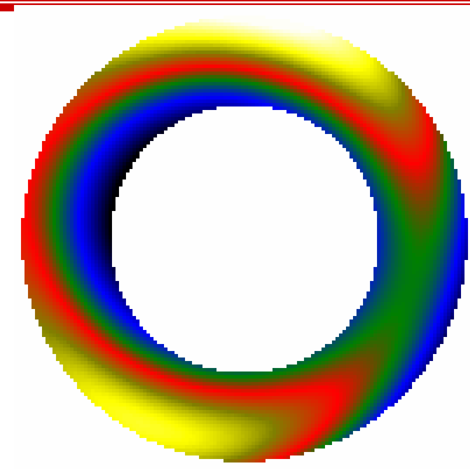




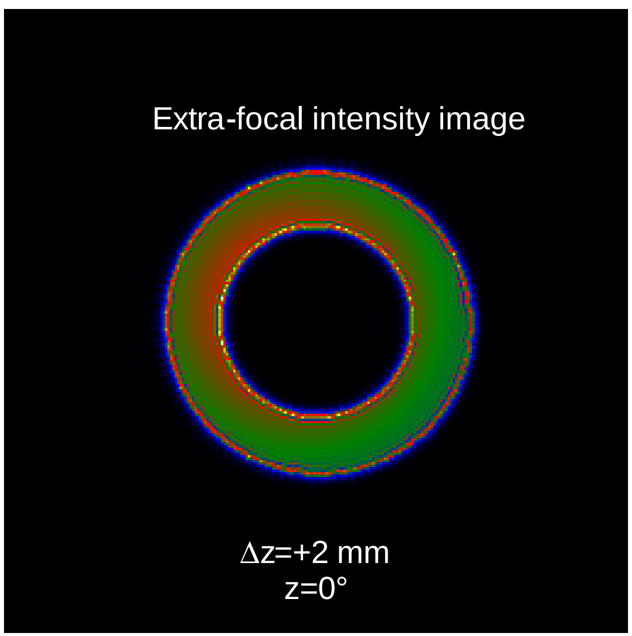
CWFS reconstruction example with no turbulence and |Δz|=2 mm

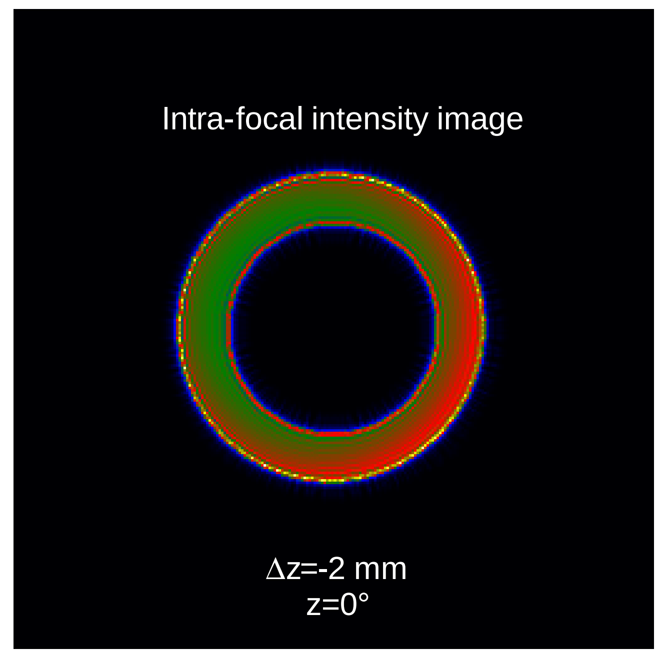


ACTUAL PUPIL PHASE
c_4 to c_9 = +400, -400, +300, -300, +200, -200 nm

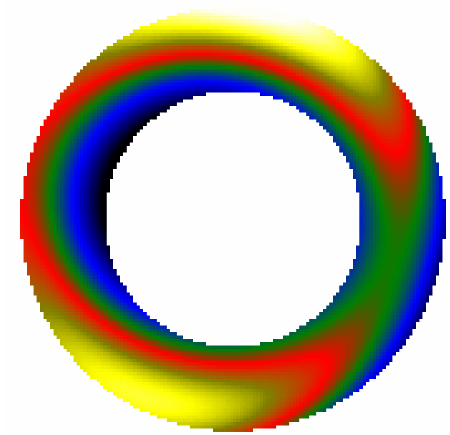


RECONSTRUCTED PUPIL PHASE
c_4 to c_9 = +397, -389, +295, -285, +191, -185 nm (z=0°)
c_4 to c_9 = +390, -393, +302, -300, +190, -179 nm (z=80°)



