Application of Cepstrum-Based Phase Retrieval to Lightning Safety Studies of Explosive and Weapons Storage Facilities

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UCRL-PRES-208102

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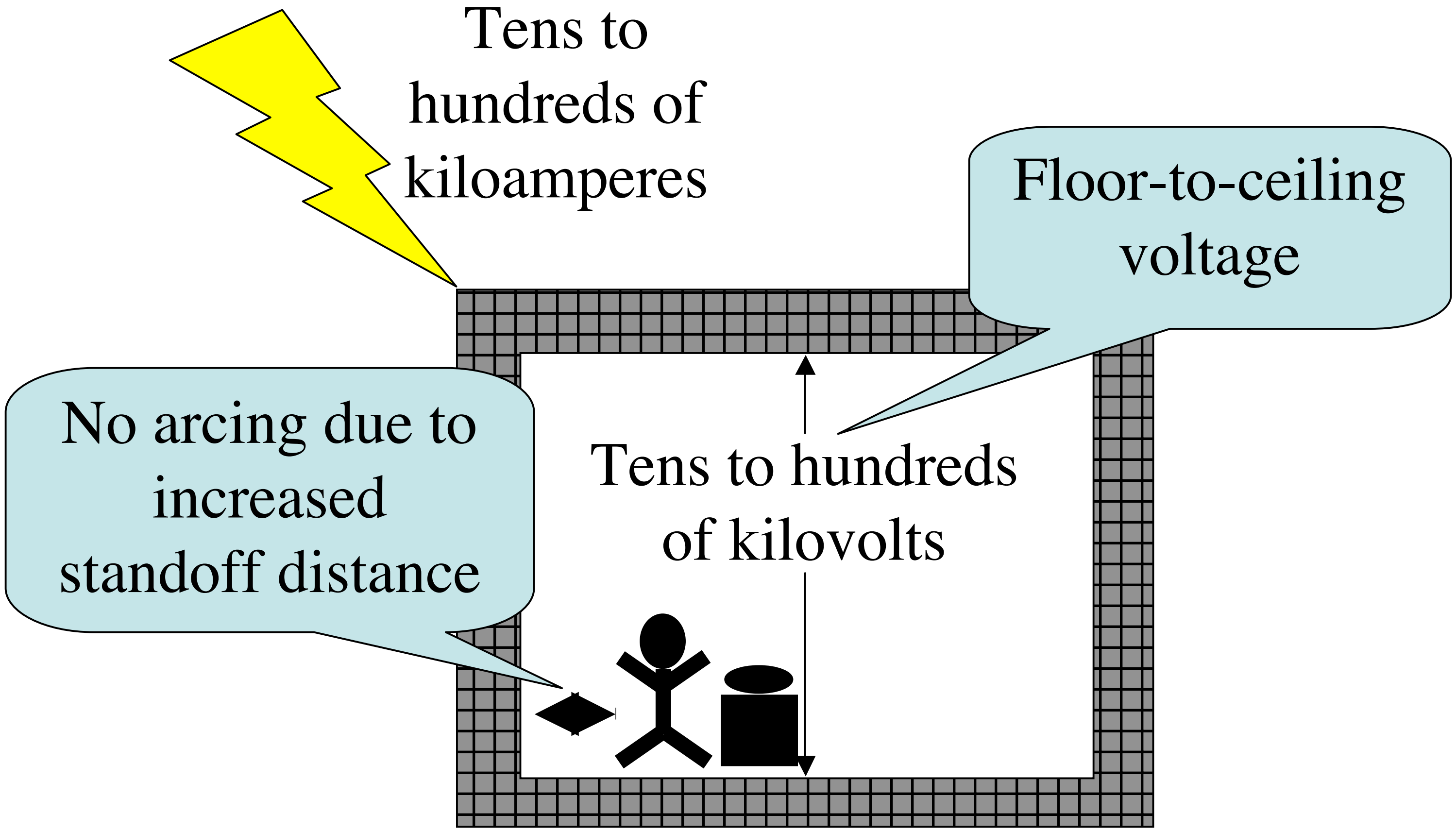
Must know floor-to-ceiling voltage to compute safe standoff distances



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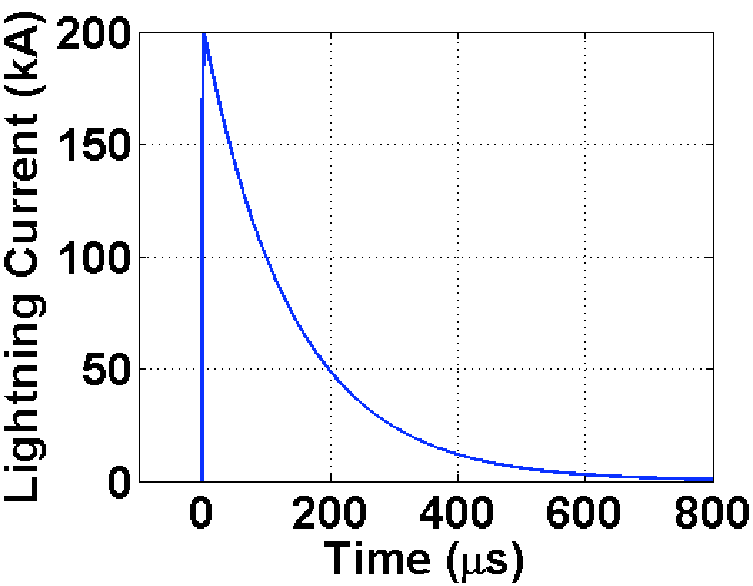
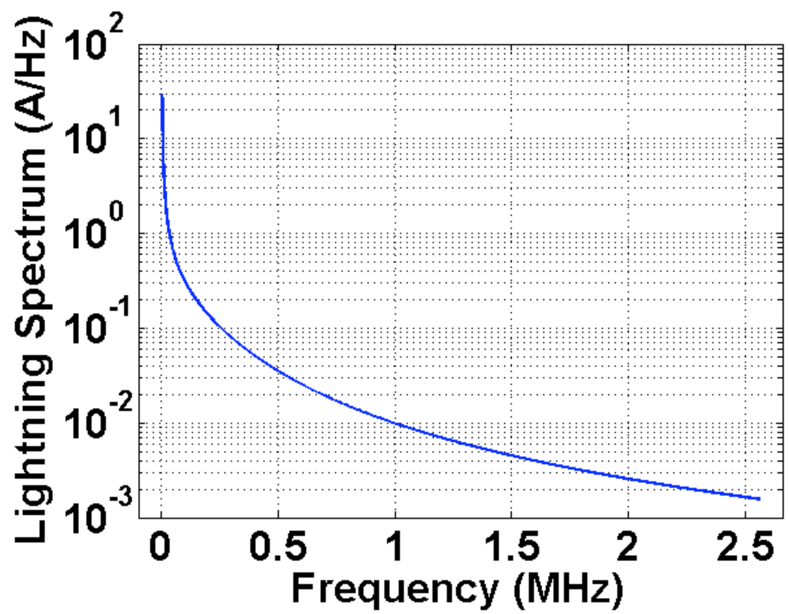
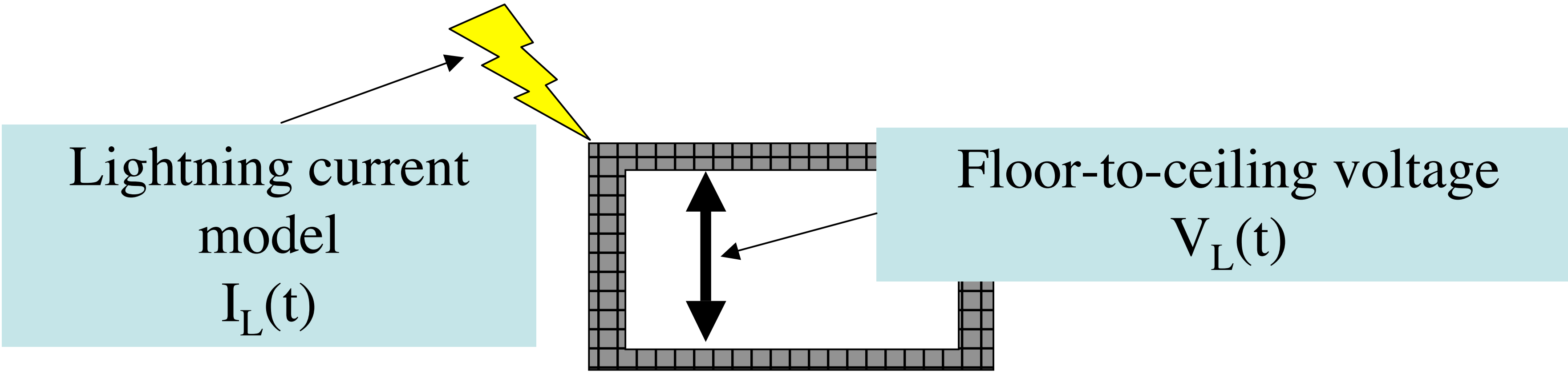
Double-exponential lightning current model

