Introduction to Particle Swarm Optimization

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November 17, 2006

CASIS Workshop

November 16–17, 2006

UCRL-PRES-226157

Acknowledgments

This work was performed under the auspices of the U.S. Department of Energy by University of California Lawrence Livermore National Laboratory under contract No. W-7405-Eng-48.

Next year: Who knows?

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- Inter-particle communication of the swarm best (global) location;
- Competition between individual particle’s best location and swarm’s best location;
- A bit of biomimicry.

Particle Swarm Optimizer (PSO) Fundamentals

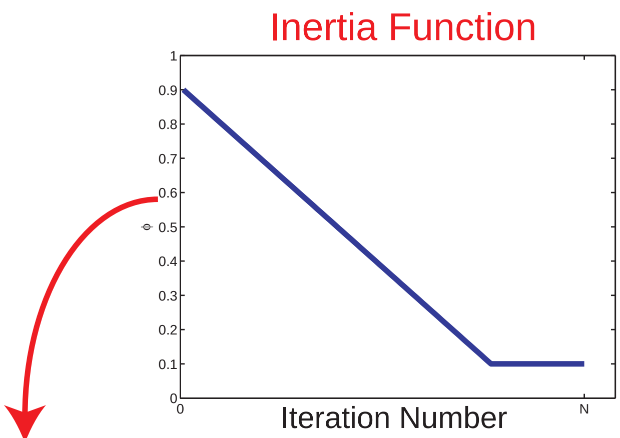
Velocity of *n*-th particle

$$v_n(t+1) = \phi(t)\, v_n(t) + \alpha_1 \gamma_{1n}(t)\, [p_n - x_n(t)] + \alpha_2 \gamma_{2n}(t)\, [G - x_n(t)] \quad (1)$$

$$x_n(t+1) = x_n(t) + v_n(t) \quad (2)$$

Location of *n*-th particle

Particle Swarm Optimizer (PSO) Fundamentals



$$v_n(t+1) = \phi(t)\, v_n(t) + \alpha_1 \gamma_{1n}(t)\left[p_n - x_n(t)\right] + \alpha_2 \gamma_{2n}(t)\left[G - x_n(t)\right] \quad (1)$$

$$x_n(t+1) = x_n(t) + v_n(t) \quad (2)$$

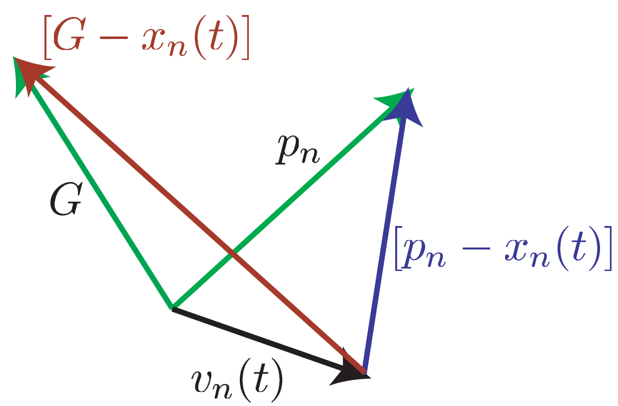
Position update

Particle Swarm Optimizer (PSO) Fundamentals

Particle best position

$$
\begin{aligned}
v_n(t+1) &= \phi(t)\, v_n(t) + \alpha_1 \gamma_{1n}(t)\, [p_n - x_n(t)] + \alpha_2 \gamma_{2n}(t)\, [G - x_n(t)] \\
x_n(t+1) &= x_n(t) + v_n(t)
\end{aligned}
$$

Swarm global best position



Particle Swarm Optimizer (PSO) Fundamentals

Acceleration constant

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$$v_n(t+1) = \phi(t)\, v_n(t) + \alpha_1 \gamma_{1n}(t)\, [p_n - x_n(t)] + \alpha_2 \gamma_{2n}(t)\, [G - x_n(t)] \quad (1)$$

$$x_n(t+1) = x_n(t) + v_n(t) \quad (2)$$

Particle Swarm Optimizer (PSO) Fundamentals

Random Random

$$v_n(t+1) = \phi(t)\, v_n(t) + \alpha_1 \gamma_{1n}(t)\, [p_n - x_n(t)] + \alpha_2 \gamma_{2n}(t)\, [G - x_n(t)] \quad (1)$$

