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A Single-Layer Network of Unscented Kalman Filters Adaptively Fused by the Mixture-of-Experts Method

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Method-of-Experts (MOE) Reduce the Effects of Filter Parameter Uncertainties in State Estimation Applications

• Given a bank of filters, the goal of the method-of-experts is to assign the greatest weight to the filter with the lowest mean-square-error (MSE), thereby minimizing the MSE of the weighted linear sum of the output (y^*) at each discrete instant of time.

• MOE is sometimes associated with/known as,

  - Multiple Model
  - Multiple Hypothesis
  - Magill Filter Bank.

• Approach used in this presentation was developed by,

  - W. S. Chaer, R. H. Bishop, J. Ghosh, “A mixture of experts framework for adaptive Kalman filtering”, IEEE Trans. Systems, Man and Cybernetics, vol. 27, no. 3, June 1997.

• Another useful paper is the following,

  - K. C. Slatton, V. Aggarawal, K. Nagarajan, “Estimating failure modes using a multiple-model Kalman filter”, University of Florida, ASPL Rep_2004-03-001.

Approach Taken to Demonstrate UKF/MOE Effectiveness

Initially given:

- Discrete Kalman filters (KFs) assume noise terms to be white zero-mean Gaussian sequences.
- Discrete unscented Kalman filters (UKFs) assume noise terms to be zero-mean, however they can be non-Gaussian sequences (single-modal).
- MOE approach assumes that residuals are of a known distribution (e.g., Gaussian).

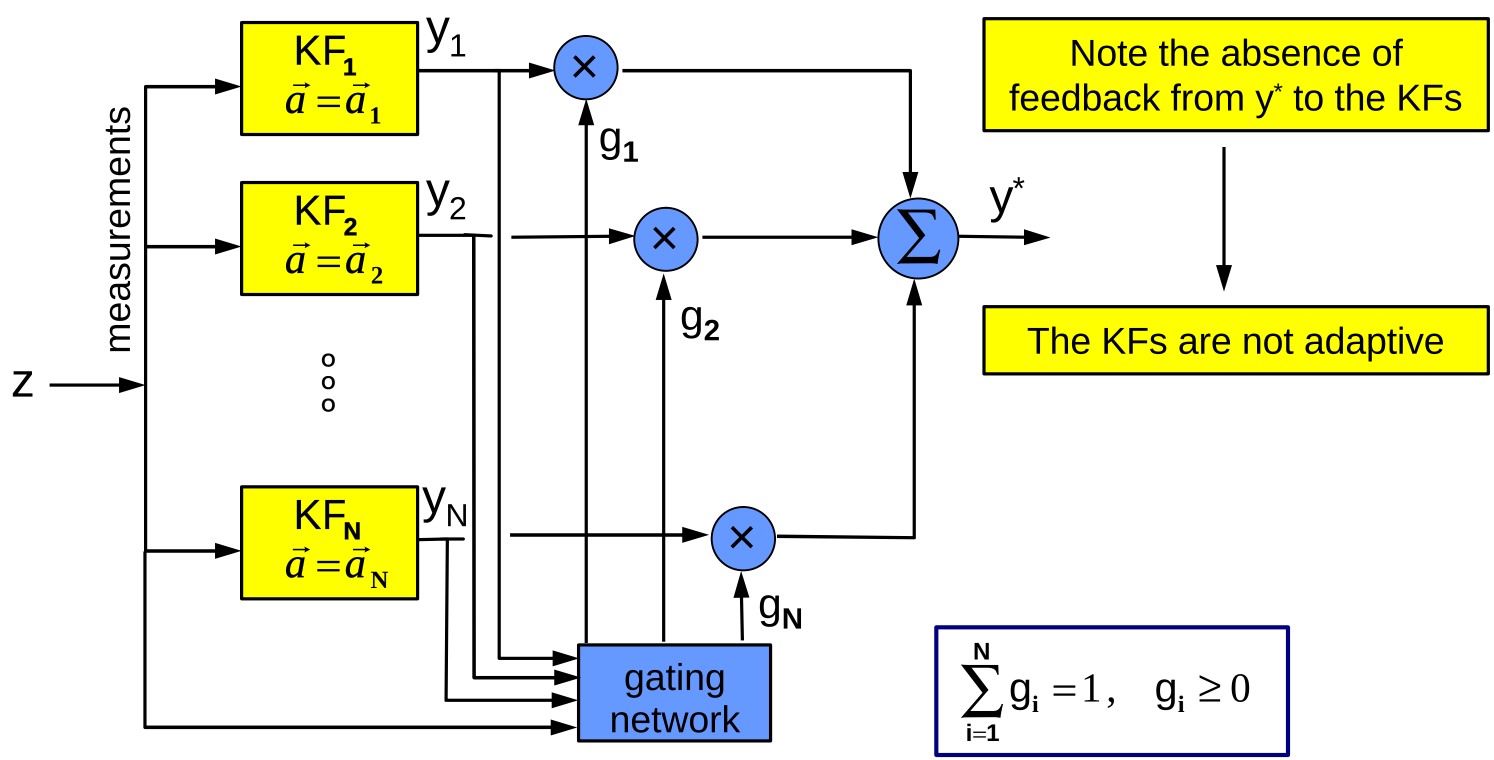
Demonstrate:

1) Assemble a single layer of N UKFs whose outputs and residuals are input to the MOE gating algorithm.
   a) Show that with white zero-mean Gaussian disturbances added to the model, the UKF/MOE system works similarly to the EKF/MOE system.