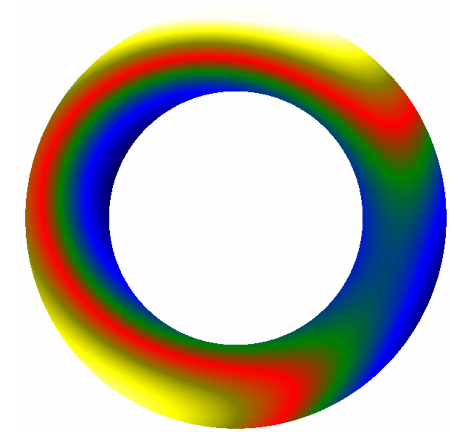


CWFS reconstruction example with turbulence and |Dz|=1 mm



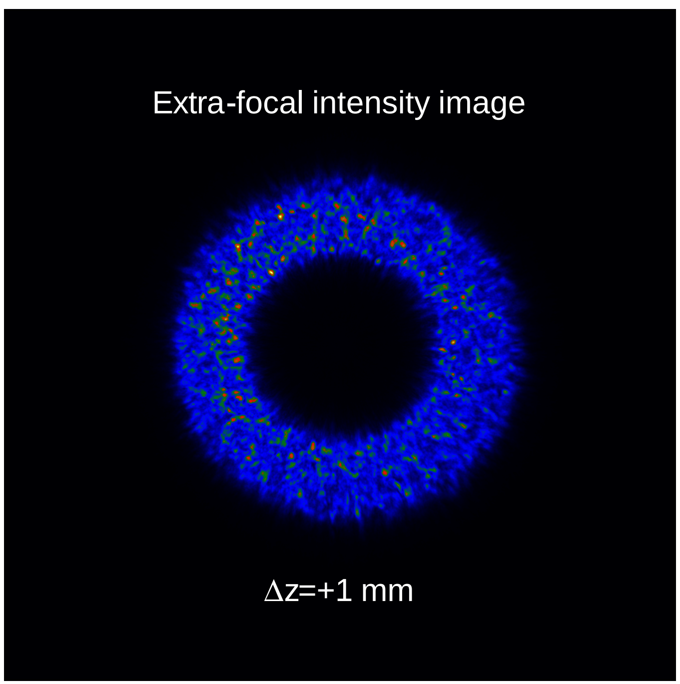
ACTUAL PUPIL PHASE

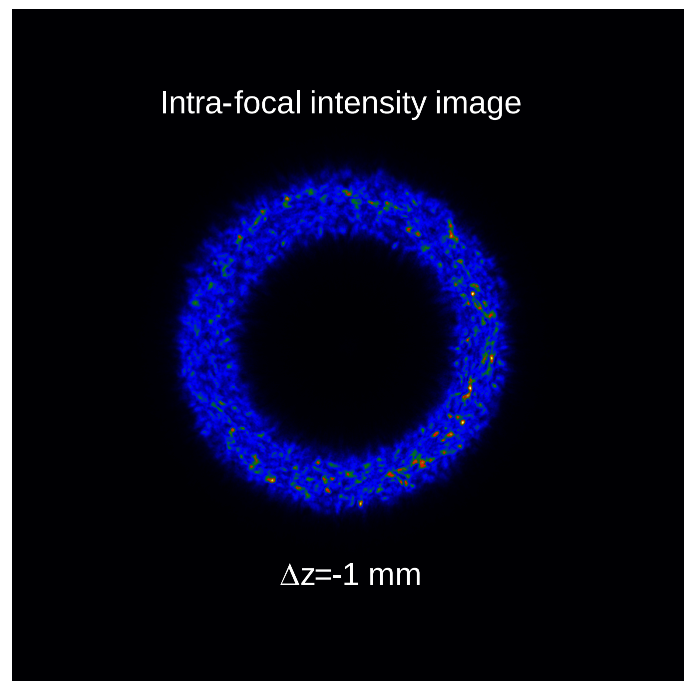
c_4 to c_9 = +400, -400, +300, -300, +200, -200 nm