200 kA peak current   400 kA/_s maximum rise rate

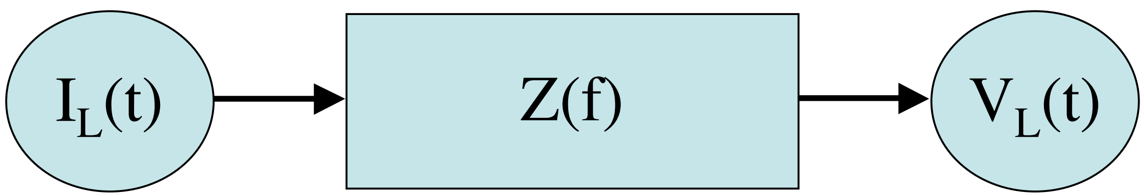




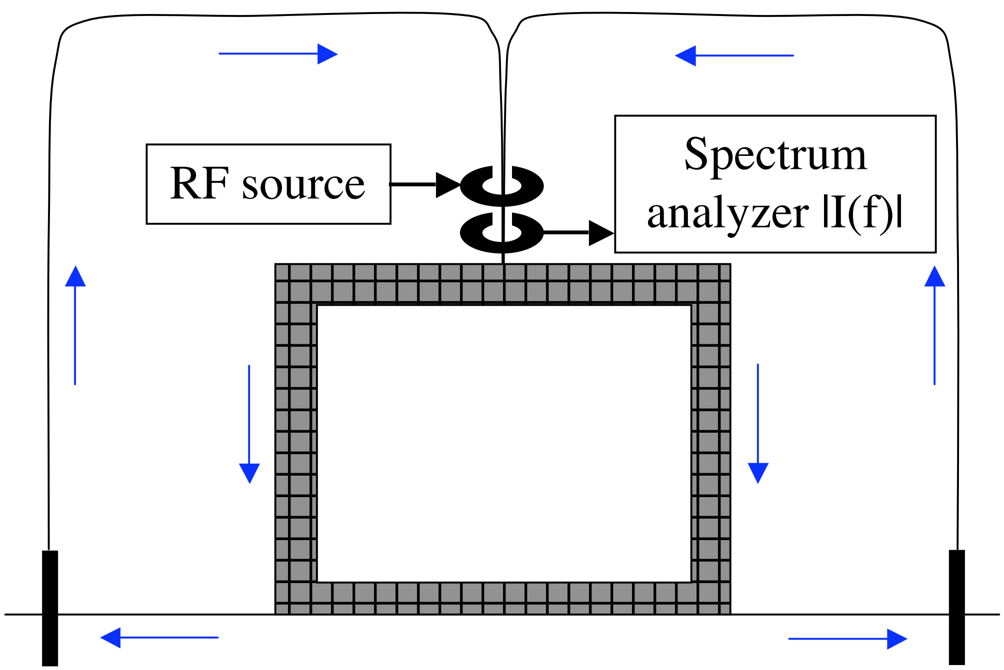
Model of floor-to-ceiling voltage

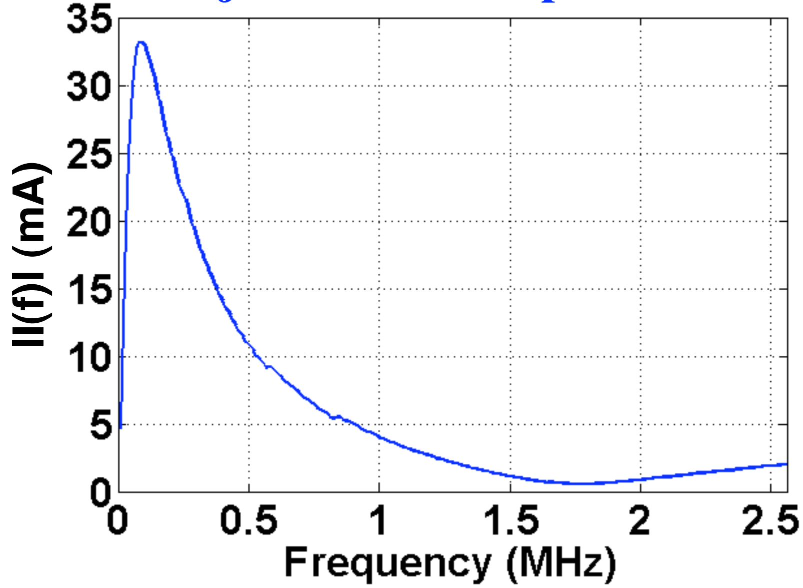


Building transfer impedance Z(f) relates $I_L(t)$ to $V_L(t)$



Measurement of the building transfer impedance: spectrum of the injected current lI(f)l





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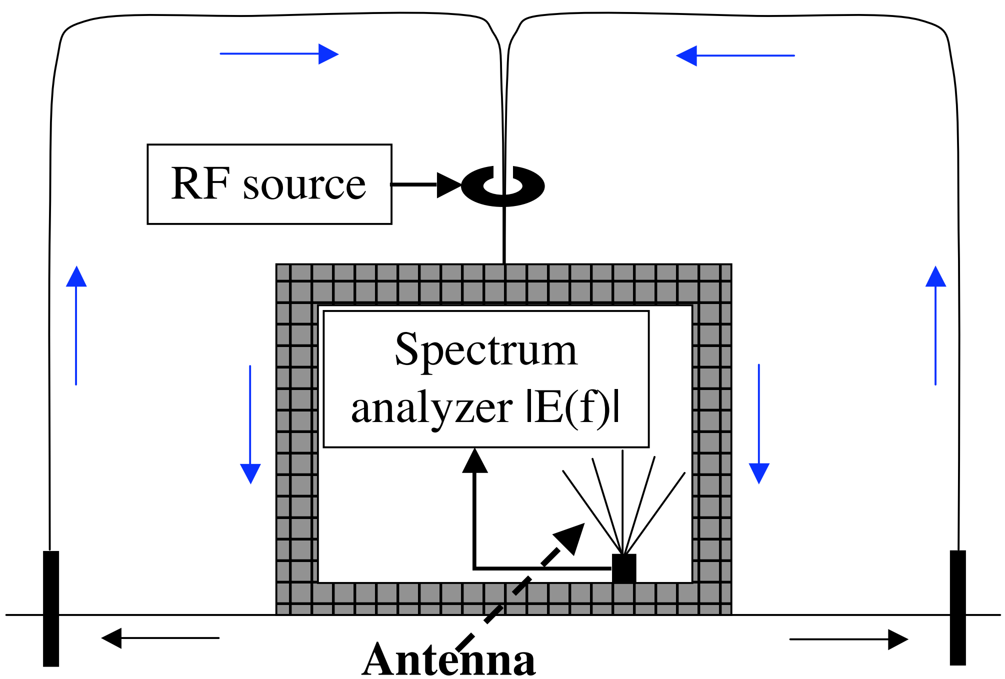
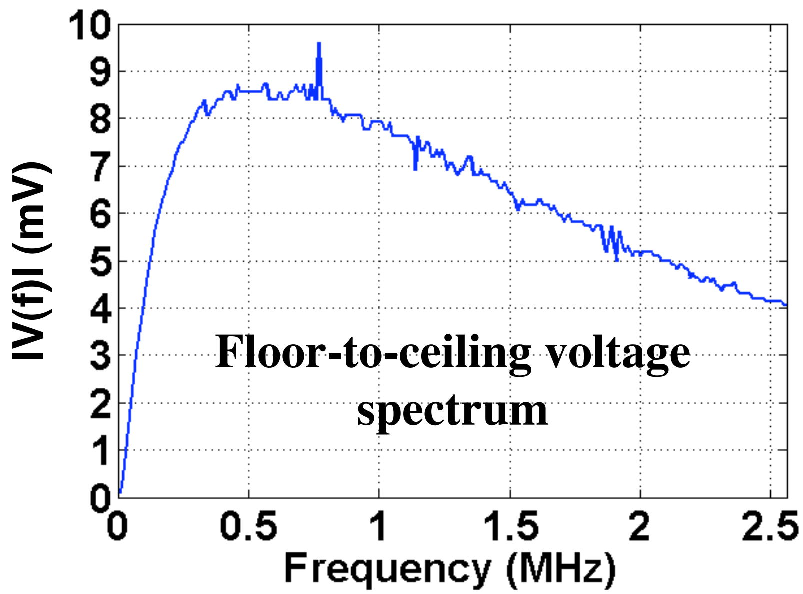
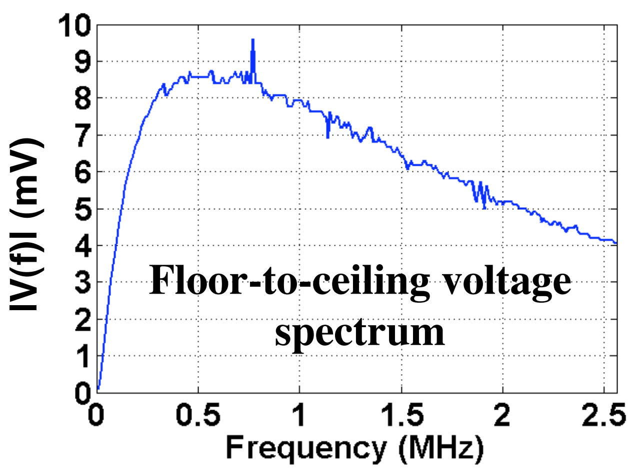
Measurement of the building transfer impedance: resulting floor-to-ceiling voltage spectrum

