Model-based Layer Estimation using a Hybrid Genetic/Gradient Search Optimization Algorithm

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Auspices

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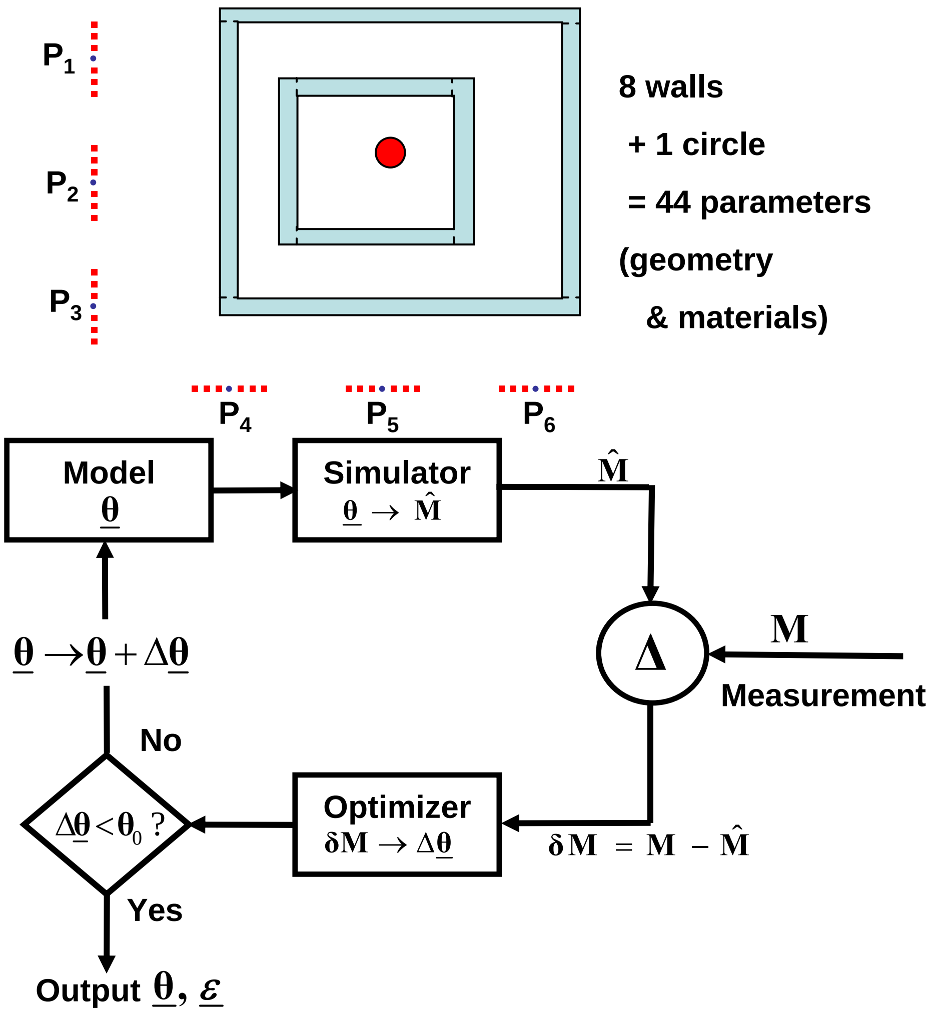
Topics

- Short survey of model-based tomographic approach
- Application to one-dimensional multilayers
- Problem of multiple minima in error functional
- Parameter studies (noise, wall characteristics)
- Monte Carlo simulations for statistical analysis

Model-based tomographic reconstruction of building structure using radar



- Building interior model is a collection of basic elements (walls, doors, ceilings, …) parameterized by geometry (position, size) and material (permittivity, conductivity)
- From model and sensor position data, calculate radar return from building using a fast propagation code

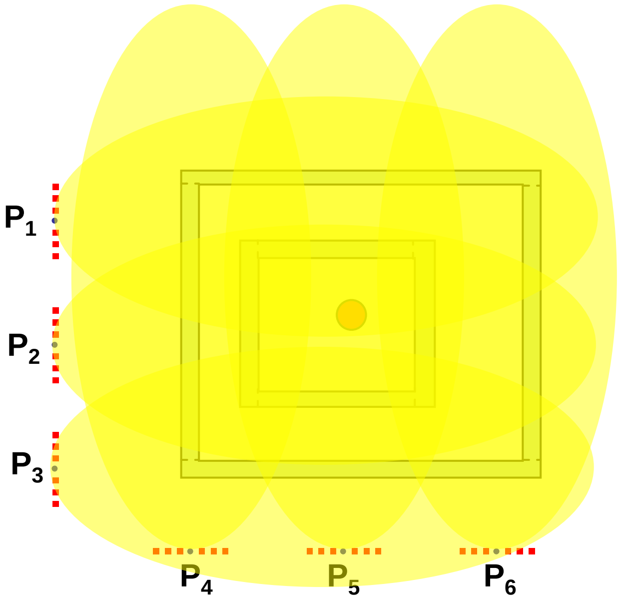
  Multistatic response matrix: $\hat{\mathbf{K}}(t;\boldsymbol{\theta})$

- Compare predicted data with measured data $\mathbf{K}(t;\boldsymbol{\theta})$

$$E(\boldsymbol{\theta};t_0,t_f) = \int_{t_0}^{t_f} \left\| \mathbf{K}(t) - \hat{\mathbf{K}}(t;\boldsymbol{\theta}) \right\|^2 dt + C(\theta)$$

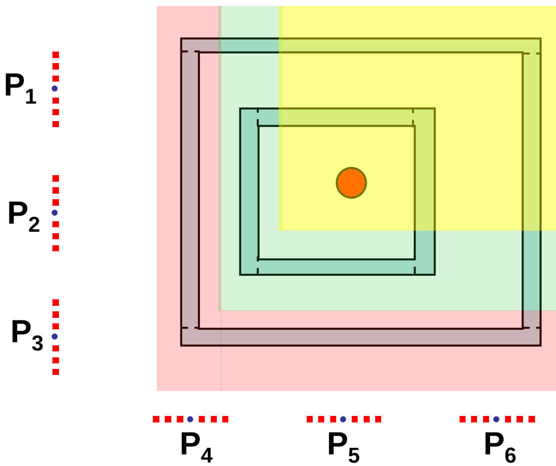
- Minimize error *E* to obtain optimal set of model parameters

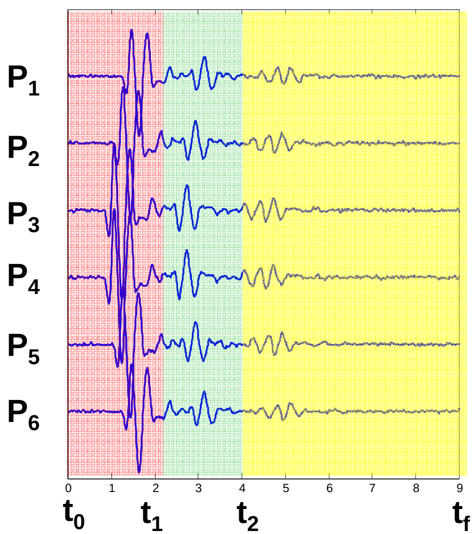
Collect radar data set for building



- Suppose we have an linear array system of one transmitter and six receivers
- Array interrogates structure at six different locations P_1 through P_6
- At location P_n the receivers record signals $K_{mn}(t)$; m=1,2,…,6
- Collection of all signals forms the 6 x 6 multistatic response matrix K(*t*)

Apply time gating and estimate parameters in sequence from outside to inside





- **Divide total time into 3 intervals $t_0 < t_1 < t_2 < t_f$**
- **First interval contains return from nearest two walls => 10 parameters θ_1, 3 constraints $C_1(\theta_1)$. Minimize $Q_1(\theta_1)$:**

$$E_1(\boldsymbol{\theta}_1) = \int_{t_0}^{t_1} \left\| \mathbf{K}(t) - \hat{\mathbf{K}}(t;\boldsymbol{\theta}) \right\|^2 dt + C_1(\boldsymbol{\theta}_1)$$

- **Next interval contains second walls => 10 parameters θ_2, 3 constraints $C_2(\theta_2)$. Minimize $Q_2(\theta_2)$:**

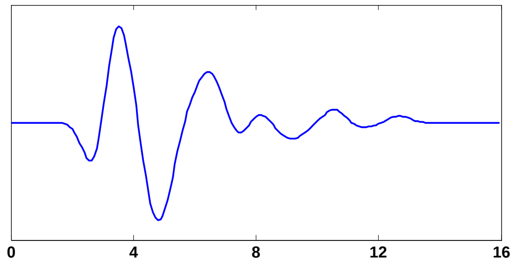
$$E_2(\boldsymbol{\theta}_2) = \int_{t_1}^{t_2} \left\| \mathbf{K}(t) - \hat{\mathbf{K}}(t;\boldsymbol{\theta}) \right\|^2 dt + C_2(\boldsymbol{\theta}_2)$$