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$$v_n(t+1) = \phi(t)v_n(t) + \alpha_1\gamma_{1n}(t)\left[p_n - x_n(t)\right] + \alpha_2\gamma_{2n}(t)\left[G - x_n(t)\right] \quad (1)$$

$$x_n(t+1) = x_n(t) + v_n(t) \quad (2)$$

where

n	is the particle number;
t	is the time step;
$x_n(t)$	is the location of the n-th particle at time t;
$v_n(t)$	is the velocity of the n-th particle at time t;
$\phi(t)$	is the “inertia” function;
α_1 & α_2	are “acceleration” constants;
$\gamma_{1n}(t)$ & $\gamma_{2n}(t)$	are $[0,\ 1]$ uniformly distributed random numbers;
p_n	is the n-th particle’s best location;
G	is the entire swarm’s best location

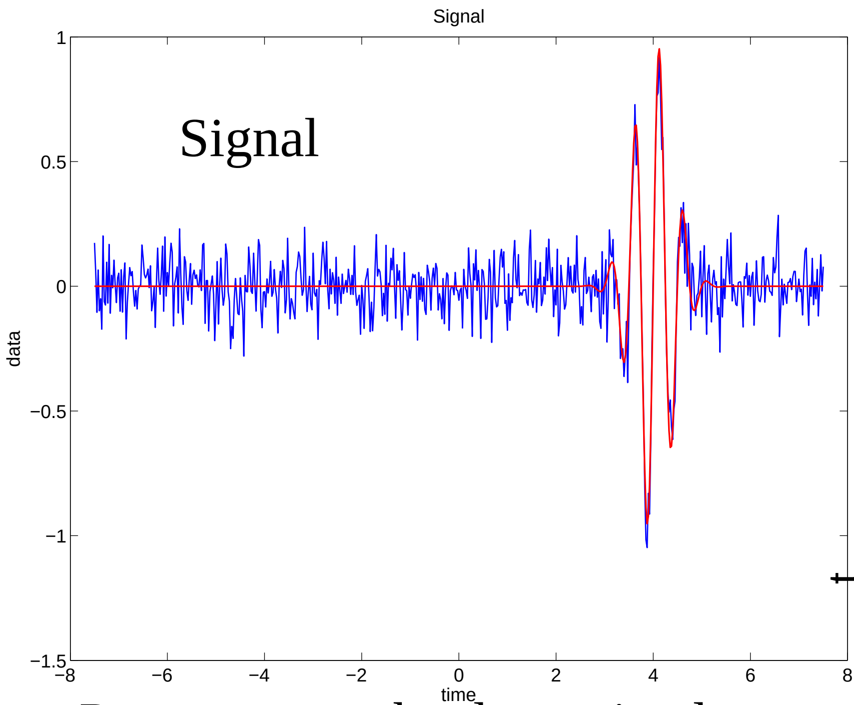
$x_n(t)$, $v_n(t)$, p_n, and G are N-dimensional vectors