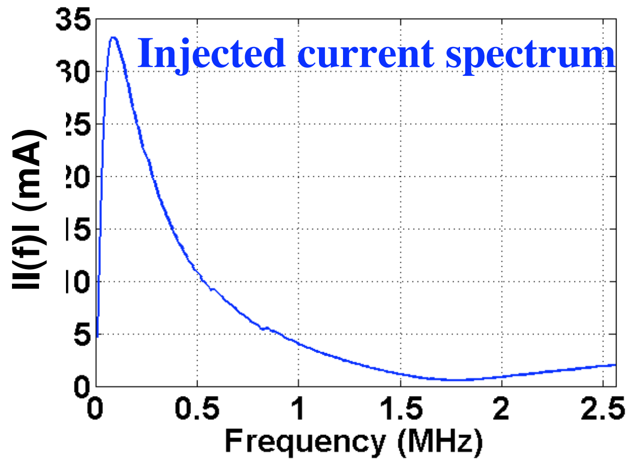
|V(f)| = |E(f)| x floor-to-ceiling height

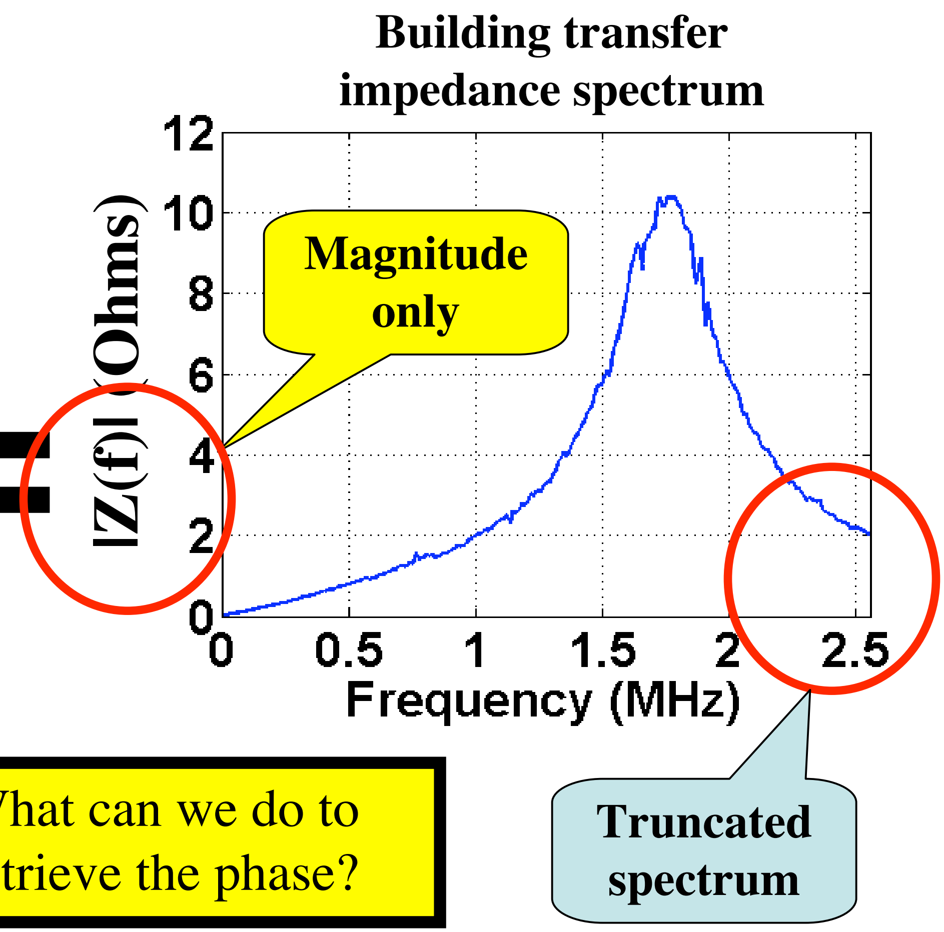




Measurement of the building transfer impedance: spectral division yields the impedance







What can we do to retrieve the phase?

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The technical problem is very difficult, because we do not have access to the phase of the measured spectra

- We are given the modulus |X(k)| of a ***complex*** Discrete Fourier Transform (DFT), and we would like to invert the full DFT to find its corresponding ***real*** time waveform x(n). Of course, we need to find x(n) uniquely.

- Unfortunately, in the absence of any underlying signal model or constraints, the loss of either phase or magnitude information of a complex function is irreversible. For our problem, this means that there is no unique inverse for the DFT.

- Surprisingly, however, under some fairly general conditions, it is possible to recover a signal from the phase of its Fourier Transform or from its magnitude.

Phase Retrieval Algorithms Have Limitations

- **Uniqueness of the Solution:**
  - ***For 2D Signals (Images)***

    The phase retrieval problem is usually ("almost always") unique, if noise issues are ignored.

  - ***For 1D Signals***

    Unfortunately, uniqueness is a big problem for 1D signals

- ***All phase retrieval algorithms are sensitive to noise***
  - **Require "regularizing" using constraints to make them effective with measured data**

- **Steps must be taken to avoid time domain aliasing errors and leakage**

- **Many algorithms have serious difficulties with slow convergence**

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Uniqueness Is More Problematic in 1D Than 2D

Theorem:

A finite-length signal which has an *irreducible z-transform* is uniquely defined (to within a sign, a time shift, and a time reversal) by the magnitude of its Fourier Transform.

2D:

It can be shown that because almost all polynomials in two or more variables are irreducible, a finite support constraint is sufficient (in most cases) to ensure uniqueness.

1D:

Unfortunately, the only one-dimensional polynomials which have irreducible z-transforms are those which are of length N = 1 or N = 2 (N is the length of the sequence).

So, this result is not particularly useful in practice.

Several Types of Algorithms Are Available

- Energy reduction algorithm (Fienup et. Al.) - Iterates with constraints between the Fourier and Time domains
- Hybrid Input-Output algorithm (Fienup et. Al.)
- Use of Higher-Order Spectra (Bispectrum), (Petropulu et. Al.)
- Wavelet-based algorithms (Yagle et. Al.)
- Homomorphic signal processing algorithms based on the complex cepstrum or the real cepstrum, Oppenheim, Schafer, et. Al.
- Some related results from the blind deconvolution literature (Stark et. Al.)
- Methods based upon the solution of systems of linear equations (Yagle et. Al.)