Retain UKF flexibility

2) Modify the MOE algorithm to numerically approximate any unknown probability density function.
3) Use the MOE algorithm and repeat (1) with non-Gaussian disturbances added to the model.
4) Use the modified MOE algorithm and repeat (1) with non-Gaussian disturbances added to the model.
   a) Observe any differences between the results of (3) and (4).

Block Diagram of Single-Layer MOE Network



$$\sum_{i=1}^{N} g_i = 1, \quad g_i \geq 0$$

Gate weights are generated by forcing the residuals and covariances to conform to a Gaussian pdf.

This is required in a standard Kalman or extended Kalman filter (EKF), and is allowable in a UKF provided that the noise terms are Gaussian.

The MOE Is Easily Applied to the UKF

- A discrete system with additive state and sensor noise terms was used.

$$x(k+1) = \Phi \cdot x(k) + n_{\mathbf{Q}}(k), \; \mathrm{E}\left[n_{\mathbf{Q}} \cdot n_{\mathbf{Q}}^{\mathrm{T}}\right] = Q$$

$$y(k) = h\big(x(k)\big) + n_{\mathbf{R}}(k), \; \mathrm{E}\left[n_{\mathbf{R}} \cdot n_{\mathbf{R}}^{\mathrm{T}}\right] = R$$

- The extended Kalman filter (EKF) relies on the Jacobian (H),

$$\hat{x}(k)^{-} = \Phi \cdot \hat{x}(k-1)^{+}$$

$$P(k)^{-} = \Phi \cdot P(k-1)^{+} \cdot \Phi^{\mathrm{T}} + Q$$

$$r(k) = z(k) - H \cdot \hat{x}(k)^{-}$$

$$W = H \cdot P^{-} \cdot H^{\mathrm{T}} + R$$

- The unscented Kalman filter (UKF) uses the nonlinear measurement function (h),

$$\mathcal{X} = func(Q, R, a, b, k), \; \mathcal{X}_i : \text{sigma points}$$

$$\mathcal{Z}(k) = h(\mathcal{X}(k)), \; h = \text{nonlinear measurement function}$$

$$\hat{z}(k)^{-} = \sum_{i=0}^{2n} C_i^{m} \cdot \mathcal{Z}(k)$$

$$W = \sum_{i=0}^{2n} C_i^{c} \cdot \left[\mathcal{Z}(k) - \hat{z}(k)^{-}\right] \cdot \left[\mathcal{Z}(k) - \hat{z}(k)^{-}\right]^{\mathrm{T}} + R$$

$$p(z_{\boldsymbol{k}} | a_{\boldsymbol{i}}) = \frac{1}{2p\sqrt{|W|}} exp\left(-\frac{1}{2} r_k^{\mathrm{T}} \cdot W^{-1} \cdot r_k\right)$$

$$g_{\boldsymbol{i}} = \frac{e^{z^{\mathrm{T}} a_{\boldsymbol{i}}}}{\sum_{\boldsymbol{j}=1}^{\boldsymbol{L}} e^{z^{\mathrm{T}} a_{\boldsymbol{j}}}}$$

$$h_{\boldsymbol{i}} = \frac{p(z_{\boldsymbol{k}} | a_{\boldsymbol{i}})}{\sum_{\boldsymbol{j}=1}^{\boldsymbol{L}} p(z_{\boldsymbol{k}} | a_{\boldsymbol{j}}) g_{\boldsymbol{j}}}$$

h : learning parameter

$$a_{\boldsymbol{i}} \leftarrow a_{\boldsymbol{i}} + h\left(h_{\boldsymbol{i}} - g_{\boldsymbol{i}}\right) z_{\boldsymbol{k}}$$