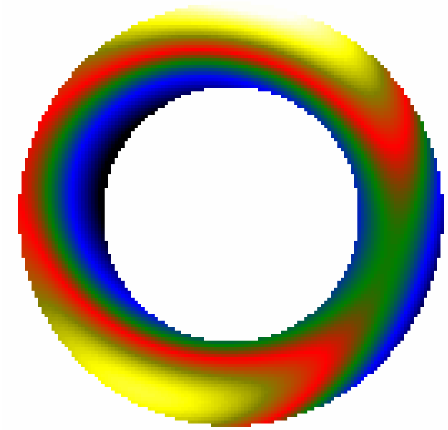
RECONSTRUCTED PUPIL PHASE

c_4 to c_9 = +385, -387, +291, -244, +167, -101 nm

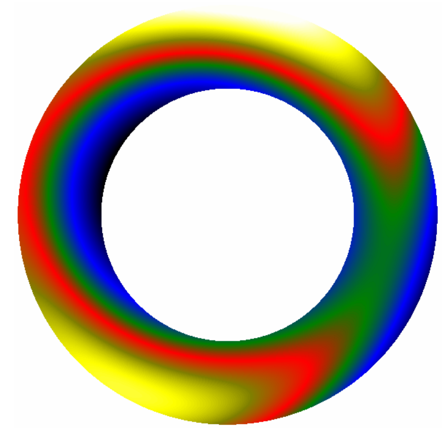




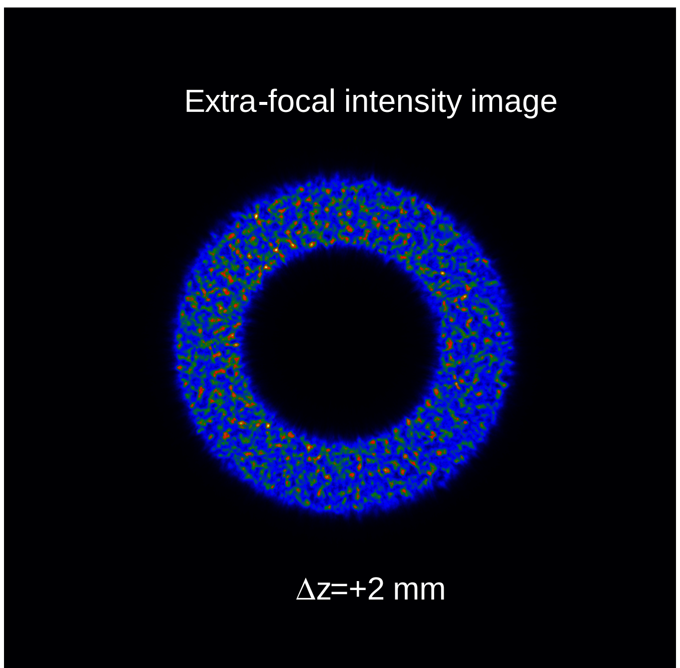
CWFS reconstruction example with turbulence and |Dz|=2 mm