Due to Tight Programmatic Budgetary and Time Constraints, We Chose to Apply the Homomorphic Approach Using the Cepstrum

Define the *Complex Cepstrum*

Given a discrete - time sequence $h(n)$ that has a corresponding z - transform $H(z)$, we can define the quantity $\hat{H}(z)$ as follows:

$$\hat{H}(z) = \log\left[H(z)\right]$$

We then define the complex cepstrum $\hat{h}(n)$ as follows:

$$\hat{h}(n) = Z^{-1}\left[\hat{H}(z)\right]$$

$$= Z^{-1}\left\{\log\left[H(z)\right]\right\}$$

We can implement this with the Discrete Fourier Transform (DFT):

$$\hat{h}(n) \quad = \mathrm{IDFT}\left[\log\left\{\mathrm{DFT}\left[\mathrm{h(n)}\right]\right\}\right]$$

Note:

- The DFT length N must be large enough to avoid cepstral aliasing.
- A complex logarithm is used, and the phase must be unwrapped appropriately.

Define the *Real Cepstrum* *(or just "Cepstrum")*

Using a real logarithm, let

$$\hat{H}(z) = \log\left|H(z)\right|$$

The real cepstrum $c(n)$ of the real sequence h(n) is then

$$c(n) = Z^{-1}\left[\hat{H}(z)\right]$$

$$= Z^{-1}\left\{\log\left|H(z)\right|\right\}$$

We can implement this with the Discrete Fourier Transform (DFT) as follows:

$$c(n) \quad = \mathrm{IDFT}\left[\log\left|\mathrm{DFT}\left[h(n)\right]\right|\right]$$

Note:

• The DFT length N must be large enough to avoid cepstral aliasing.

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The Name *"Cepstrum"* Was Coined by Bogert, Healy and Tukey in 1963.

B.P. Bogert, M.J.R. Healy and J.W. Tukey, *"The Quefrency Alanysis of Time Series for Echoes: Cepstrum, Pseuso-Autocovariance, Cross-Cepstrum, And Saphe Cracking,"* Symp. Time Series Analysis, M. Rosenblatt, Ed., New York, John Wiley and Sons, Inc., New York, 1963, p.. 209-243.

- *They were processing signals containing echoes.*
- *They found that the log of the power spectrum of a signal containing an echo has an additive periodic component due to the echo.*
- *So, the Fourier Transform of the log-power spectrum should have a peak at the echo delay.*
- *They called the Inverse Fourier Transform of the log-power spectrum the "cepstrum" (from "spectrum").*
- *"In general, we find ourselves operating on the frequency side in ways customary on the time side and vice versa."*
- *They also coined "lifter," "alanysis," "quefrency," etc., but only "cepstrum" has been adopted widely.*

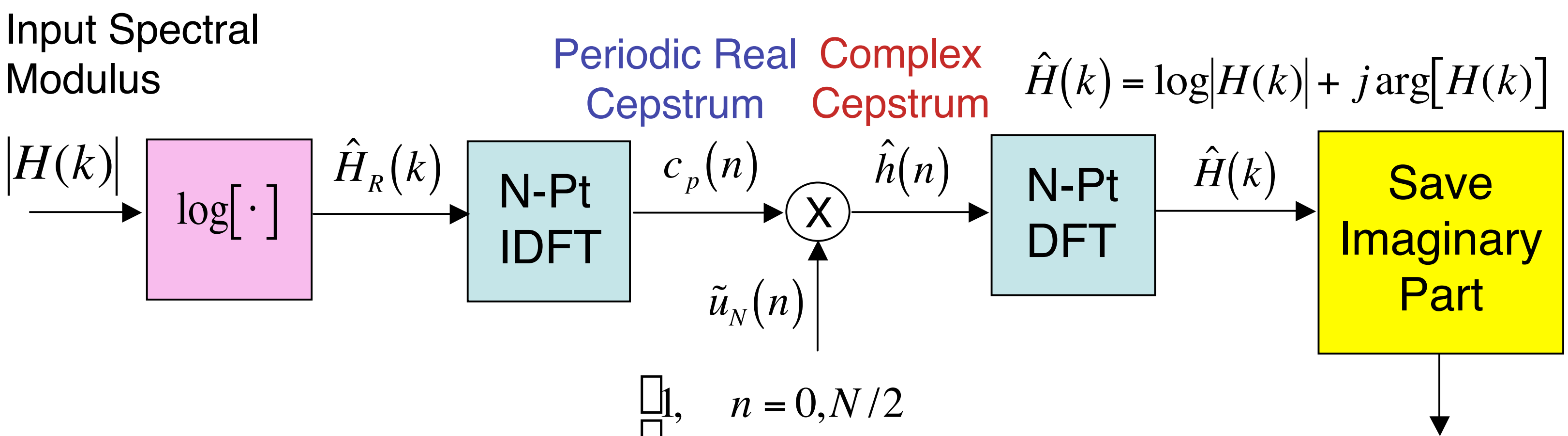
The Phase Retrieval Algorithm Exploits Several Theoretical Conditions/Properties That Are Often Reasonable in Practice:

- ***Causality***: Real and Imaginary Part Sufficiency for Causal Sequences: If *h(n)* is causal, then it is possible to recover *h(n)* from:
  1. Only the even part $h_e(n)$ of h(n)
  2. Only the odd part $h_o(n)$ of h(n) ***for*** $n \neq 0$

- The underlying signal/system *H(z)* is assumed to obey the ***Minimum Phase*** condition:
  1. $\log|H(z)|$ and $\arg[H(z)]$ are Hilbert Transforms of each other
  2. H(z) has no poles or zeros outside the unit circle
  3. There exists a causal, stable inverse system with system function H^{-1} such that $H(z)H^{-1}(z) = 1$

Given Only Spectral Modulus $|H(k)|$, We Can Reconstruct Spectral Phase $\arg[H(k)]$ Using the DFT

$$\hat{h}_{cp}(n) = c_p(n)\tilde{u}_N(n) \approx \hat{h}(n) \text{ for large N}$$



$\arg[H(k)]$

=

Reconstructed Phase of h(n)

Cepstral Window $\tilde{u}_N(n) = \begin{cases} 1, & n = 0, N/2 \\ 2, & n = 1, 2, \ldots, (N/2) - 1 \\ 0, & n = (N/2+1), \ldots, N-1 \end{cases}$

$\hat{h}(n)$ = A *minimum phase* reconstruction of a finite-length, real, causal, stable sequence h(n) corresponding to the measured input spectral modulus |H(k)|
= An estimate of the *complex cepstrum* of h(n) for large N