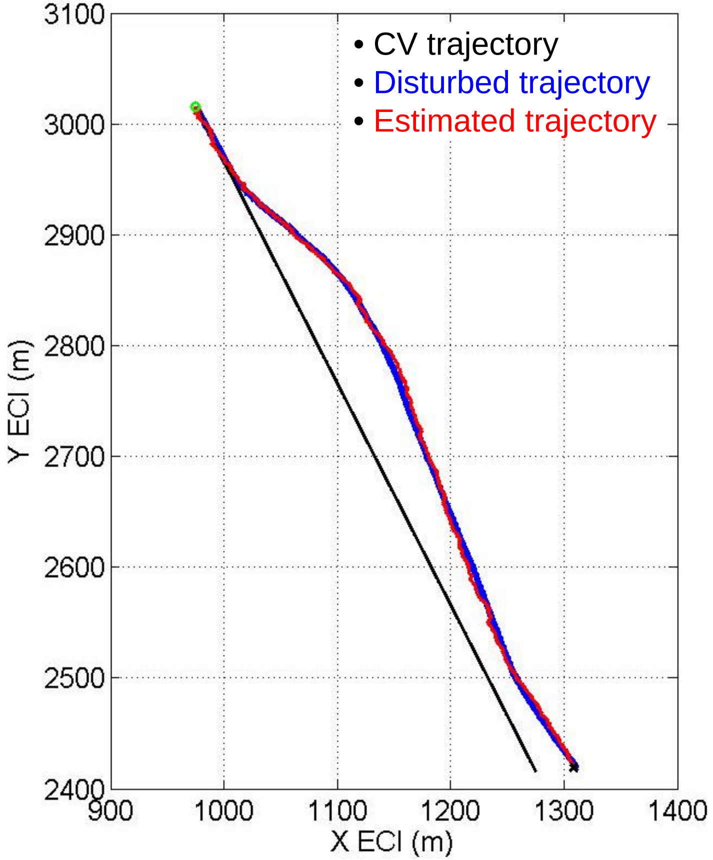
A Constant Velocity Discrete Model Was Used for the Simulations

$$\begin{bmatrix} x \\ y \\ Vx \\ Vy \end{bmatrix}_{k+1} = \begin{bmatrix} 1 & 0 & T & 0 \\ 0 & 1 & 0 & T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ Vx \\ Vy \end{bmatrix}_{k} + \begin{bmatrix} T^2/2 & 0 \\ 0 & T^2/2 \\ T & 0 \\ 0 & T \end{bmatrix} \cdot \begin{bmatrix} n_{\mathbf{Q}x} \\ n_{\mathbf{Q}y} \end{bmatrix}$$

Measurements were angle and range

$$z = \begin{bmatrix} q \\ \|r\| \end{bmatrix} = \begin{bmatrix} atan2(y,x) + n_{\mathbf{R}q} \\ \sqrt{x^2+y^2} + n_{\mathbf{R}r} \end{bmatrix}$$