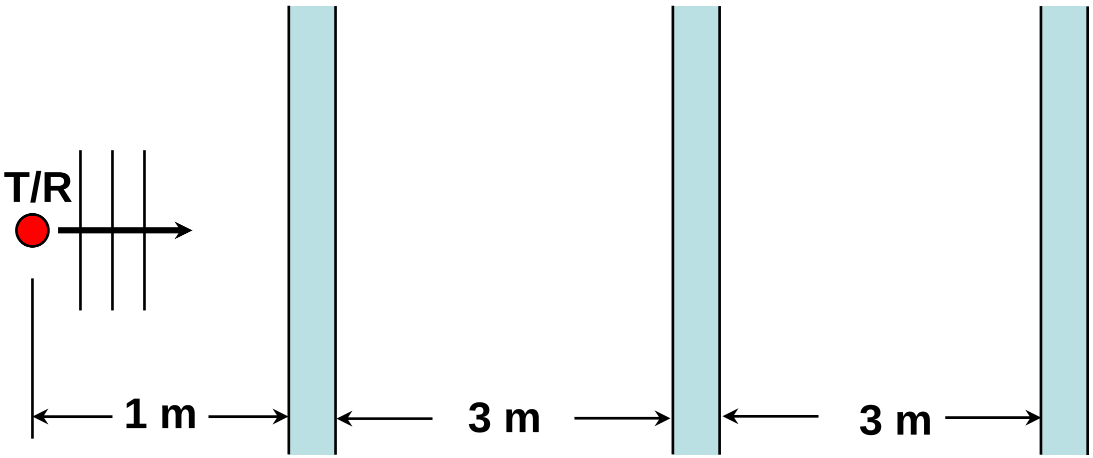
- **Remaining time interval => 24 parameters θ_3, 8 constraints $C_3(\theta_3)$**

$$E_3(\boldsymbol{\theta}_3) = \int_{t_2}^{t_f} \left\| \mathbf{K}(t) - \hat{\mathbf{K}}(t;\boldsymbol{\theta}) \right\|^2 dt + C_3(\boldsymbol{\theta}_3)$$

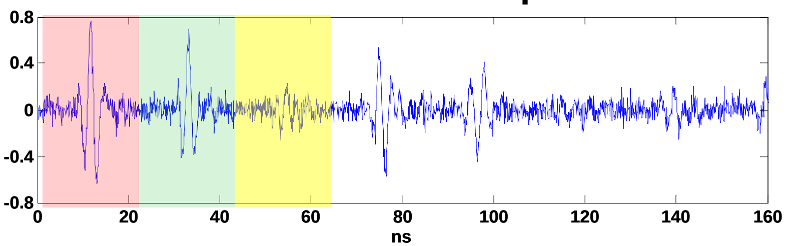
Model-based reconstruction – 1d
Three walls (ε_r = 5.593, σ/ε_0 = 0.025, T = 0.1 m)

Input pulse, length 16 ns





Generate Received pulse



Add noise (23 dB WGN)

Estimate wall parameters using range gating

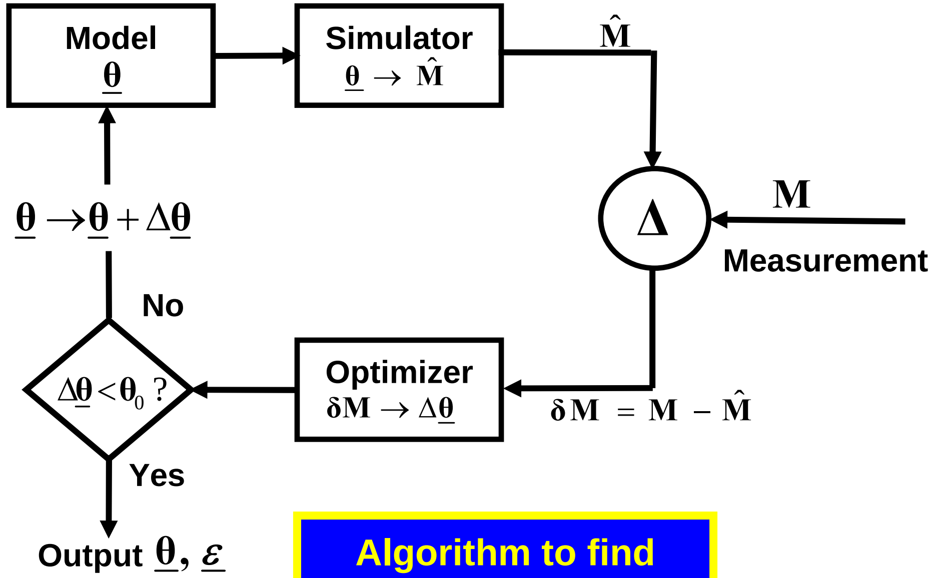
	Range	Thickness	ε_r	σ/ε_0
Wall 1	1.0009	0.1007	5.53	-0.028
Wall 2	4.098	0.107	4.89	-0.009
Wall 3	7.204	0.11	4.59	-0.003

Construction of model-based 1D multilayer reconstruction algorithm

For each layer:

- Range
- Thickness
- Permittivity
- Conductivity

1D propagation through multilayer



Algorithm to find minimum in error function

1D multilayer propagation

- **An incoming electric field $E_i(z,t)$ is reflected from a stack of N material layers beginning at z=0**

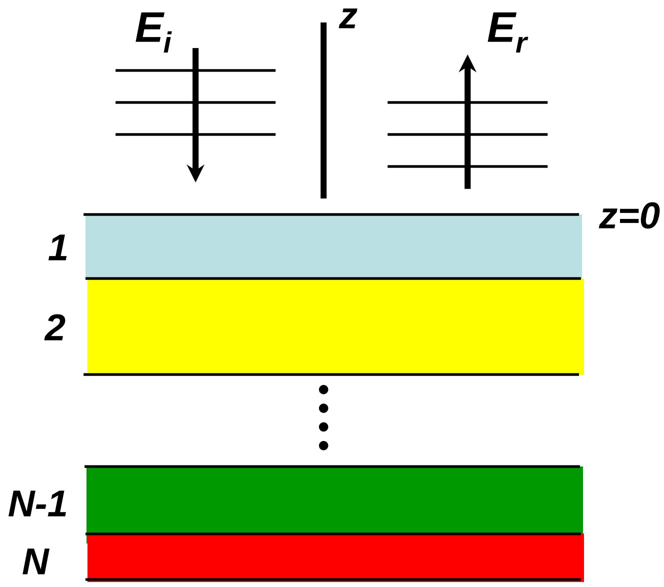
- **At z=0, the reflected field is given by**

$$E_r(0,t) = \int_{-\infty}^{\infty} \hat{E}_i(0,\omega) R_{eff}(\omega) e^{i\omega t} \, d\omega$$

where R_{eff} is the effective reflection coefficient

- **If $E_i(z,t)$ is generated by a pulse $p(t)$ emitted at z_s, then the received pulse at z_s is**

$$E_r(z_s,t) = \int_{-\infty}^{\infty} \hat{p}(\omega) e^{-2i\omega z_s/c_0} R_{eff}(\omega) e^{i\omega t} \, d\omega$$



$\hat{p}(\omega)$ **is the Fourier transform of** $p(t)$

R_{eff} is obtained iteratively

- Define $R_{eff} = \tilde{R}_{0,1}$

  where $\tilde{R}_{0,1}$ is obtained by solving the backwards iterative equation

$$\tilde{R}_{j,j+1} = \frac{R_{j,j+1} + \tilde{R}_{j+1,j+2} X_{j+1}^2}{1 + R_{j,j+1} \tilde{R}_{j+1,j+2} X_{j+1}^2} \qquad X_j = e^{-2i\omega\ell_j / c_j}$$

  starting with $R_{N+1,N+2} = 0$

- $R_{j,j+1}$ are the Fresnel reflection coefficients

$$R_{j,j+1} = \frac{n_j - n_{j+1}}{n_j + n_{j+1}} \qquad n_j = \frac{c_0}{c_j} = \sqrt{\varepsilon_j + \frac{i\sigma_j}{\varepsilon_0 \omega}} \qquad n_0 = n_{N+1} = 1$$

  jth layer characterized by permittivity ε_j, conductivity σ_j, and thickness ℓ_j

LM (gradient) error minimization algorithm

- **Problem: Find parameters θ that minimize the error function**

$$E(\boldsymbol{\theta};t_0,t_f)=\int_{t_0}^{t_f}\left\|\mathbf{K}(t)-\hat{\mathbf{K}}(t;\boldsymbol{\theta})\right\|^2 dt+C(\theta)$$

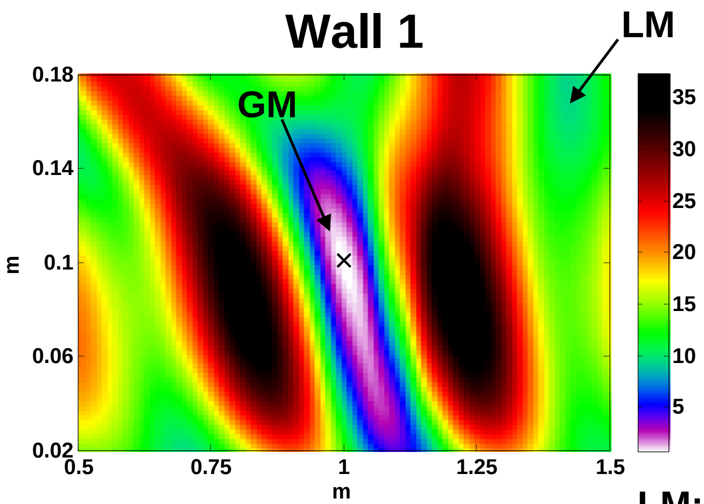
- **Levenberg-Marquardt – based on Newton's method for finding roots**
  - **Discrete form for error:** $E(\boldsymbol{\theta};t_0,t_f)=\frac{1}{2}\sum_{n=1}^{N}\left\|\mathbf{r}_n(\boldsymbol{\theta})\right\|^2 ,\quad \mathbf{r}_n(\boldsymbol{\theta})=\mathbf{K}(t_n)-\hat{\mathbf{K}}(t_n;\boldsymbol{\theta})$
  - **Start with initial estimate θ_0**
  - **Recursively update using**