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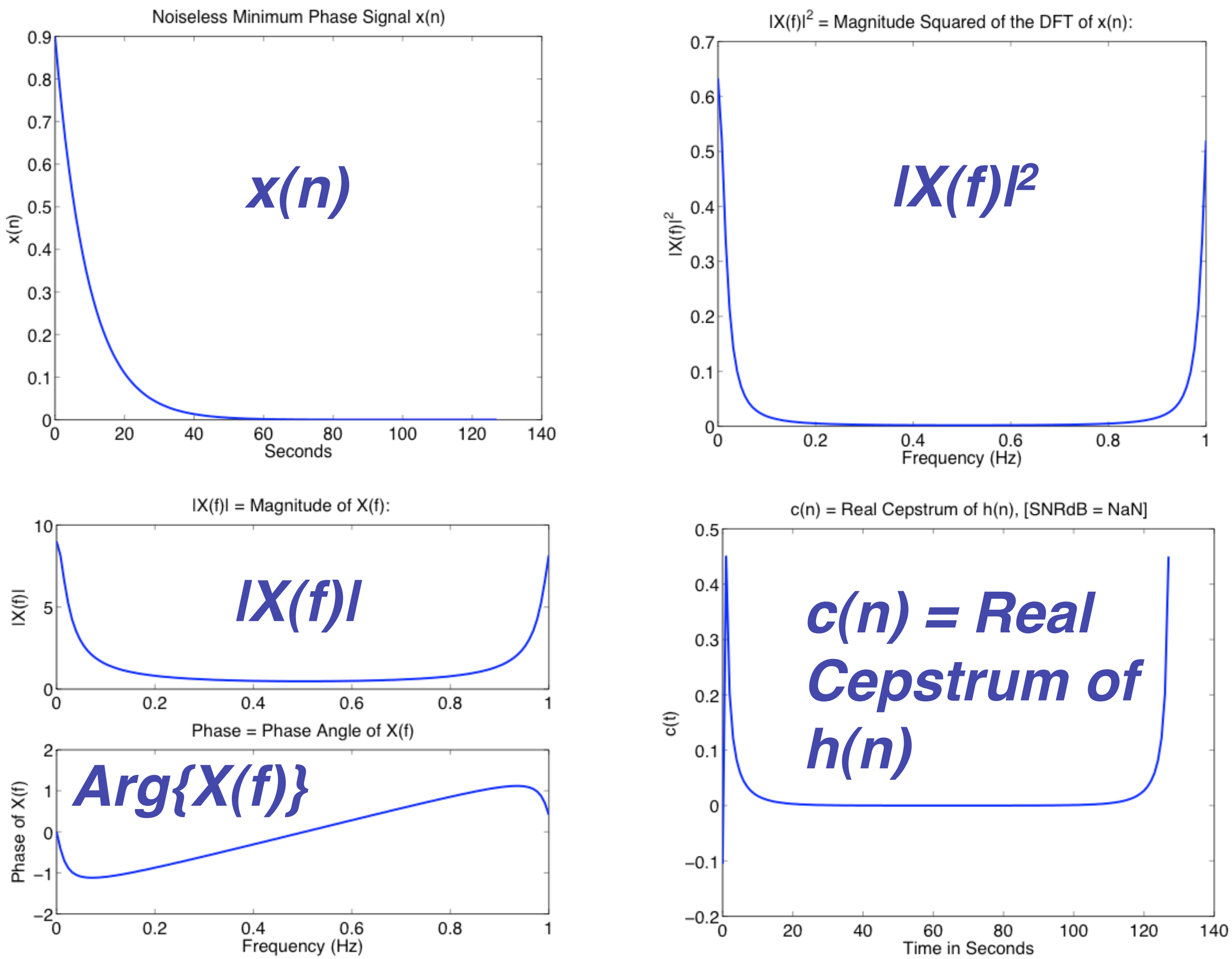
Example: Simulated noiseless minimum phase signal x(n), |X(f)|^2 , |X(f)|, Arg{X(f)}, c(n)



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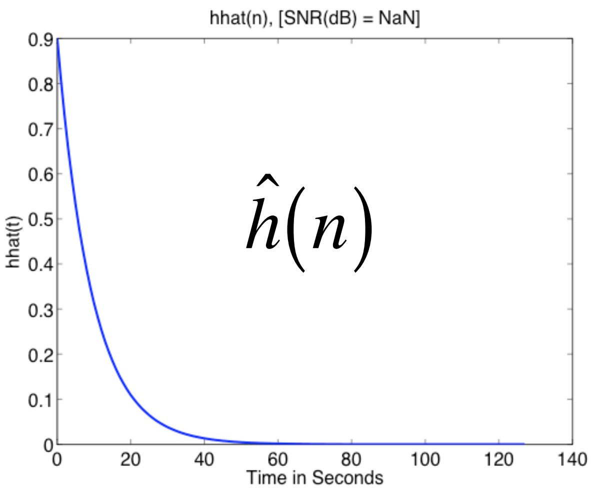
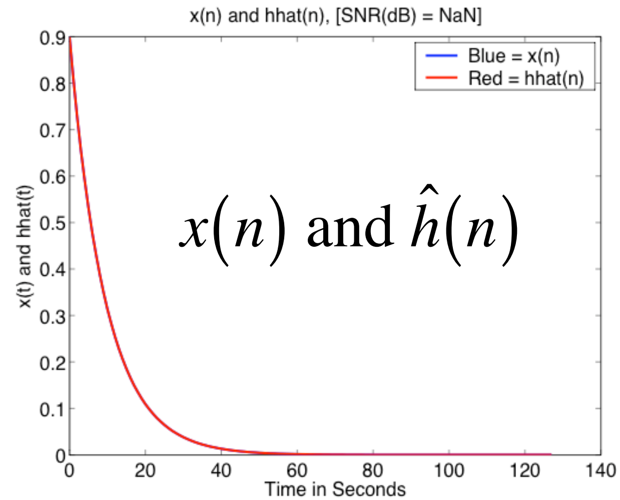
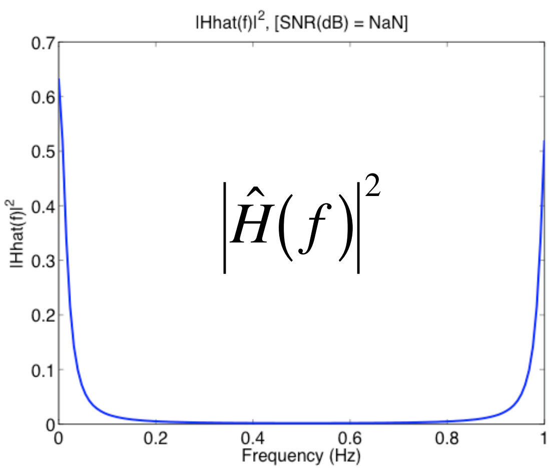
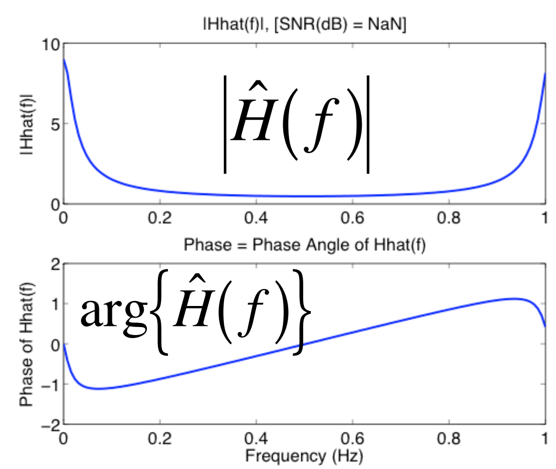
Example A: The reconstructed minimum phase signal $\hat{h}(n)$ matches the original signal x(n)