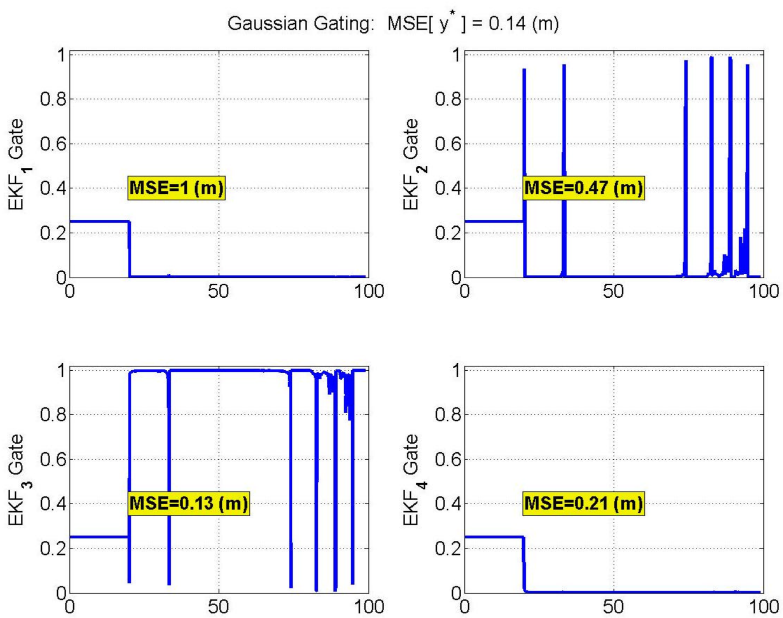
Typical simulation run

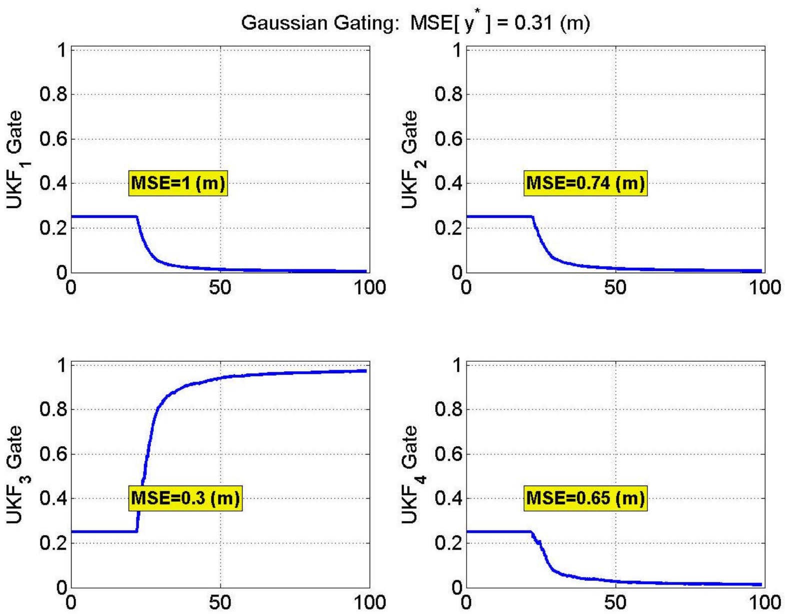


EKF and UKF Gate Weights for Gaussian Disturbances

- Shown in yellow are the normalized mean-squared-errors of each filter
- The goal of the MOE was to assign the greatest weight to the filter with the lowest MSE
- performance metric = y^{*}/ min { MSE_i }

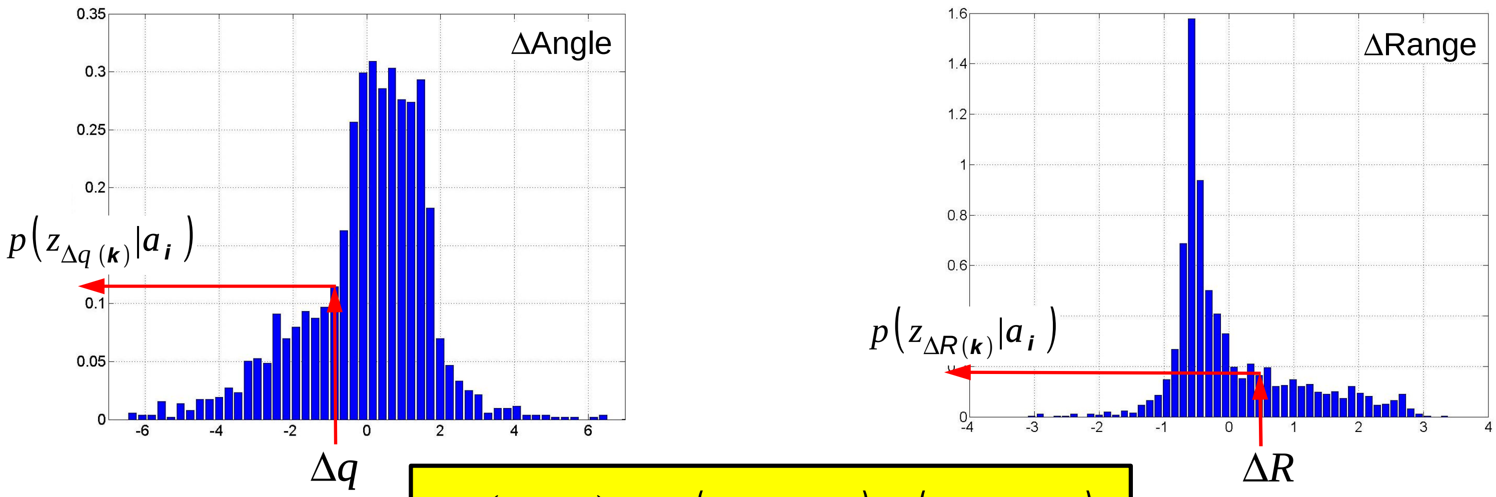
EKF_{pm} = 1.08 ← Nearly identical performance metric → UKF_{pm} = 1.03





Numeric Calculation of PDF of Residuals

- UKF cannot rely on residuals conforming to a strictly Gaussian distribution.
- Numerically approximate the residual distributions using some method; in this case a histogram was used (requires an application with many measurements).
- Resolution or granularity becomes an issue.
- Assume that the noise processes are wide sense stationary (WSS) due to the time lag associated with the histogram.
- Noise processes are independent but not necessarily identically distributed.
- Assuming a restricted set of probability functions, like generalized Gaussians ($p=1$:Laplacian, $p=2$:Gaussian, …, $p=8$:nearly uniform; p shaping parameter), then the residuals may be decorrelated.