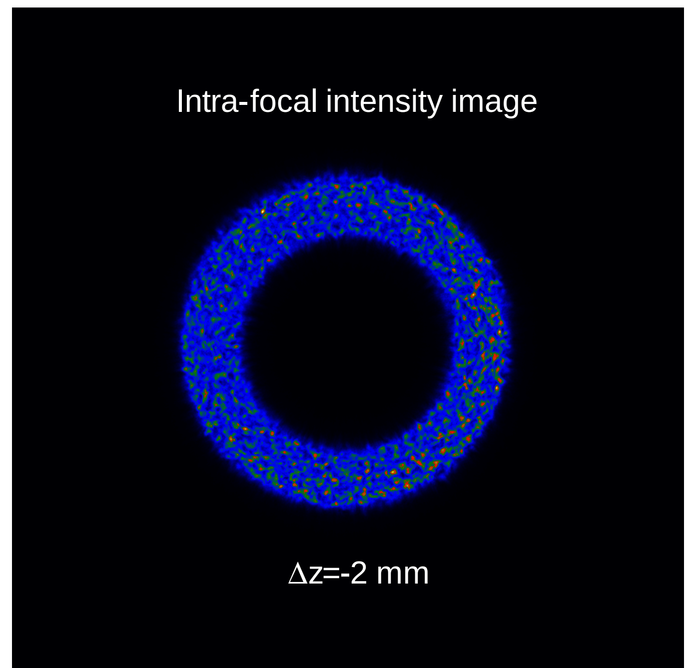


ACTUAL PUPIL PHASE
c_4 to c_9 = +400, -400, +300, -300, +200, -200 nm



RECONSTRUCTED PUPIL PHASE
c_4 to c_9 = +398, -394, +296, -281, +190, -166 nm





CWFS simulation in the Telescope AO code

- **Wave Optics Image Generation:**
  - The phase and amplitude are specified by equations. Can include any aberrations, vignetting, secondary spider, etc.
  - Can specify the optical system parameters including the spectral band
  - Can average the intensity for multiple wavelengths covering the spectral band
  - Can average over Kolmogorov turbulence with wind
  - Can include photon and read noise. Several additional variables must be specified, such as the star's bolometric magnitude and surface temperature, the integration time, and the optical system's average transmission in the spectral band.
  - Can display the image on the CCD grid as well as relayed to object space

- **Solving the TIE**
  - Can reconstruct using either the circular or annular Zernikes
  - Registration is done iteratively: The starting registration registers either the intensity centroid or the area centroid using a fractional intensity threshold. This is refined by finding the registration that makes the tilts zero in the reconstruction.
  - Masking

Averaging over turbulence with wind

- You can't just generate the average phase screen and use that to produce the images
- You must calculate the intensity multiple times with turbulence and average those intensity images together
- However, then you've added an average unknown amount of turbulence and you have two variables you are testing simultaneously – the effect of the turbulence on the reconstruction and the effect of the new phase errors of the turbulence.
- To fix this problem, repeat the above steps with the inverse phase screen at each of the locations
- This produces lots of images (especially if you are averaging multiple wavelengths) and can be slow

Vignetting

- The curvature sensors for LSST are located at the edge of the field, where vignetting will occur
- Using two images with different vignetting causes the reconstruction to not work
- However, if you know the vignetting (meaning you know the location of the stars on the image plane and you can calculate the amount of vignetting), you can calculate the effect it causes on the reconstructed Zernike coefficients
- Then you can use those coefficients calculated in the step above as correction factors

Vignetting and turbulence – actual coefficients

- **You need to apply a correction if you have different vignetting between the two images**
  - **Example below with 9 terms (Performance is similar with 22 terms)**

VIGNETTED IN -X AT Z=-53.345 KM AND IN -Y AT Z=+53.345 KM
WITH TURBULENCE
dz_CCD=2 mm +/- 53.345 km

Zernike terms	Applied Zernike coefficients	Coefficients from TIE algorithm: Vignetted LSST annulus — No phase applied "Correction factors"	Coefficients from TIE algorithm: Vignetted LSST annulus — Phase: power law with first 9	Reconstruction error before correction (Difference between applied and calculated coefficients)	Corrected vignetted LSST annulus with power law phase on first 9 coefficients	Reconstruction error after correction (Difference between applied and calculated		
0	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00		
1	0.00E+00	-1.92E-03	-1.93E-03	1.93E-03	-6.57E-06	6.57E-06		
2	0.00E+00	1.92E-03	1.91E-03	-1.91E-03	-7.77E-06	7.77E-06		
3	-2.25E-04	6.70E-08	-2.22E-04	-3.98E-06	-2.22E-04	-3.92E-06		
4	-1.19E-04	7.59E-04	6.42E-04	-7.61E-04	-1.17E-04	-2.46E-06		
5	1.58E-04	3.51E-08	1.54E-04	3.43E-06	1.54E-04	3.47E-06		
6	-7.85E-05	1.44E-04	6.96E-05	-1.48E-04	-7.42E-05	-4.36E-06		
7	-9.52E-05	-1.44E-04	-2.34E-04	1.38E-04	-9.00E-05	-5.15E-06		
8	5.04E-05	-1.60E-07	4.54E-05	5.00E-06	4.55E-05	4.84E-06		
			RMS ERROR	7.88E-04	mm	1.01E-05	mm	
			RMS ERROR	7.88E-01	um	1.01E-02	um	
			variance	6.20E-01	um^2	1.02E-04	um^2	
			variance	41.26128707	rad^2	0.006805441	rad^2	(At 770.5nm)
			Strehl	**0.000000**		**0.999954**		