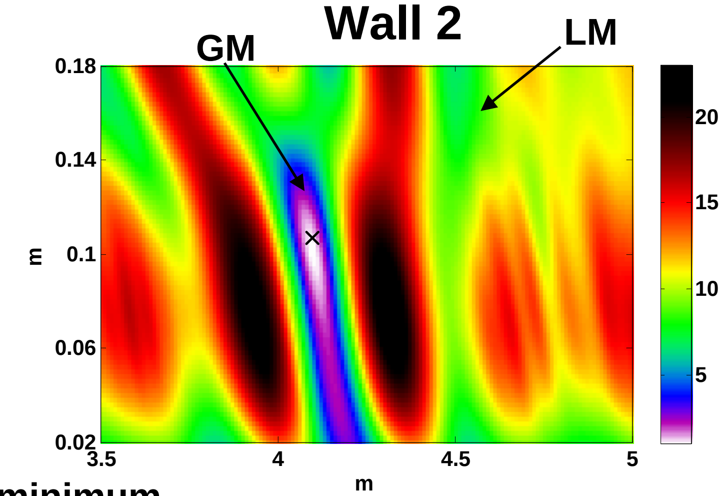
$$\boldsymbol{\theta}_{k+1}=\boldsymbol{\theta}_k-\left[J(\boldsymbol{\theta}_k;\mathbf{K})^T J(\boldsymbol{\theta}_k;\mathbf{K})+\lambda\mathbf{I}\right]^{-1} J(\boldsymbol{\theta}_k;\mathbf{K})^T\mathbf{r}(\boldsymbol{\theta}_k)\ ,\ \nabla_\theta E(\boldsymbol{\theta}_k)=J(\boldsymbol{\theta}_k;\mathbf{K})^T\mathbf{r}(\boldsymbol{\theta}_k)$$

  - **Convergence when error is minimum or θ_k no longer changes signficantly**

Gradient methods can fail when there are multiple local minima in *E*

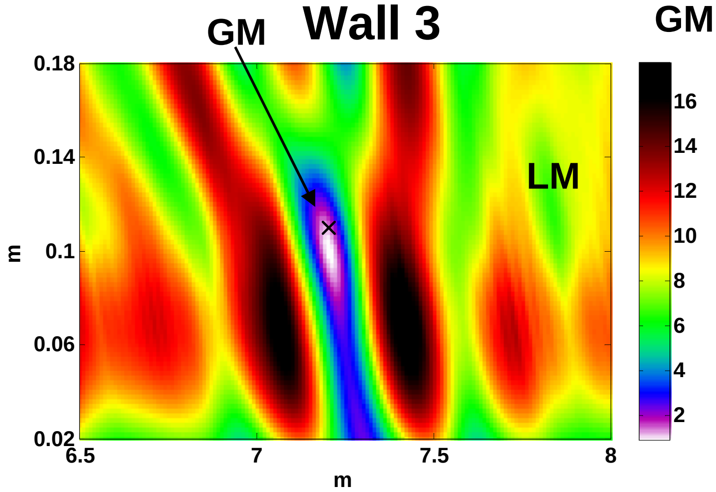
Range-thickness slices of error functions show several local minima around global minimum





LM: Local minimum

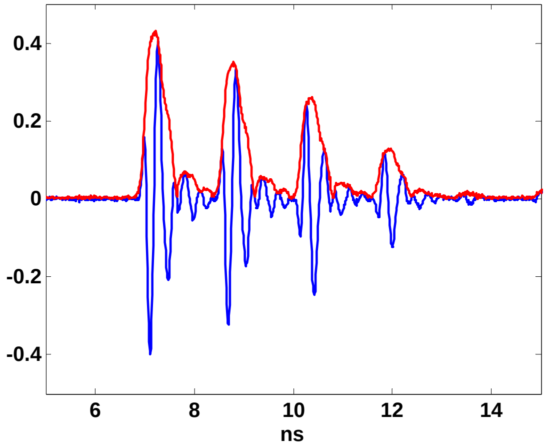
GM: Global minimum

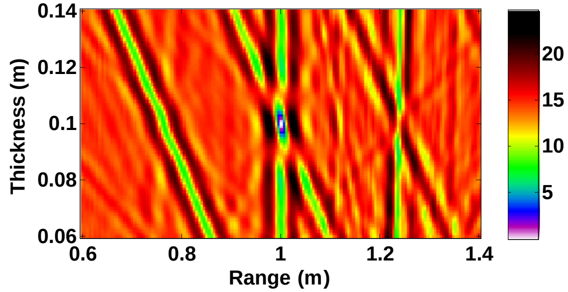


Width of global minimum scales with the autocorrelation of the transmitted pulse

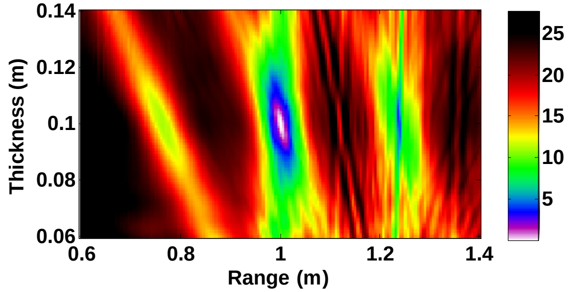
Preprocessing of data can smooth error surfaces or tighten search region

- **Match envelopes**





Range-thickness error surface for full signal



Range-thickness error surface for envelope

- **Initialize range estimate using correlation-based time-of-arrival estimates**

These help but still can wander off the GM.

Can we find a more robust method?

Particle Swarm Optimization (PSO)

- PSO searches for the global minimum of a multi-dimensional error function within a given hyper-volume
- Start by generating a random set of sampling points (particles) within the volume
- Calculate error function at particle positions $x_i(0)$. Update particle positions based on results: $x_i(k+1) = x_i(k) + v_i(k+1)$

$$v_i(k+1) = \phi(k)v_i(k) + \alpha_1\gamma_{1i}(k)\left[p_i(k) - x_i(k)\right] + \alpha_2\gamma_{2i}(k)\left[G(k) - x_i(k)\right]$$

$\phi(k)$: inertia term $\gamma_{1,2}$: random numbers uniform on (0,1)

p_i: position of minimum along trajectory of *ith* particle (single particle best)

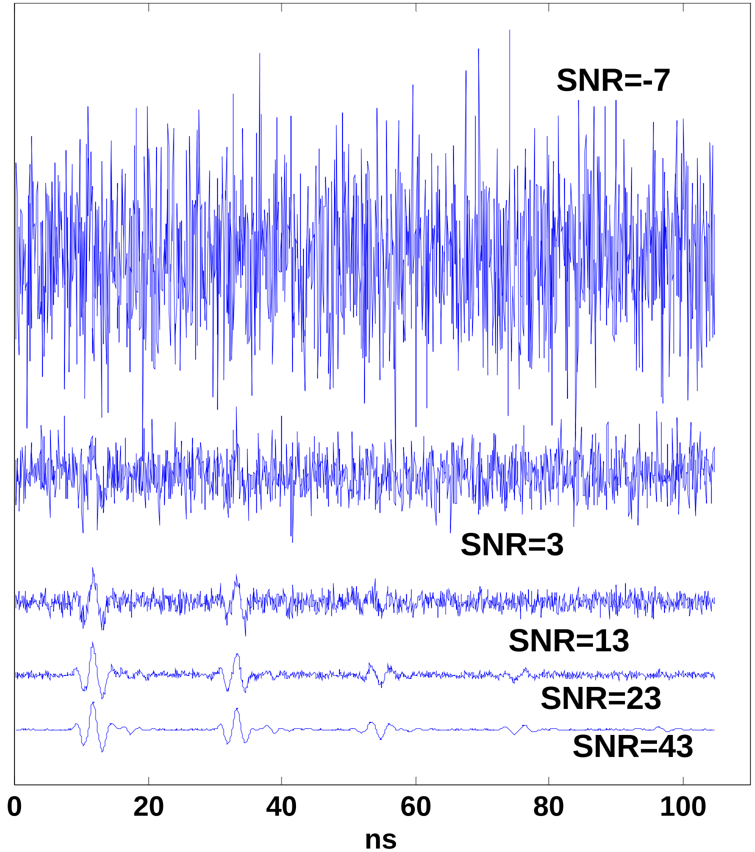
$G(k)$: position of minimum over all particles (swarm best)

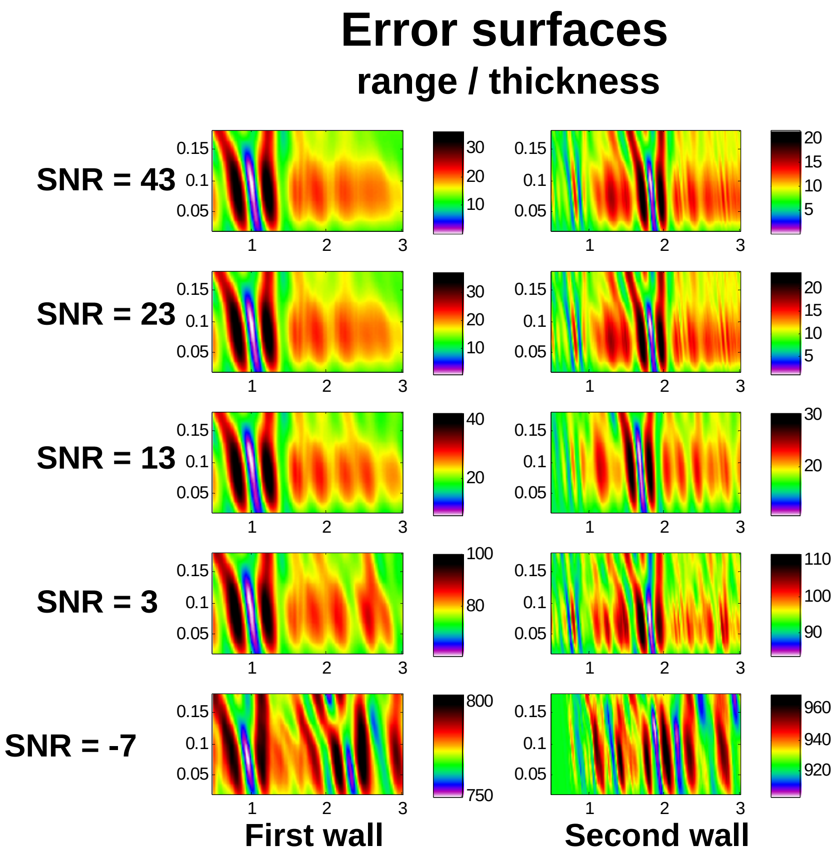
Acceleration constants $\alpha_{1,2}$ selected so that each particle executes a biased random walk that is attracted to the global minimum.