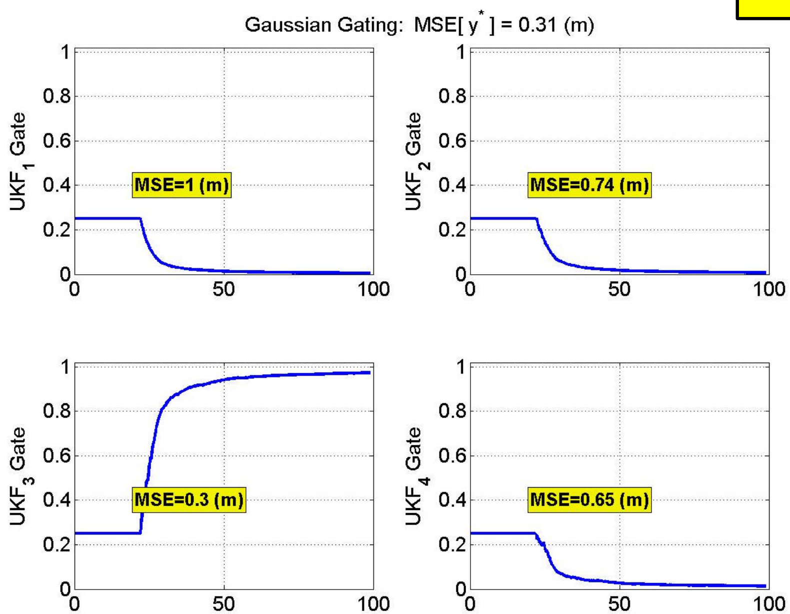


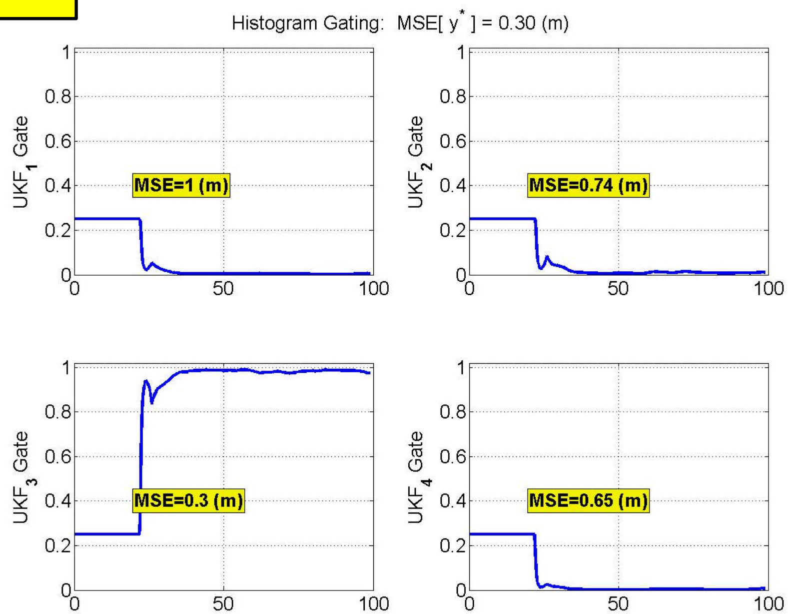
$$p\left(z_{\boldsymbol{k}} | a_{\boldsymbol{i}}\right) \propto p\left(z_{\Delta q\,(\boldsymbol{k})} | a_{\boldsymbol{i}}\right) \cdot p\left(z_{\Delta R\,(\boldsymbol{k})} | a_{\boldsymbol{i}}\right)$$

UKF Gate Weights for Gaussian Disturbances Calculated by MOE and Modified MOE

- An identical set of noise sequences were saved and run through each filter; the same set for the two systems.
- The performance is equivalent with the exception that the learning gains could be adjusted for the two systems.
- A comprehensive Monte Carlo set has yet to be made.

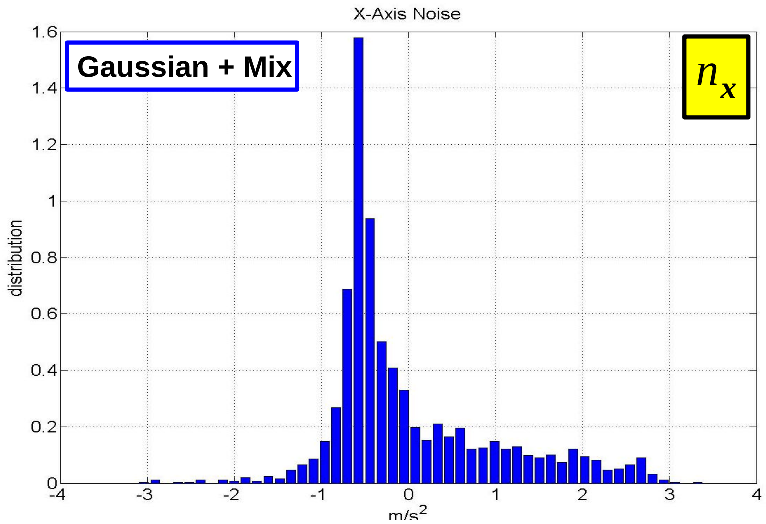
UKF_{pm} (Gauss fit) = 1.03 ← **Nearly identical performance metric** → UKF_{pm} (Numerical fit) = 1.00