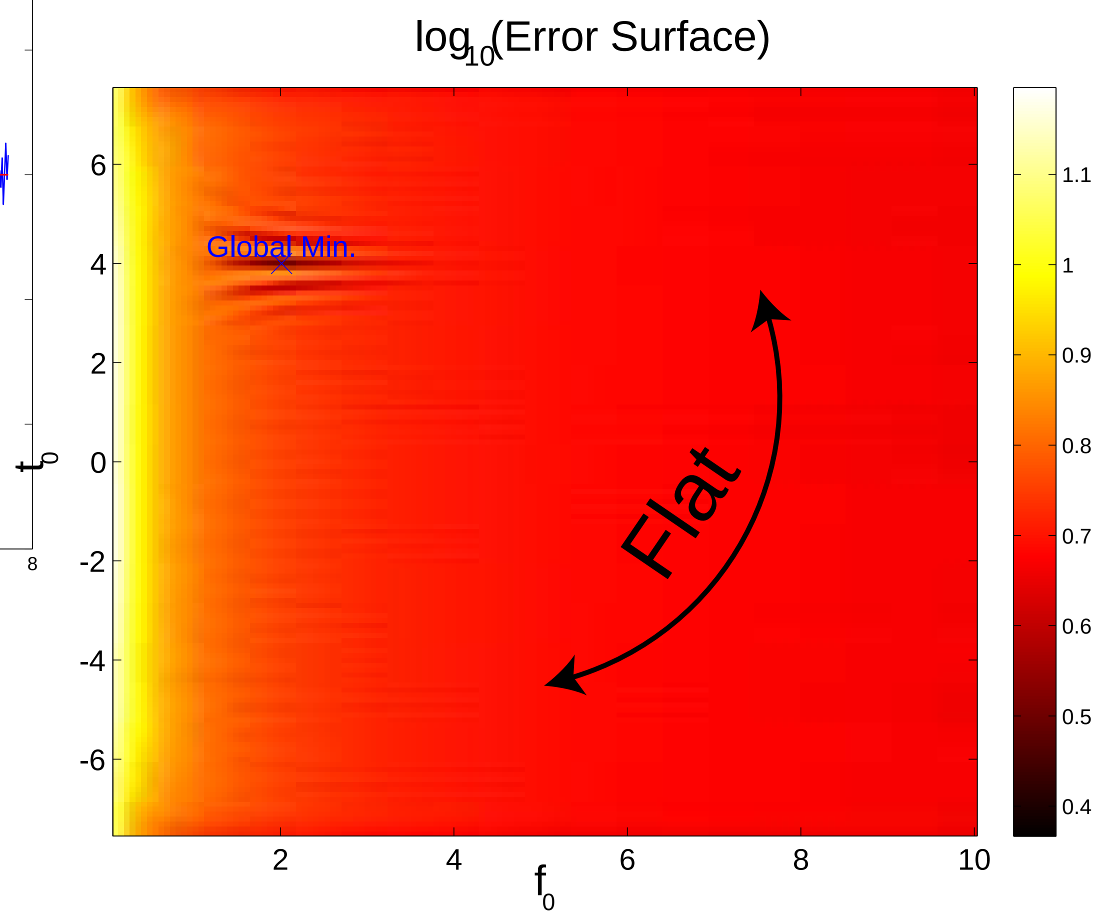
Codes

- See `www.mathworks.com` user contributed codes;
- My `flight.m`

MATLAB example...