PSO performance

- Algorithm consistently finds global minimum
- Requires many evaluations of the error function (# particles X # iterations)
- Increase efficiency by limiting PSO to finding good start point for gradient algorithm (~40 iterations)
- Hybrid PSO/LM algorithm is robust and requires significantly fewer evaluations of the error compared to a pure PSO algorithm

Looked at degradation of reconstruction with noise level (additive White Gaussian) – 2 walls





SNR based on energy of first reflected pulse

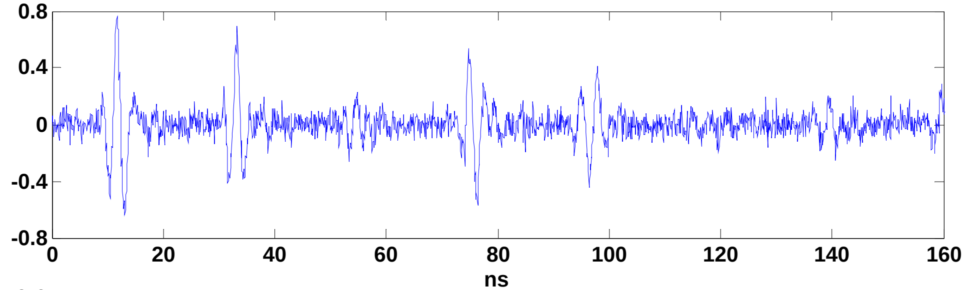
Accuracy of range and thickness for both walls are acceptable for SNR of ~23 or greater

		Truth	SNR 43	SNR 23	SNR 13	SNR 3	SNR -7
Wall 1	Range	1.0000	1.0000	1.0025	1.0081	0.9795	2.3278
	Thickness	0.1000	0.0983	0.0954	0.2527	0.0922	0.0375
	ε_r	5.5931	5.7460	5.8575	5.8799	7.0000	7.0000
	σ/ε_0	0.0246	0.0551	0.0485	0.1000	0	0
Wall 2	Range	4.1000	4.0992	4.0937	3.8669	4.0688	4.1814
	Thickness	0.1000	0.0968	0.0940	0.1069	0.0828	0.1242
	ε_r	5.5931	5.9874	6.6671	4.5950	7.0000	7.0000
	σ/ε_0	0.0246	0.0289	0.0171	0.0355	0.1000	0

Investigate reconstructions for four combinations of conductivity and wall thickness

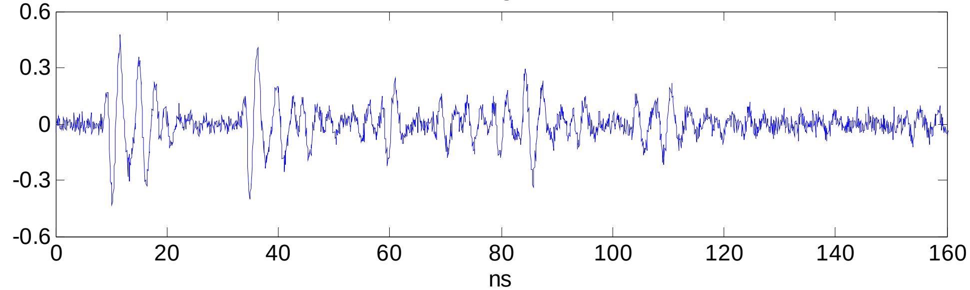
Thickness: 10 cm

σ/ε_0=0.0246



Thickness: 30 cm

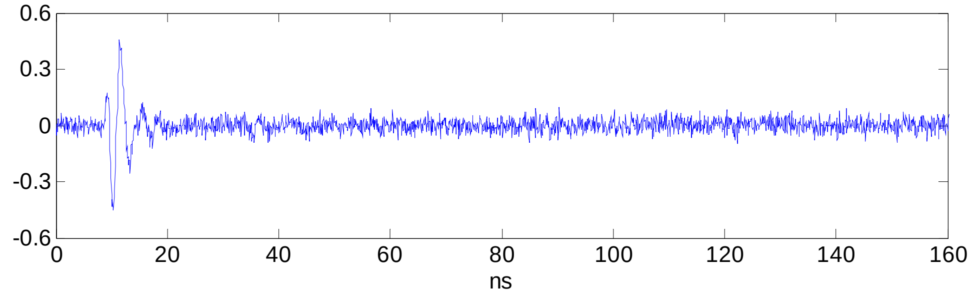
σ/ε_0=0.0246



SNR = 23 dB

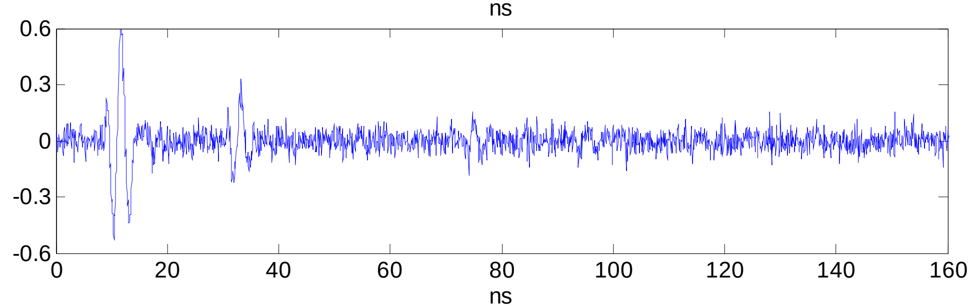
Thickness: 30 cm

σ/ε_0=0.89



Thickness: 10 cm

σ/ε_0=0.89



Accuracy of range and thickness for first two walls are acceptable for SNR of ~20 or better

		Truth	Thin/Trans	Thick/Trans	Thick/Att	Thin/Att
Wall 1	Range	1.0000	1.0009 (0.0010)	0.9990 (0.0003)	1.0007 (0.0004)	1.0011 (0.0007)
	Thickness	0.1/ 0.3	0.1007 (0.0011)	0.2990 (0.0015)	0.3004 (0.0020)	0.0999 (0.0011)
	ε_r	5.5931	5.53 (0.11)	5.645 (0.057)	5.742 (0.068)	5.57 (0.12)
	σ/ε_0	0.0246/ 0.89	-0.028 (0.018)	0.0312 (0.0029)	0.790 (0.016)	0.775 (0.046)
Wall 2	Range	4.1000	4.0978 (0.0012)	4.2963 (0.0003)	4.3021	4.0973 (0.0017)
	Thickness	0.1/ 0.3	0.1070 (0.0018)	0.2910 (0.0015)	0.5232	0.1129 (0.0024)
	ε_r	5.5931	4.89 (0.16)	5.947 (0.061)	4.1118	4.05 (0.16)
	σ/ε_0	0.0246/ 0.89	-0.009 (0.028)	0.0350 (0.0027)	0.4208	0.548 (0.069)

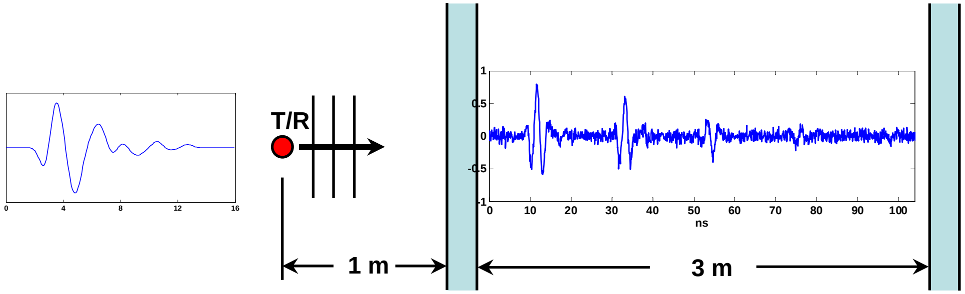
Third wall results are reasonable for thin and transparent walls; uncertain for thick, attenuating case

		Truth	Thin/Trans	Thick/Trans	Thick/Att	Thin/Att
Wall 3 ghost	Range	7.2000	5.5740 (∞)	5.3498 (∞)	7.225 (0.026)	6.831 (0.016)
	Thickness	0.1/ 0.3	-0.0048 (∞)	-0.0055 (∞)	0.196 (0.091)	0.0024 (0.0015)
	ε_r	5.5931	4.6382 (∞)	4.5270 (∞)	6.2 (5.1)	3.99 (0.61)
	σ/ε_0	0.0246/ 0.89	0.0910 (∞)	0.1212 (∞)	0.9 (1.2)	0.06 (0.11)
Wall 3 true	Range	7.2000	7.2042 (0.0006)	7.5878 (0.0003)	7.246 (0.019)	7.2185 (0.0024)
	Thickness	0.1/ 0.3	0.1098 (0.0009)	0.2940 (0.0013)	0.170 (0.055)	0.1242 (0.0032)
	ε_r	5.5931	4.59 (0.78)	5.841 (0.051)	6.6 (4.1)	2.94 (0.12)
	σ/ε_0	0.0246/ 0.89	-0.0031 (0.13)	0.024 (0.023)	-0.36 (0.11)	0.332 (0.043)