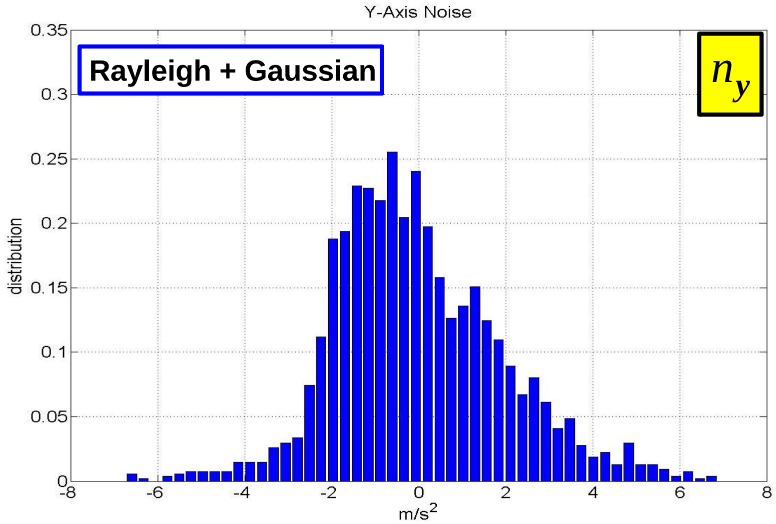




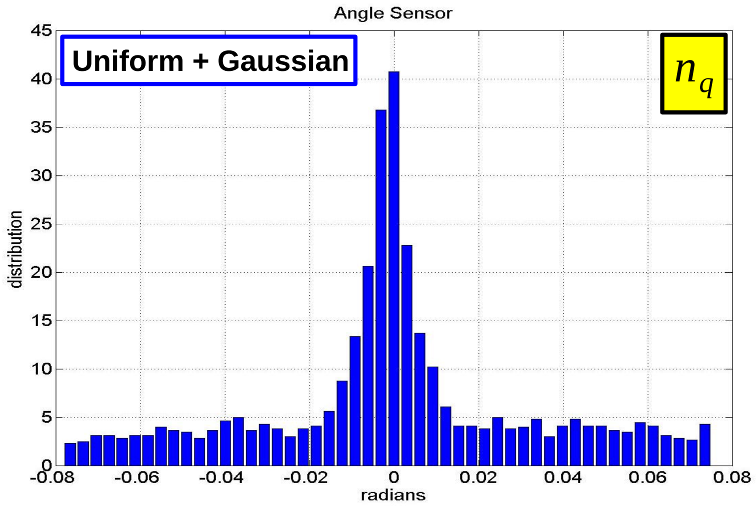
Aggregate Non-Gaussian Disturbances

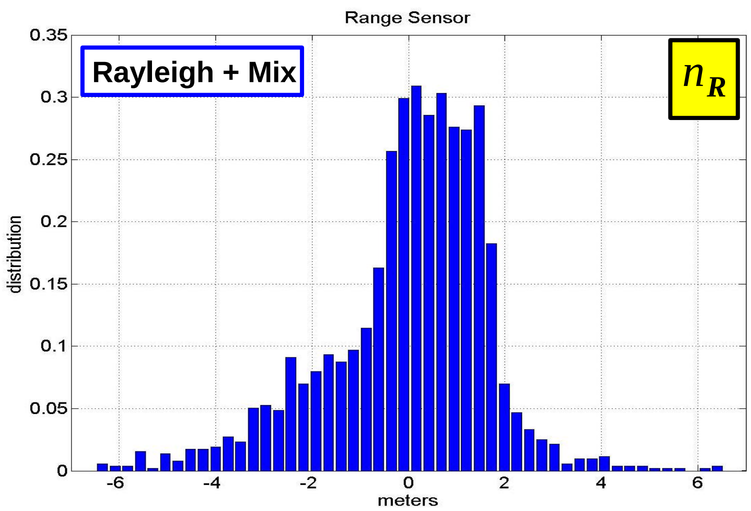
Process noises (Q)





Sensor noises (R)