Determine Frequency & Time Shift Of Gaussian Windowed Sinusoid



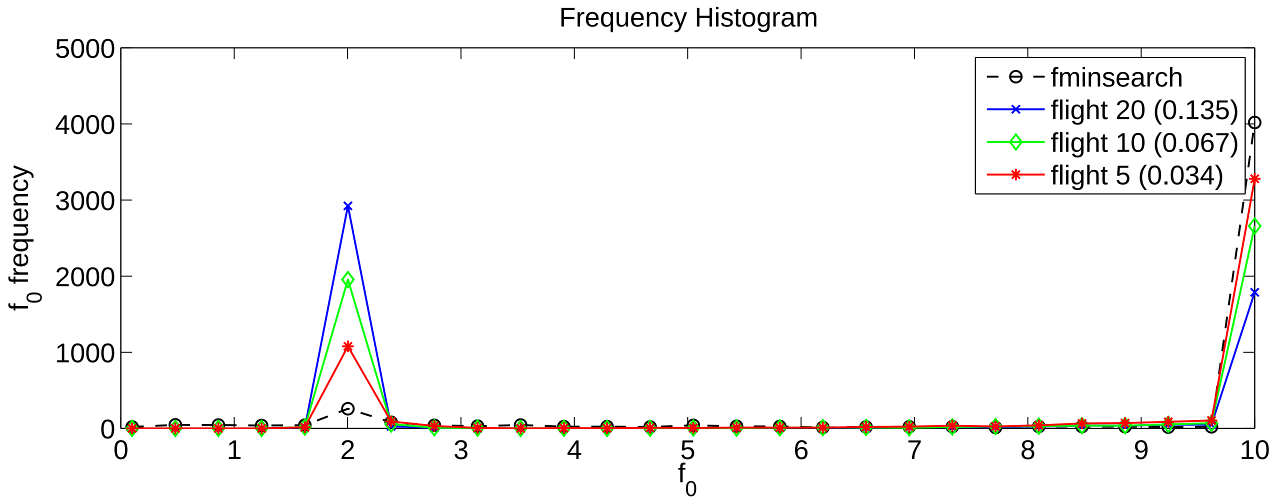


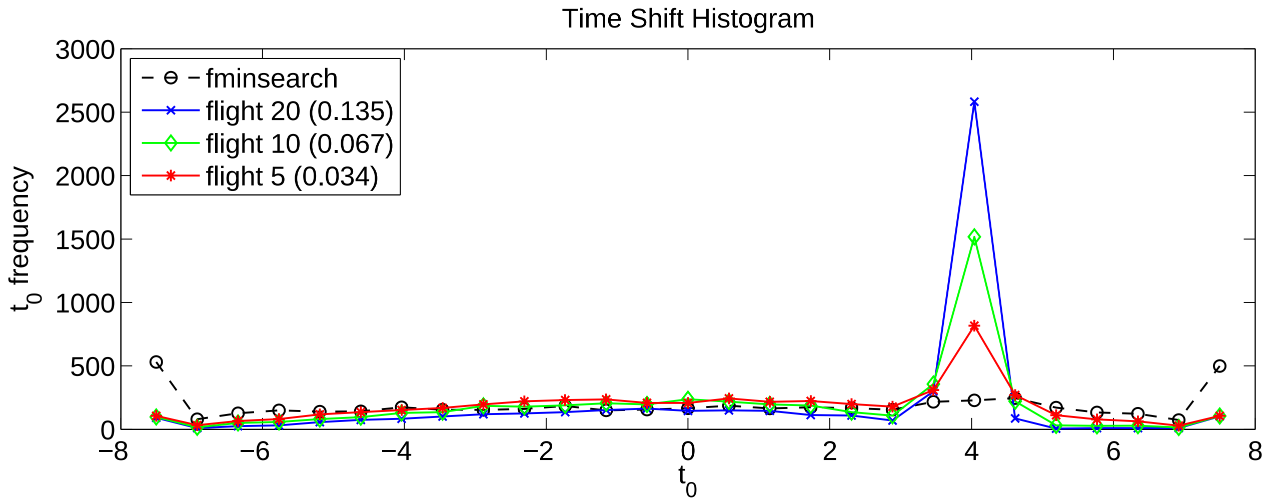
Parameters to be determined:

- Frequency, $f_0 = 2$;
- Time Shift, $t_0 = 4$;
- 20 dB SNR.