Monte Carlo simulations – two wall case

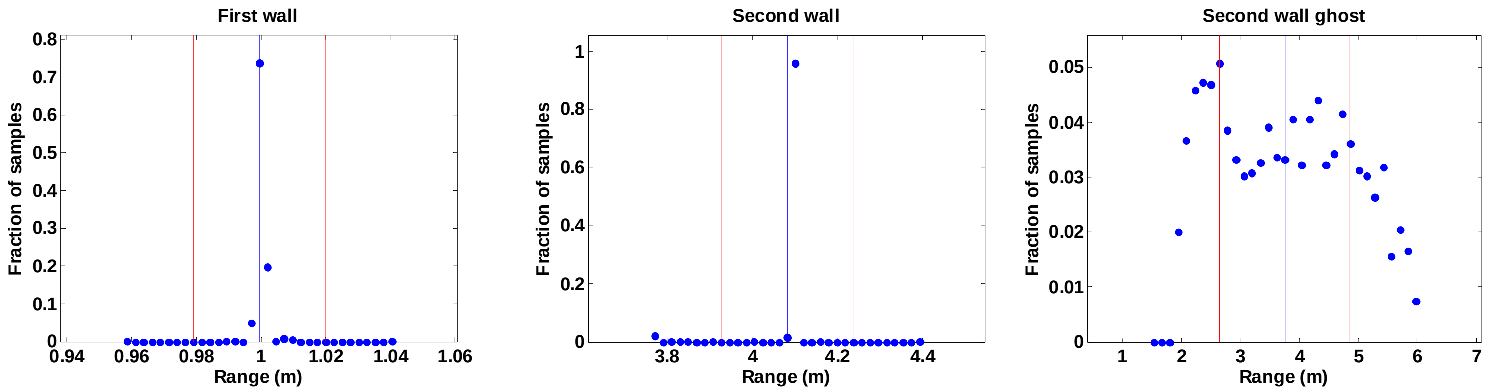
- Use Monte Carlo method to investigate two issues
  - statistical measure of the accuracy of wall parameter estimates
  - sufficient statistic for detection of the second wall

- Monte Carlo approach
  - generate 2048 noise realizations and add to simulated reflections for one wall and two walls (2 sets of simulated data)
  - apply 1D reconstruction algorithm to both sets of data. Look for two walls for both cases – generating both true and false positives.
  - search for parameters that can be used to detect the presence of a second wall



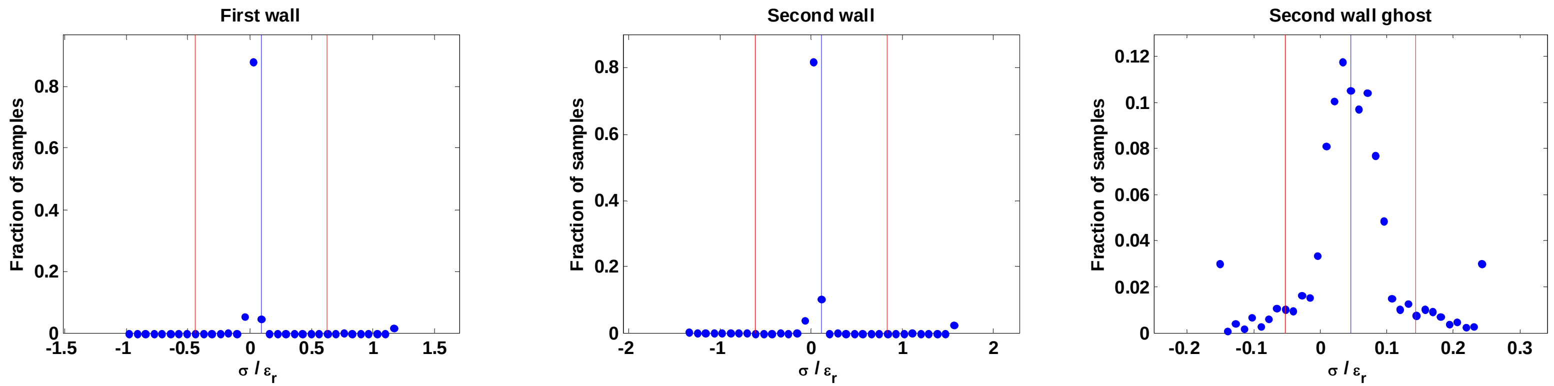
SNR = 20 dB

Statistics of range estimates



- **True ranges are 1 m for first wall, 4.1 m for second wall**
- **Probability that estimate lies in 0.5 m interval around true range is 0.998 for first wall, 0.976 for second wall**
- **Probability that estimate lies in 1.0 m interval around true range is greater than 0.9995 for first wall, 0.991 for second wall**

Statistics of conductivity estimates (true value is 0.0246)



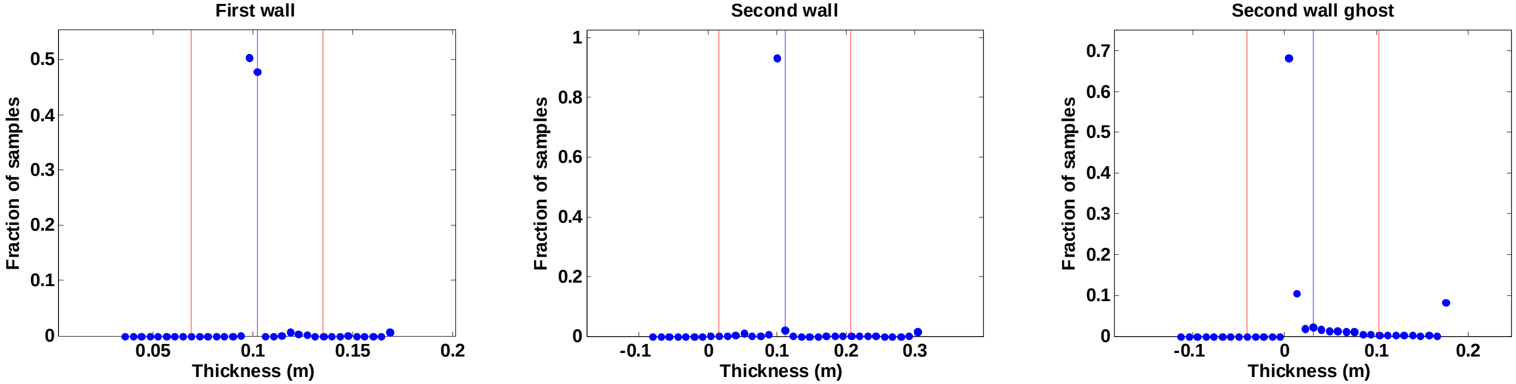
Large standard deviations due to outliers

Summary of mean and standard deviations (red) for each wall

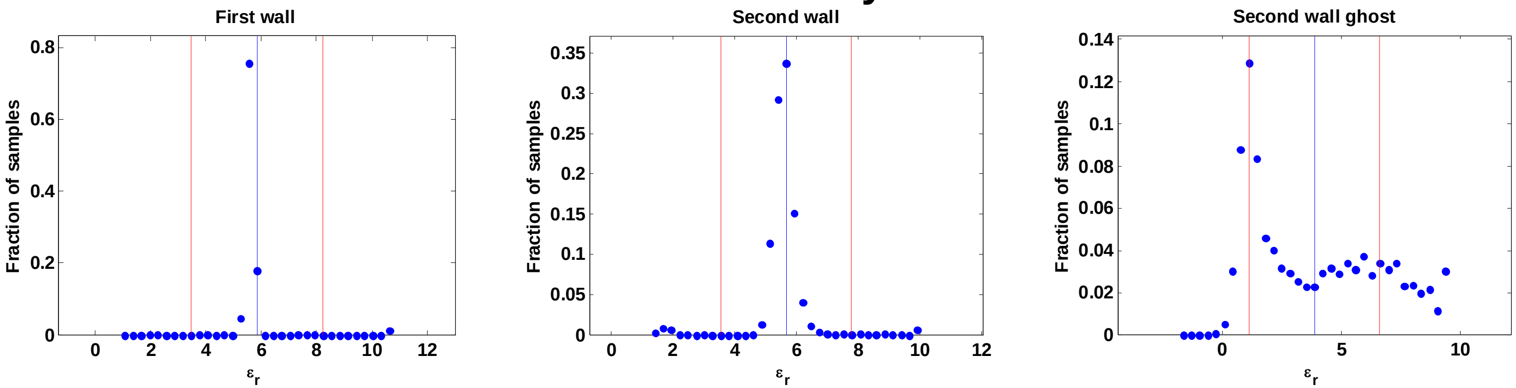
	Truth	Wall 1	Wall 2	Wall 2 ghost
Range	1.0/ 4.1	1.00 (0.02)	4.1 (0.02)	3.7 (1.1)
Thickness	0.1	0.10 (0.03)	0.1 (0.1)	0.03 (0.07)
ε_r	5.5931	5.9 (2.4)	5.6 (2.1)	3.9 (2.8)
σ/ε_0	0.0246	0.09 (0.53)	0.1 (0.7)	0.05 (0.1)

Statistics of thickness and permittivity

Thickness



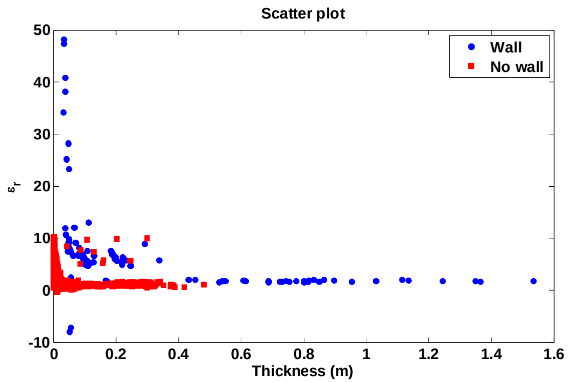
Permittivity



Note that thicknesses cluster around zero and permittivities around one for ghost wall

Detect presence of second wall using OPD

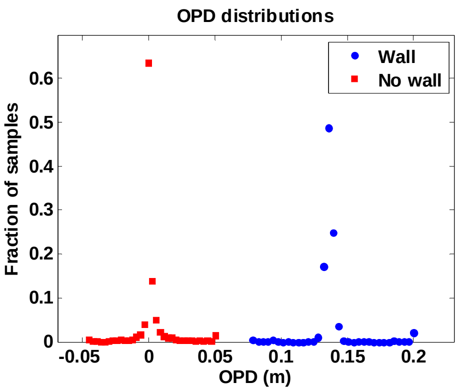
Scatter plot shows clean separation between presence and absence of second wall