Registration

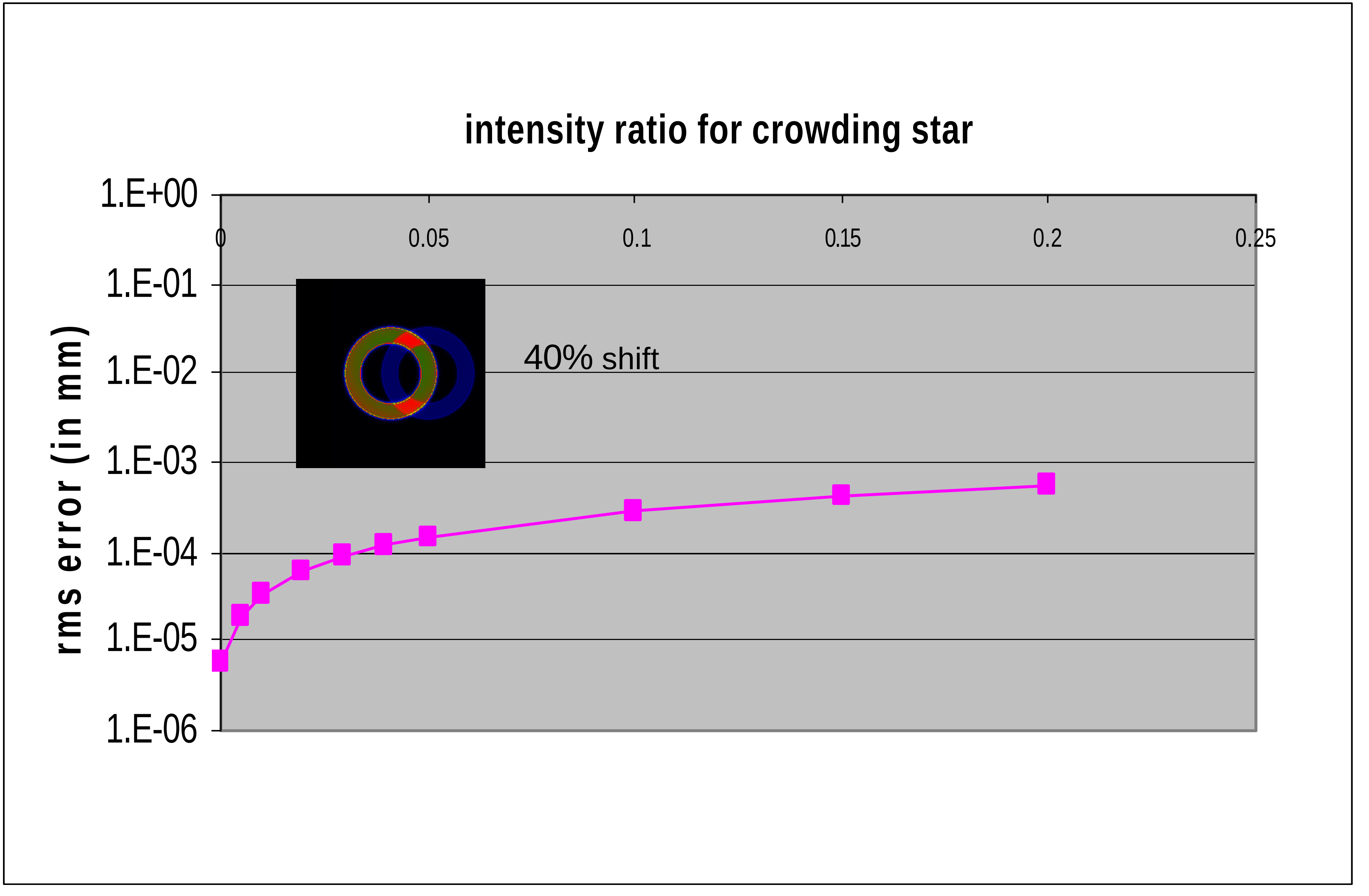
- If the CWFS focal plane is split rather than moving in z, then the registration will be unknown.