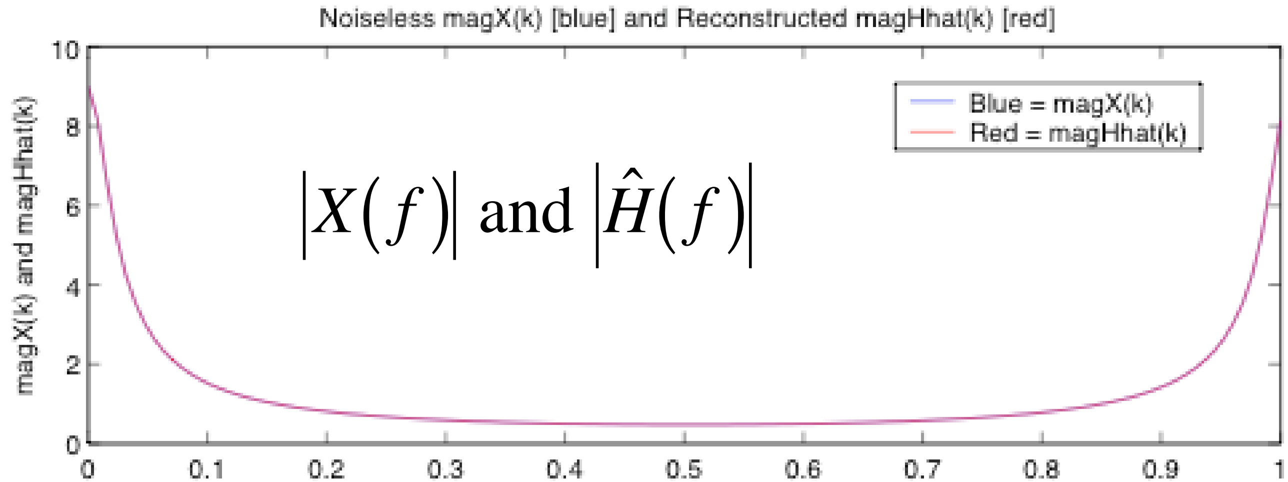


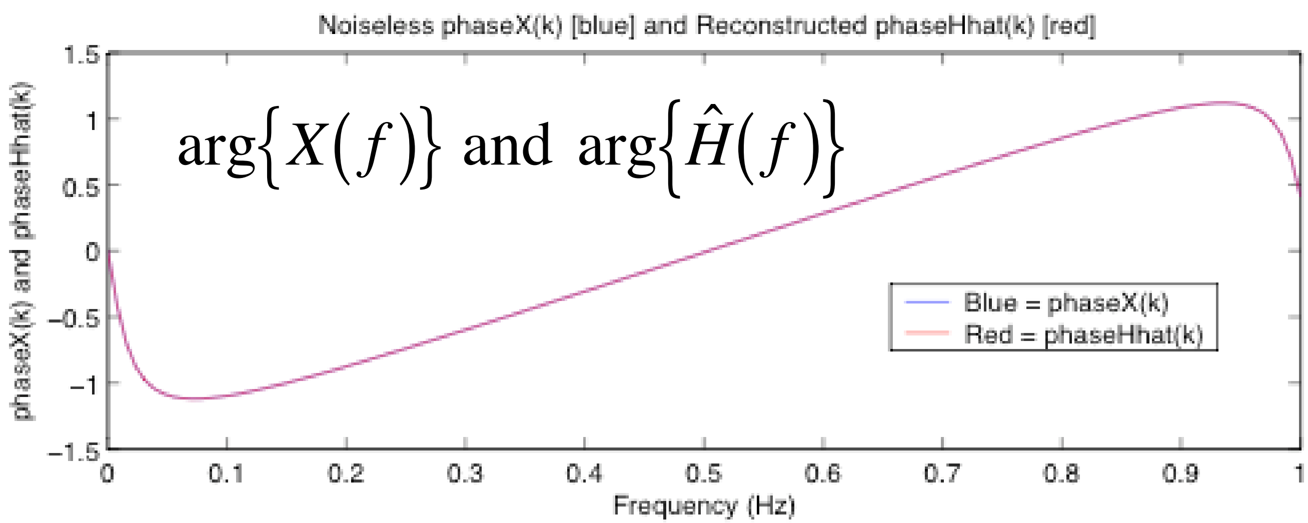






Example A: The agreement between the original and retrieved signals is excellent (in both magnitude and phase)





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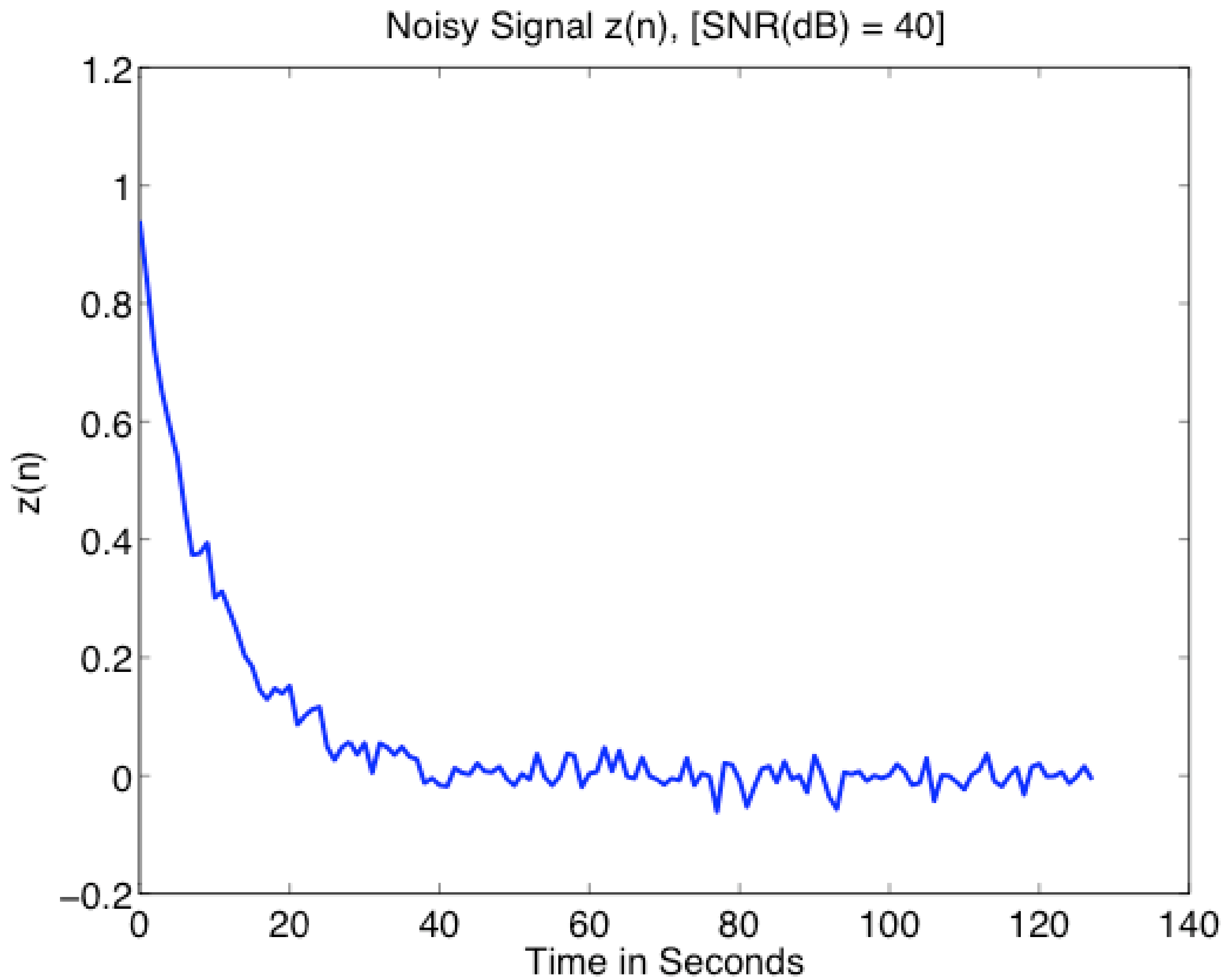
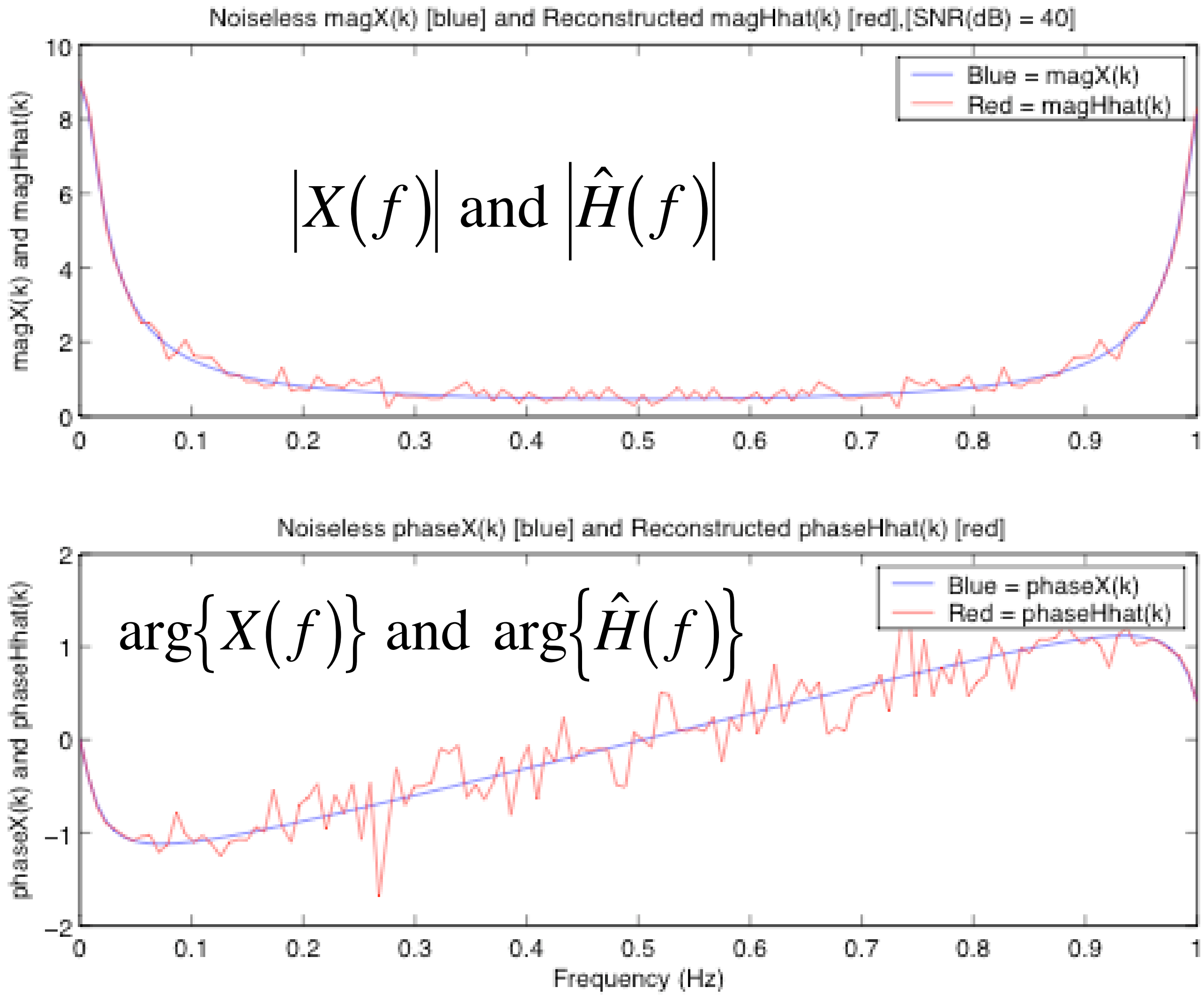
Example C: SNR = 40 dB: $z(n) = x(n) + v(n)$, where $v(n) \sim N[0, 4.26e\text{-}4]$