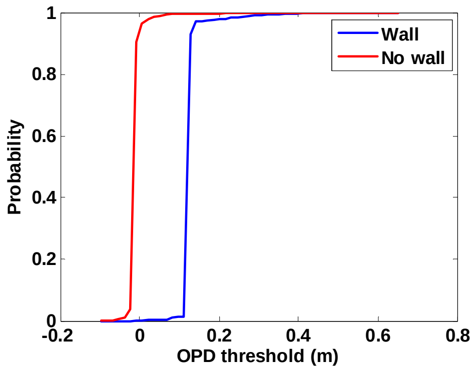
Calculate optical path difference (OPD)

$$OPD = t_h(\sqrt{\varepsilon_r} - 1)$$

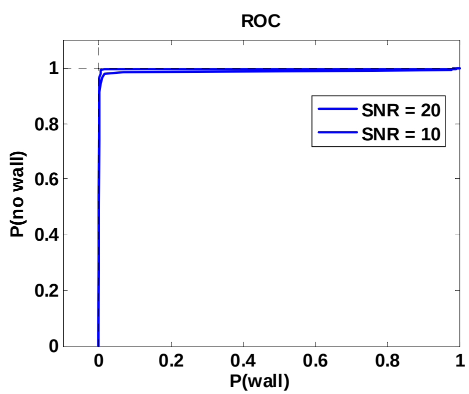
Distributions of OPD are clearly separated, SNR = 20 dB

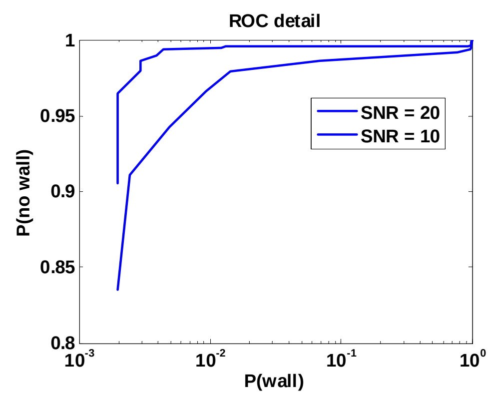


Probabilities of detection using OPD for SNR = 20 dB



Excellent performance for both SNR = 20 dB and SNR = 10 dB



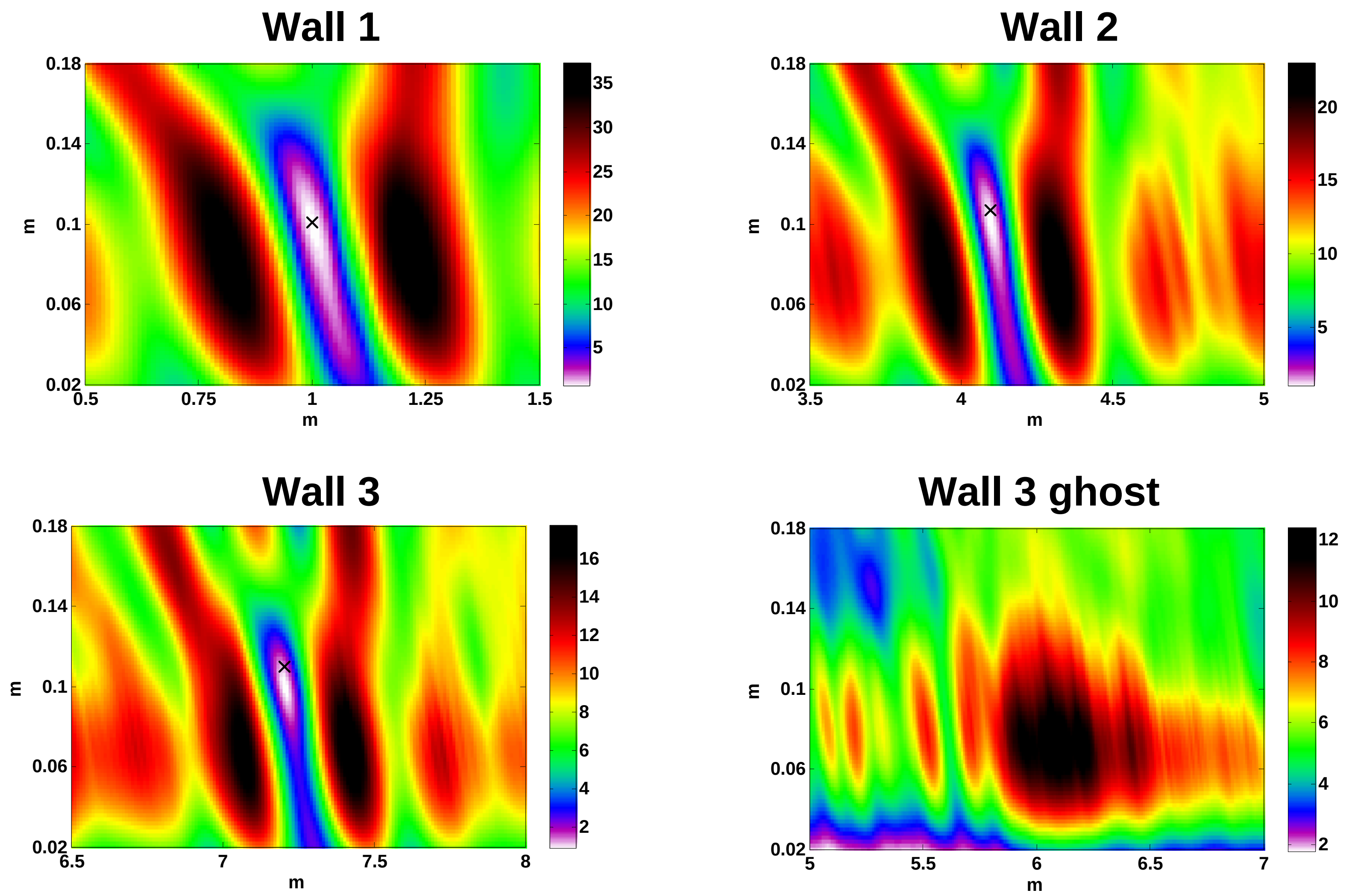


Conclusions

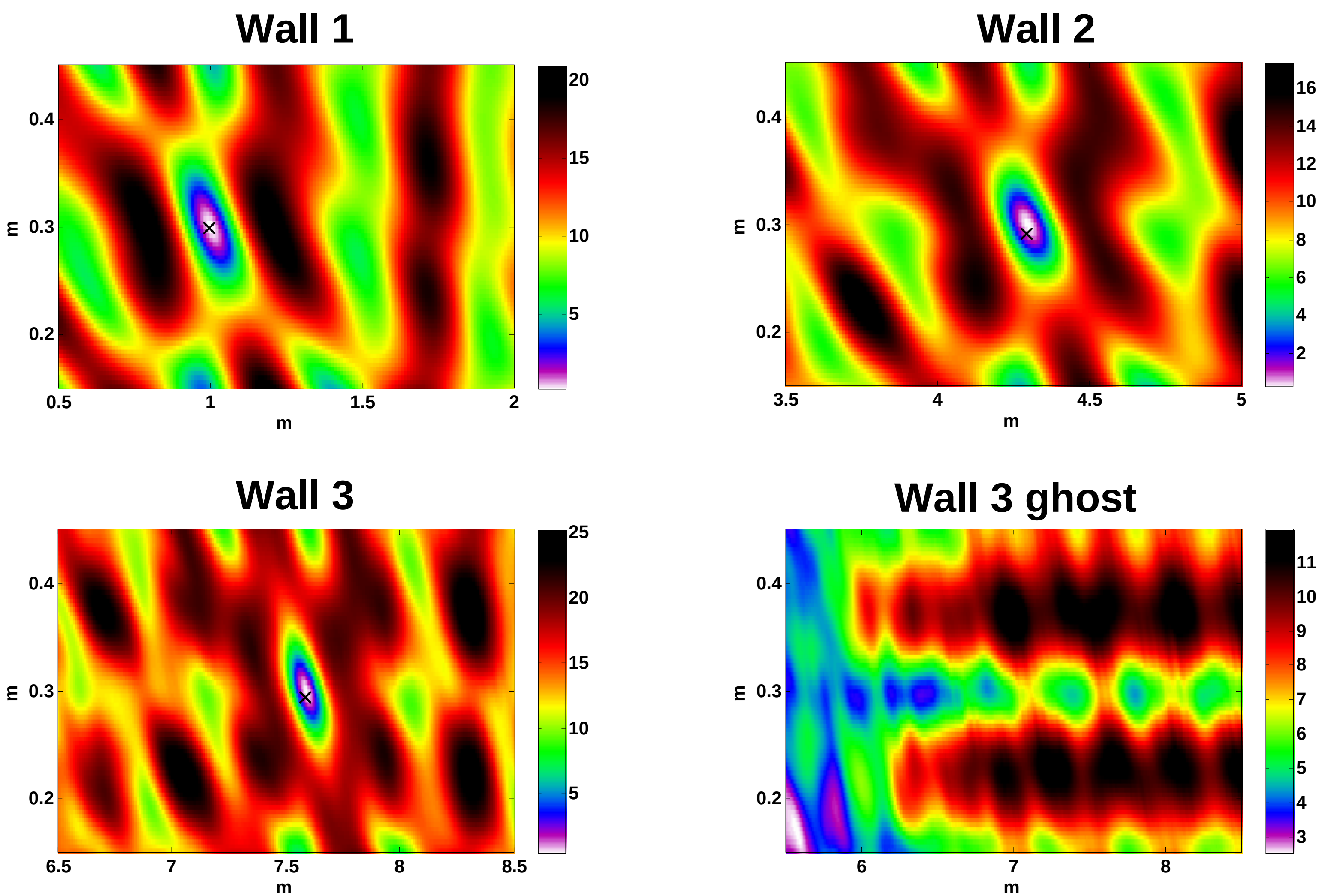
- One-dimensional reconstruction code working and giving good results for a variety of wall sequences and noise levels
- Combination of particle swarm optimization and Levenberg-Marquardt is robust and avoids difficulties of local minima
- Preliminary analysis of experimental data show good agreement between 1d model and data
- Optical path difference can be used to determine the presence or absence of a wall in a given range bin