- Two methods of initially registering the images are used:
  - By computing the intensity centroid with a threshold
  - By computing the area centroid with a threshold

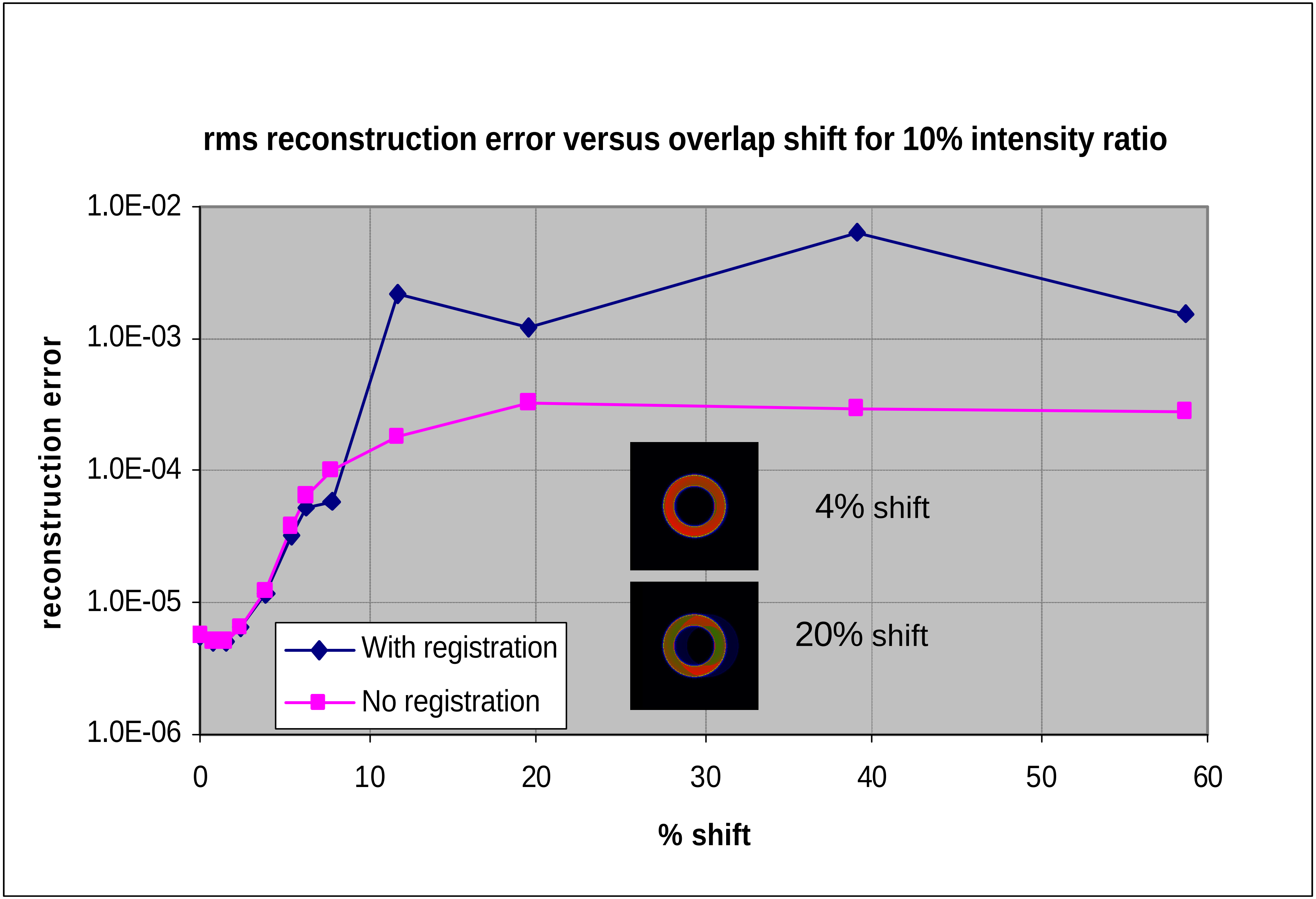
- Solving the TIE with the initial registration will give non-zero tilt coefficients

- Do a trial solution +5 pixels in x and +5 pixels in y to test effect on the tilts, then do a second solution with the images shifted accordingly to make the tilts vanish.