Example C: SNR = 40 dB:
z(n) = x(n) + v(n), where *v(n) ~ N[0, 4.26e-4]*



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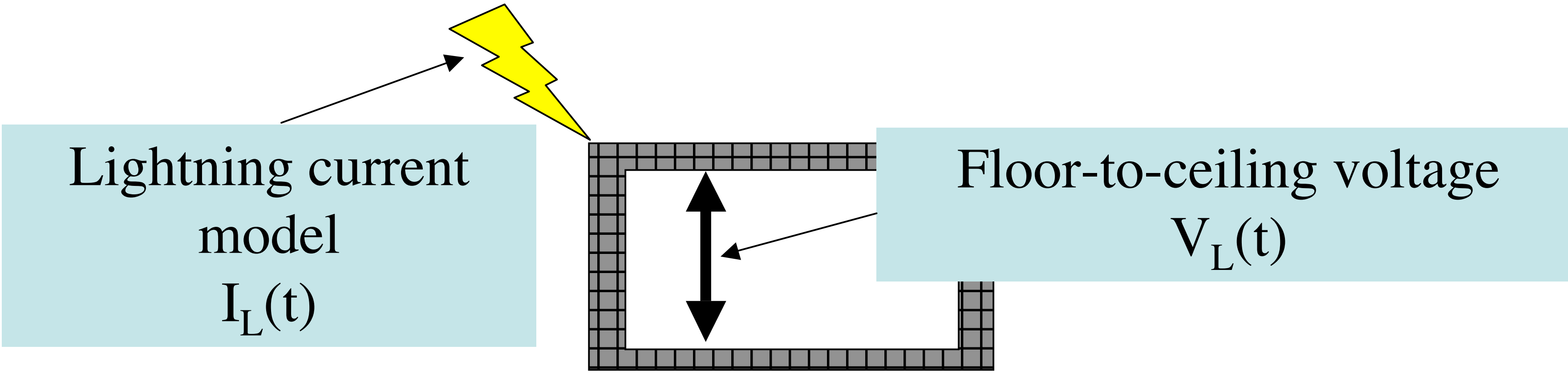
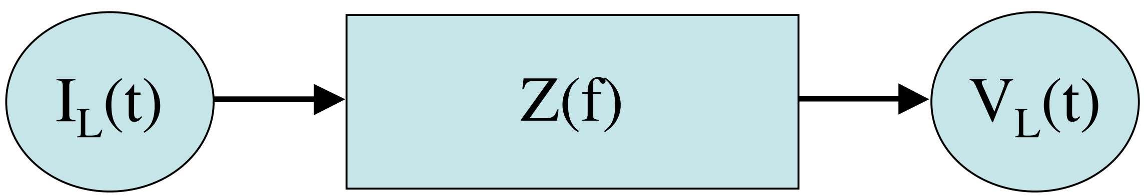
Simulation of floor-to-ceiling voltage

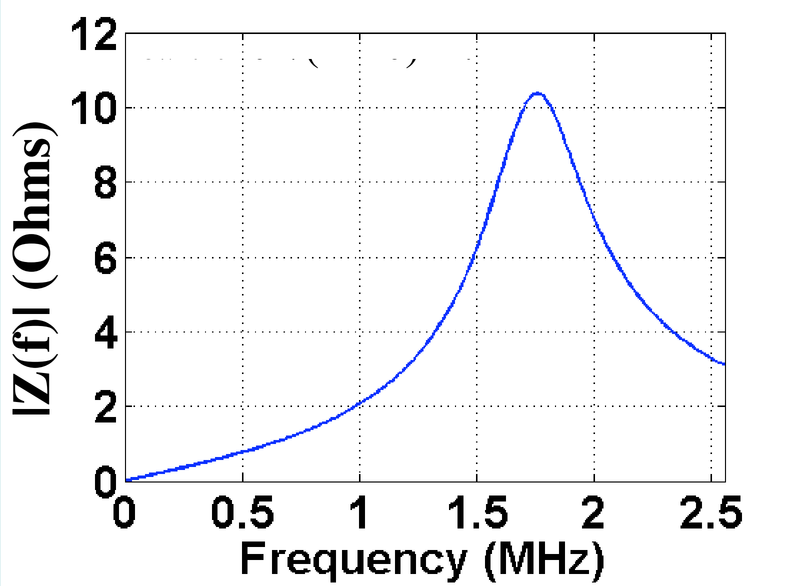


Building transfer impedance Z(f) relates $I_L(t)$ to $V_L(t)$

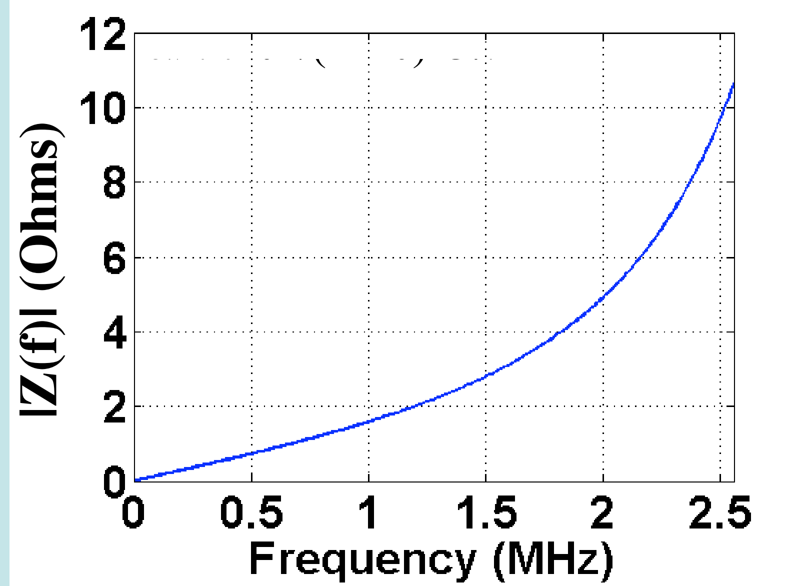


Impedance models used in simulations

Low resonant frequency model



High resonant frequency model



Evaluation of floor-to-ceiling voltage model in simulations

Exact (closed-form solution)

$Z(s)\, I_L(s)$ → Inverse Laplace transform → $V_L(t)$

Modeled phase (truncated spectrum)

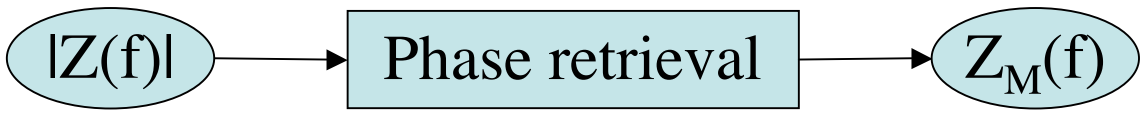
$Z(f)\, I_L(f)$ → Inverse Fourier transform (implemented using IFFT) → $V_L(t)$