Backups

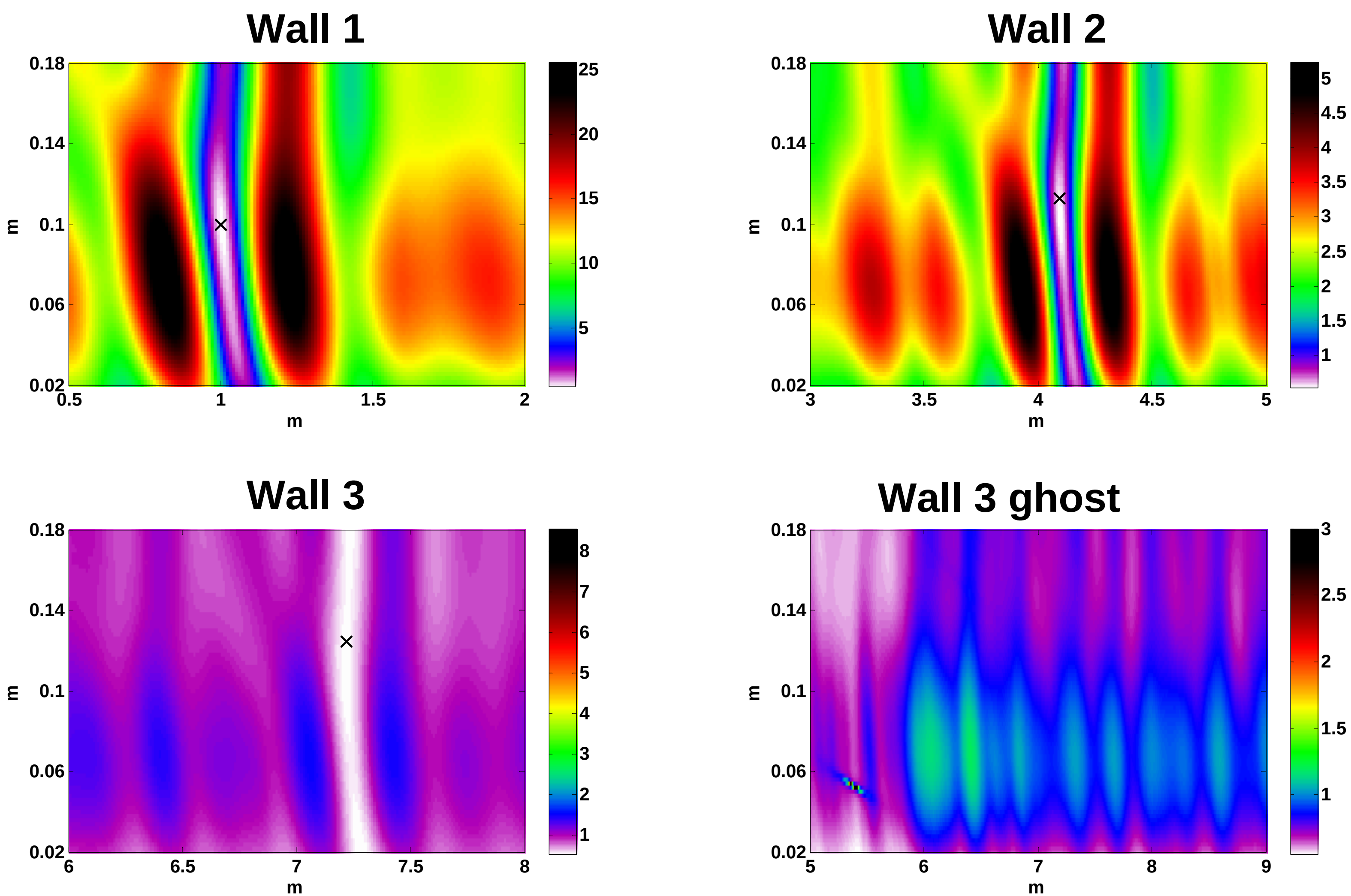
Error surfaces for 10 cm walls, $\sigma/\varepsilon_0 = 0.0246$



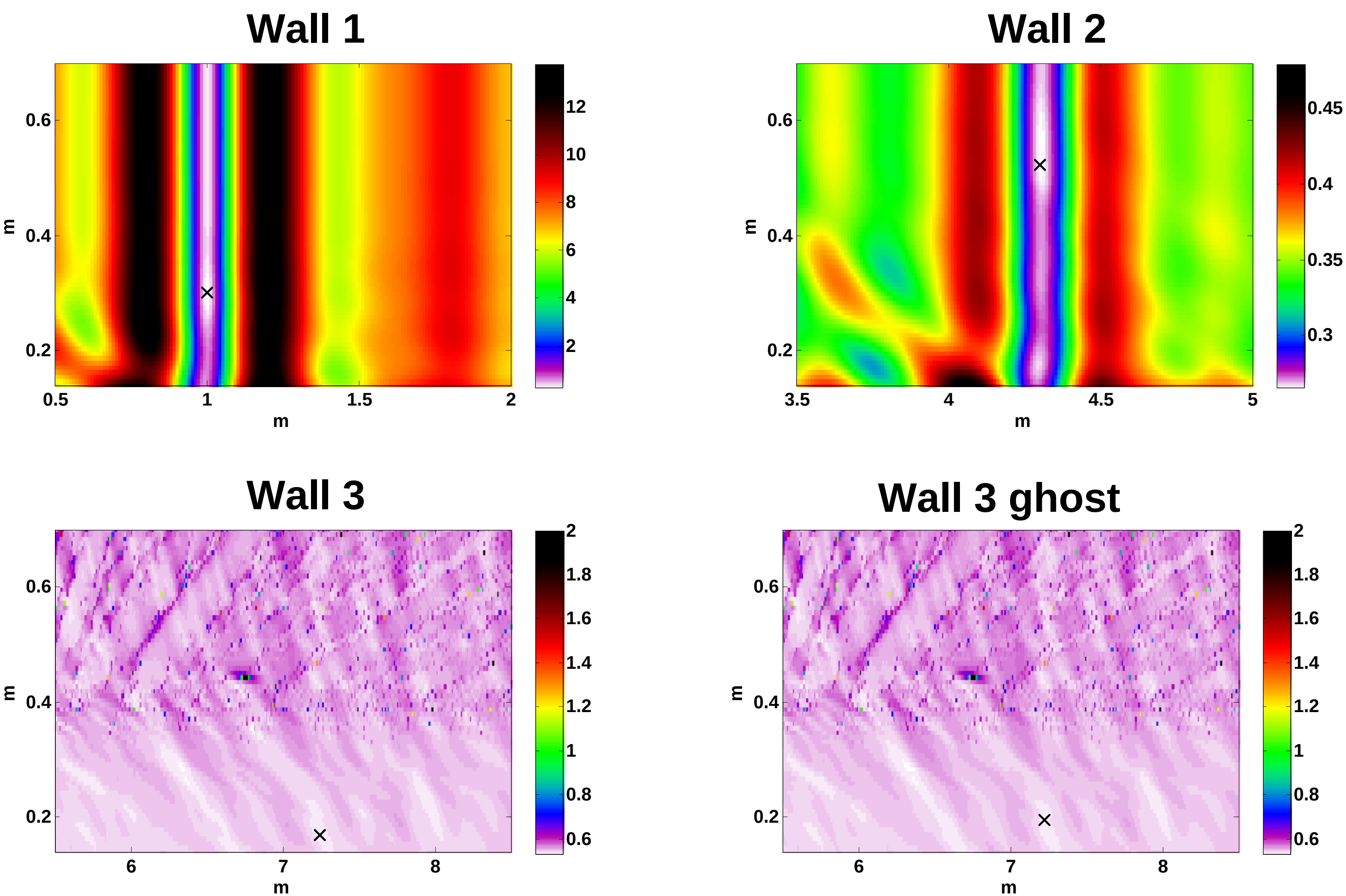
Error surfaces for 30 cm walls, $\sigma/\varepsilon_0 = 0.0246$



Error surfaces for 10 cm walls, $\sigma/\varepsilon_0 = 0.89$



Error surfaces for 30 cm walls, $\sigma/\varepsilon_0 = 0.89$



Parameter estimates for a thick, attenuating wall for SNR = 20, 30, and 40 : First 2 walls

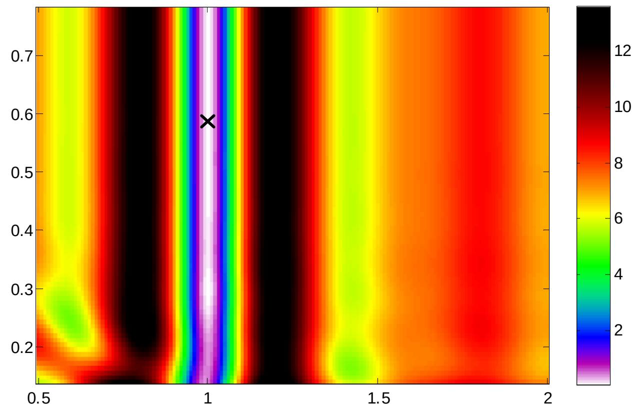
		Truth	SNR = 20	SNR = 30	SNR = 40
Wall 1	Range	1.0000	1.0007 (0.0004)	0.9998 (0.0004)	1.0001 (0.00004)
	Thickness	0.3	0.3004 (0.0020)	0.5876 (0.0066)	0.3002 (0.0002)
	ε_r	5.5931	5.742 (0.068)	5.610 (0.055)	5.6079 (0.0074)
	σ/ε_0	0.89	0.790 (0.016)	0.880 (0.019)	0.8788 (0.0019)
Wall 2	Range	4.1000	4.3021	3.891 (0.016)	4.2993 (0.0005)
	Thickness	0.3	0.5232	0.55 (0.27)	0.3049 (0.0024)
	ε_r	5.5931	4.1118	6.7 (3.4)	5.366 (0.073)
	σ/ε_0	0.89	0.4208	0.96 (0.96)	0.832 (0.018)