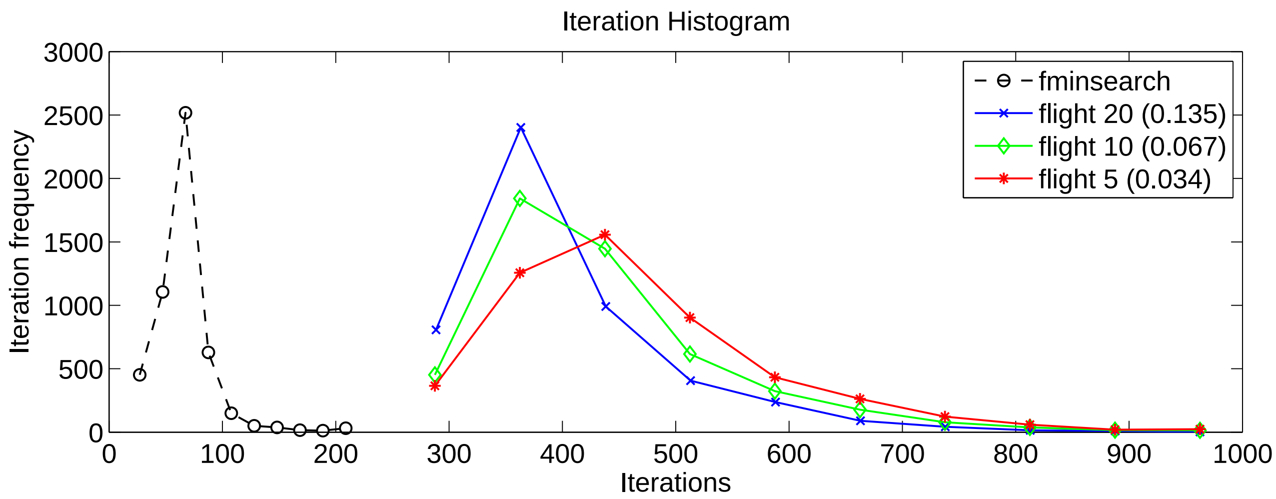
Frequency & Time Shift Estimation Results

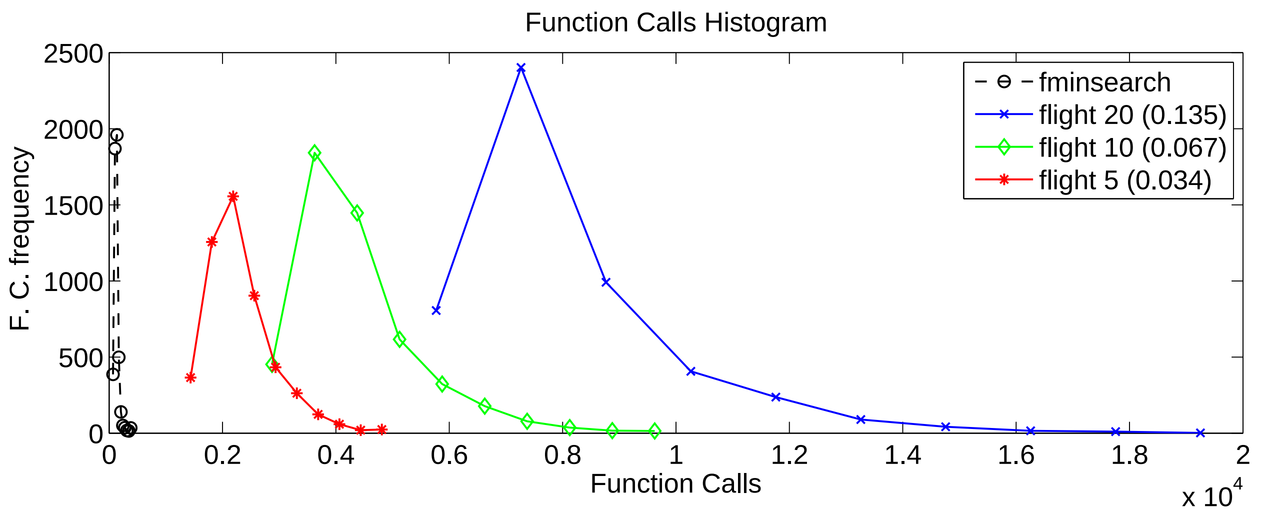
- PSO vs Gradient-based method (`fminsearch`);
- 5001 Monte Carlo runs;
- 20 dB SNR;
- Legend:
  - Gradient-based method;
  - 20 particles, density of 0.14;
  - 10 particles, density of 0.07;
  - 5 particles, density of 0.03
- `flight` out performs `fminsearch`...