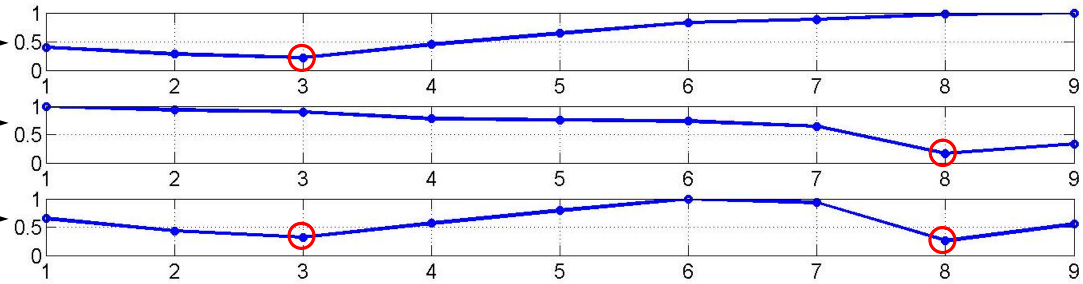
UKF Gate Weights for Non-Gaussian Disturbances Calculated by MOE and Modified MOE: Q & R Both Centered About Different UKFs

$$MSE_{i=1:9} = [\,0.65 \quad 0.43 \quad 0.32_{Q} \quad 0.57 \quad 0.79 \quad 1.00 \quad 0.93 \quad 0.26_{R} \quad 0.55\,]$$

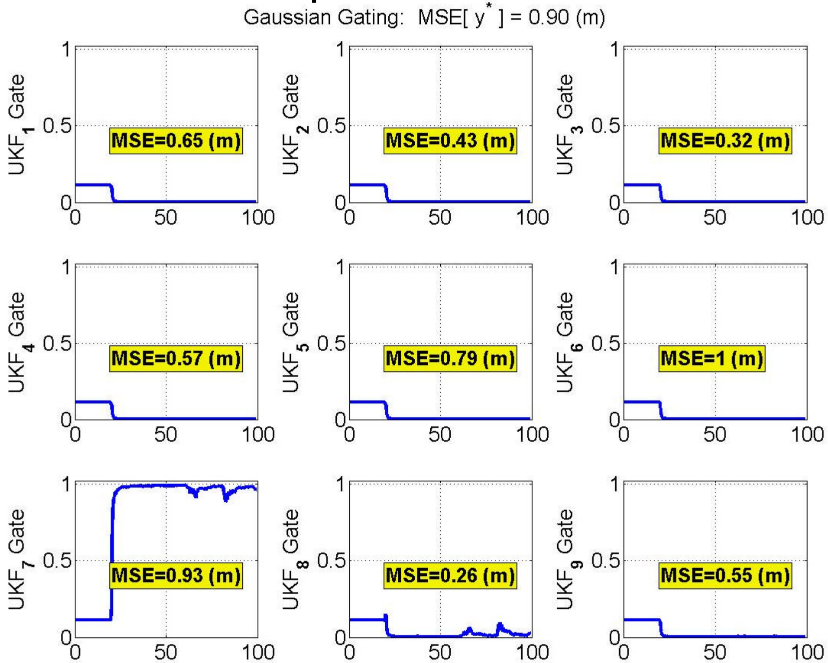
$UKF\_Q(i_{Min}=3) = Q_{TRUE}$

$UKF\_R(i_{Min}=8) = R_{TRUE}$

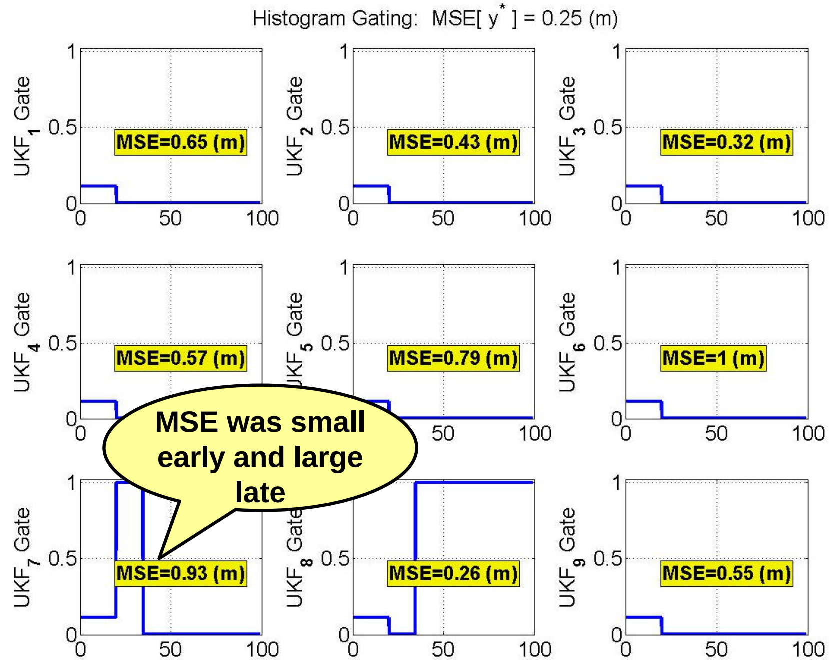
$UKF\_Q_3\_R_8\ (i=1{:}9)$ system under test