Parameter estimates for a thick, attenuating wall for SNR = 20, 30, and 40 : Third wall

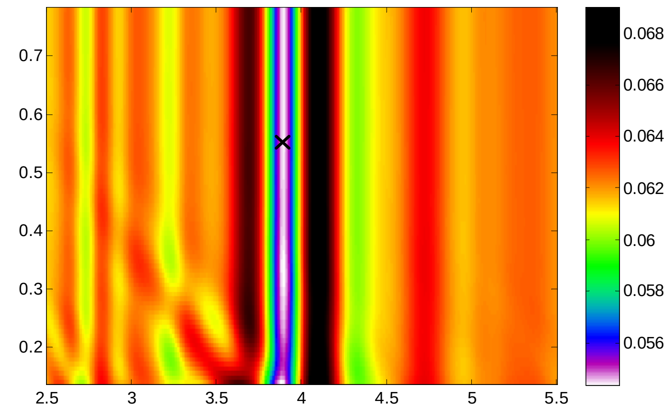
		Truth	SNR = 20	SNR = 30	SNR = 40
Wall 3 false	Range	--	7.225 (0.026)	8.17 (0.67)	6.513 (0.017)
	Thickness	--	0.196 (0.091)	0.5 (0.8)	0.810 (0.056)
	ε_r	--	6.2 (5.1)	5 (20)	2.07 (0.26)
	σ/ε_0	--	0.9 (1.2)	0.12 (0.52)	0.0225 (0.014)
Wall 3 true	Range	7.2000	7.246 (0.019)	6.77 (0.68)	7.5908 (0.0035)
	Thickness	0.3	0.170 (0.055)	0.0974 (0.60)	0.469 (0.022)
	ε_r	5.5931	6.6 (4.1)	5 (41)	4.78 (0.42)
	σ/ε_0	0.89	-0.36 (0.11)	0.15 (0.92)	0.384 (0.068)

Error surfaces for 30 cm walls, $\sigma/\varepsilon_0 = 0.89$ *SNR = 30*

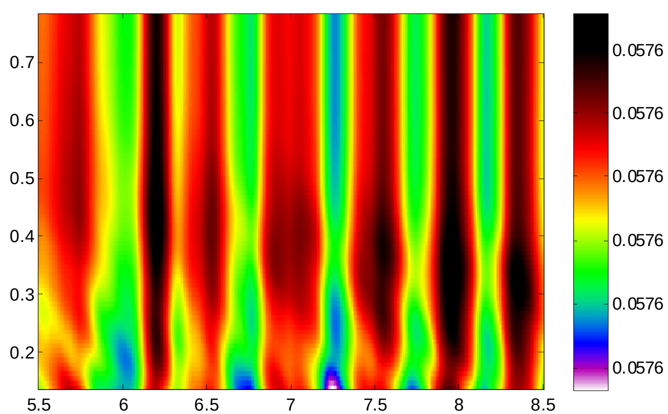
Wall 1



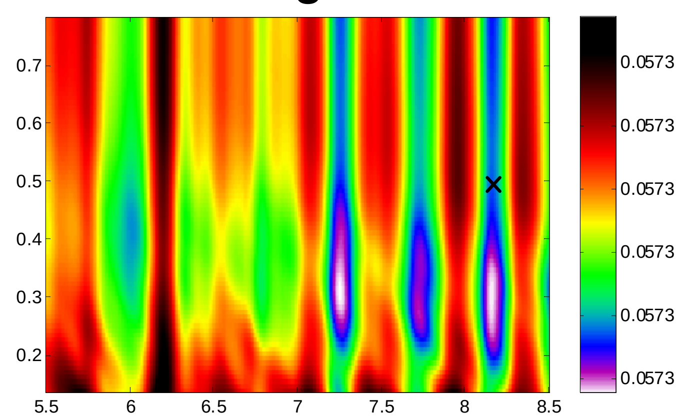
Wall 2



Wall 3

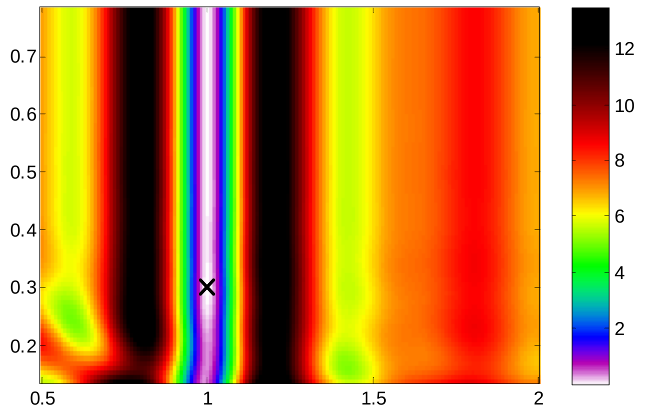


Wall 3 ghost

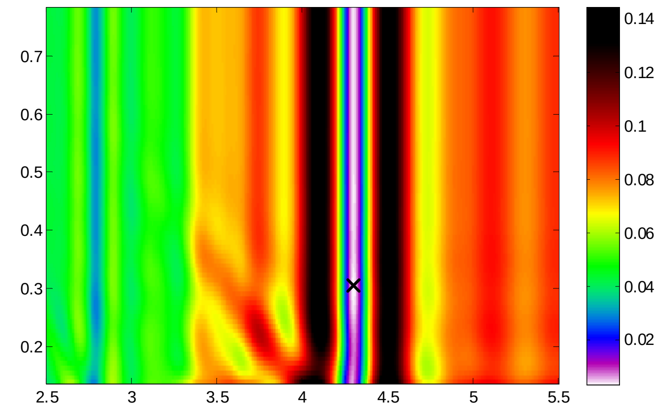


Error surfaces for 30 cm walls, σ/ε_0 = *0.89*
SNR = 40

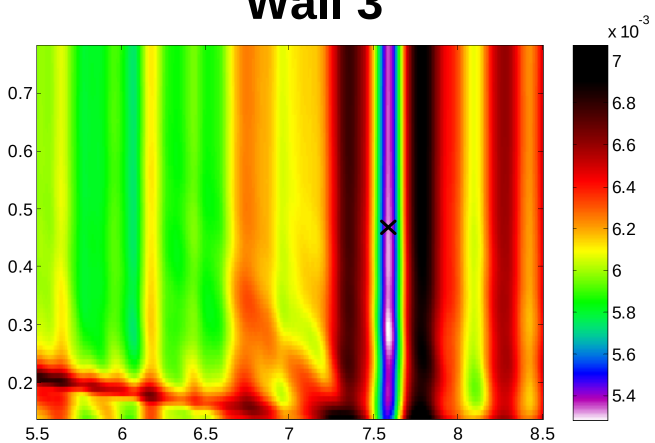
Wall 1



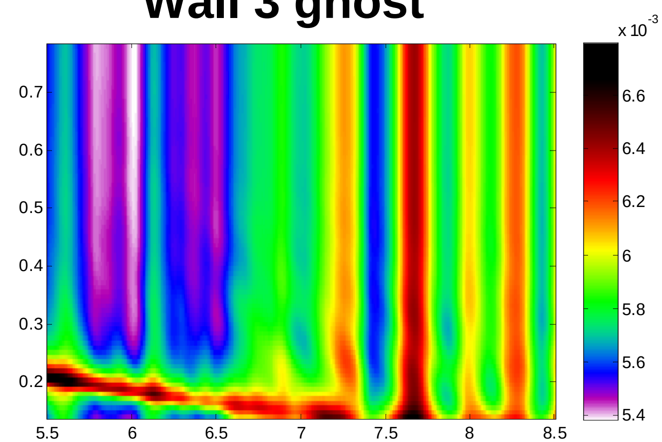
Wall 2



Wall 3

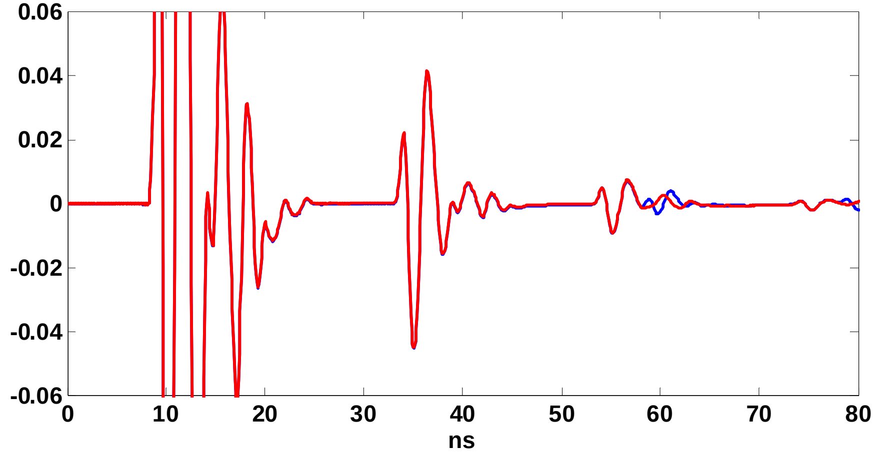


Wall 3 ghost



Input data for thick, attenuating walls
SNR = 40

No noise



SNR = 40