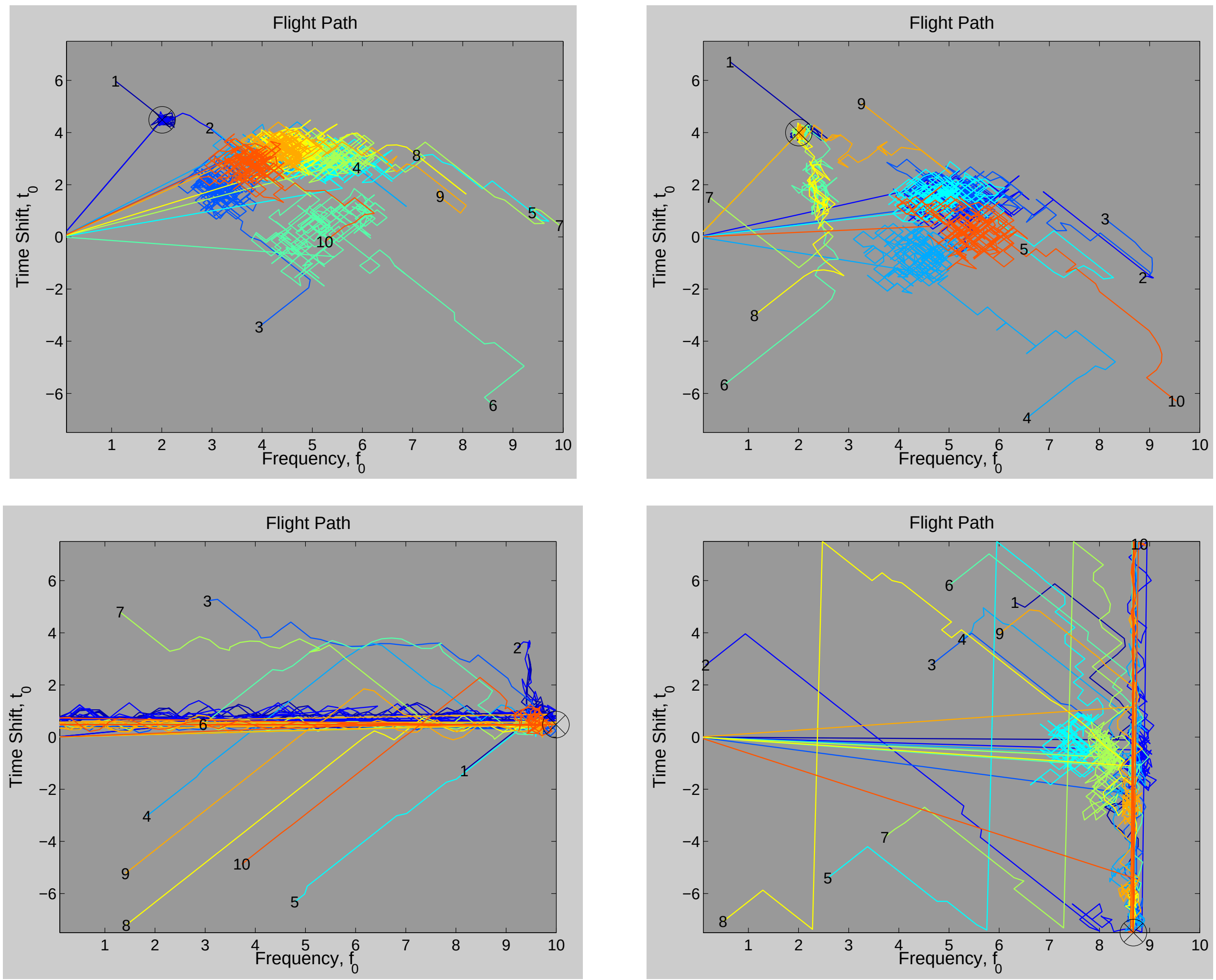


`fminsearch` Has Fewer Iterations & Function Calls





Sample Flight Paths



Conclusion

- If we had an infinite particle density, we could explore the space in one iteration;
- Short of that, use a “handful” of particles and terminate the flight early using the global best location as the initial guess for the gradient-based method.