UKF_{pm} (Gauss fit) = 3.46



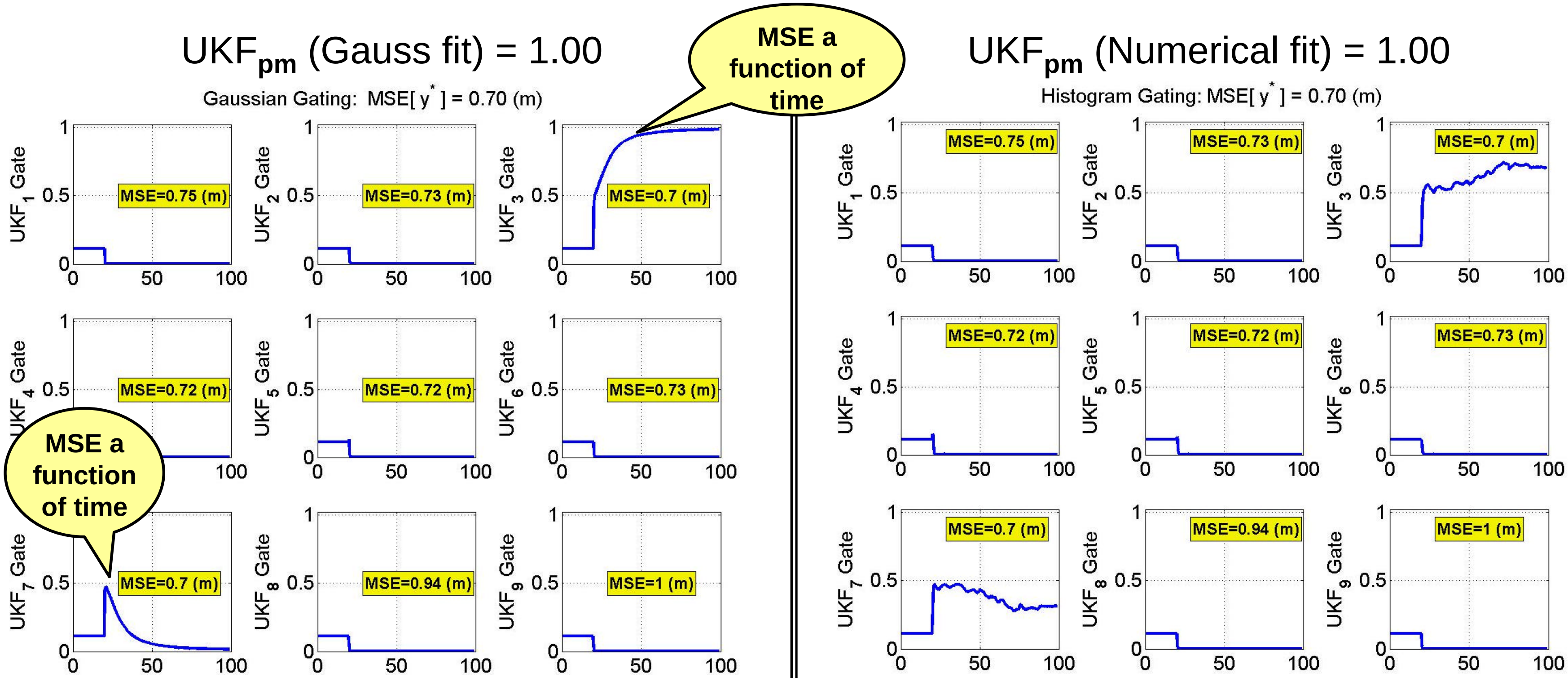
UKF_{pm} (Numerical fit) = 0.96



UKF Gate Weights for Non-Gaussian Disturbances: k and b Varied to Create Different UKF Sigma Points for Each Filter (Q & R Accurate)

- Sigma Points were varied for each filter by varying k and b.
- Scenario was intentionally set to have two minima equal to three significant digits, as well as other MSE values that were only a few percentage points larger. This tested the sensitivity of the UKF/MOE system.
- Unmodified Gaussian-fit MOE worked qualitatively better.

$$MSE_{i=1:9} = [\,0.76 \quad 0.73 \quad 0.71 \quad 0.72 \quad 0.73 \quad 0.74 \quad 0.71 \quad 0.95 \quad 1.00\,]$$



Conclusions Based On A Single-Layer Network of UKFs With Additive Noise Disturbances

• The MOE algorithm has been shown to work with unscented Kalman filters with additive Gaussian disturbances.

• A modified MOE algorithm has been shown to work with unscented Kalman filters with non-Gaussian additive disturbances.

• For the cases where the process and measurement covariance terms were varied in order to force the UKFs off of their nominal operating points, the modified MOE approach performed considerably better than the unmodified Gaussian MOE approach for non-Gaussian disturbances.

• For all other cases, including varying the UKF sigma points with non-Gaussian disturbances, the two approaches performed equivalently.