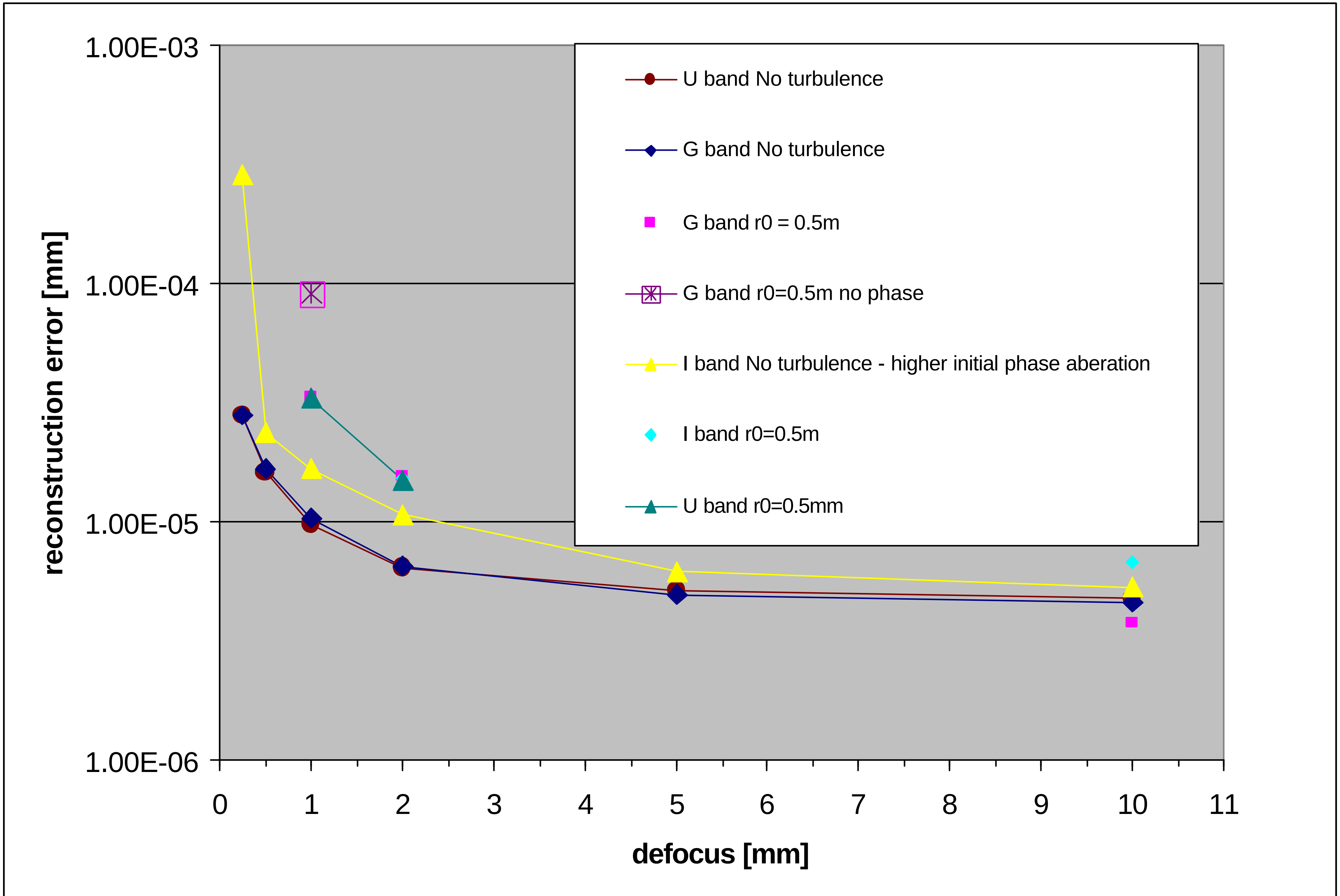
Effect of crowding on the CWFS reconstruction accuracy



Crowding simulations



Effect of turbulence for the different spectral bands



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