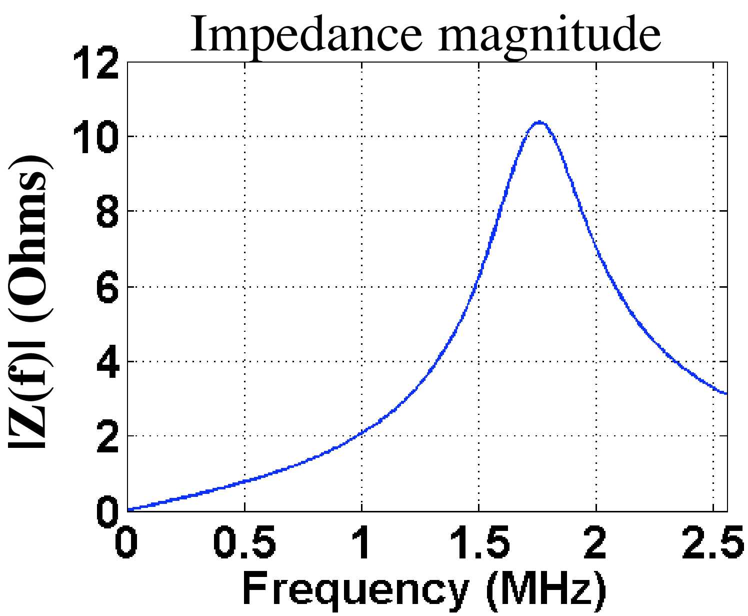
Retrieved phase (truncated spectrum)

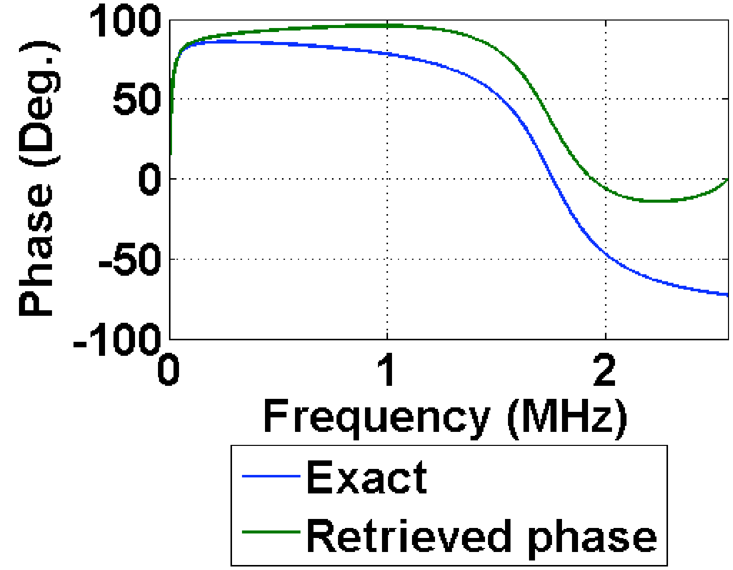
$|Z(f)|$ → Phase retrieval → $Z_M(f)$

$Z_M(f)\, I_L(f)$ → Inverse Fourier transform (implemented using IFFT) → $V_L(t)$

Cepstrum-based phase retrieval simulation: low resonant frequency model







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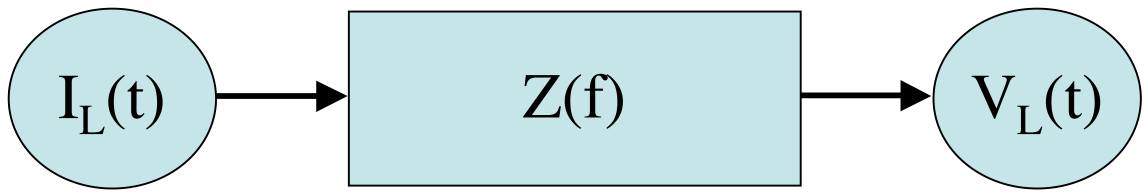
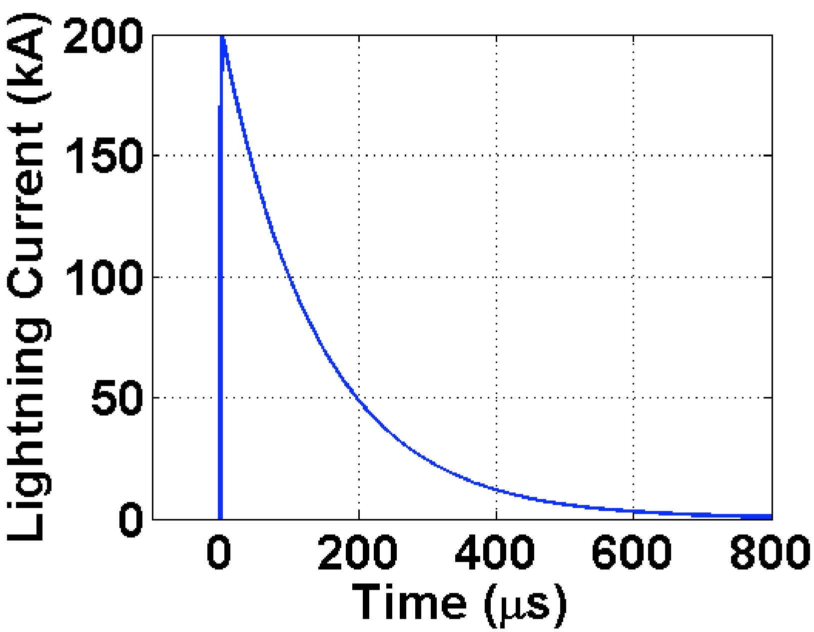
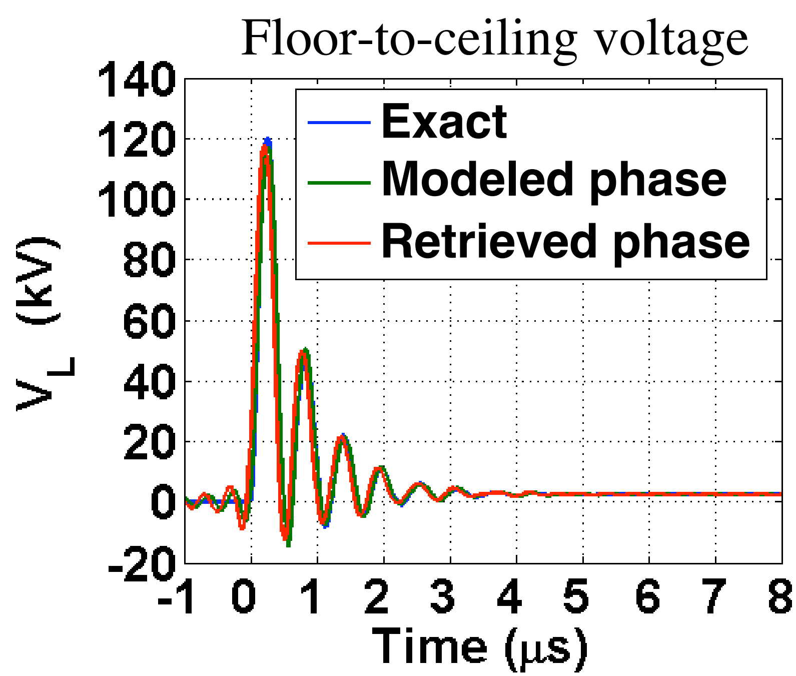
Floor-to-ceiling voltage comparison: low resonant frequency model simulation







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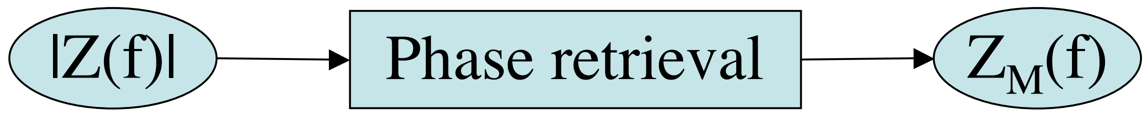
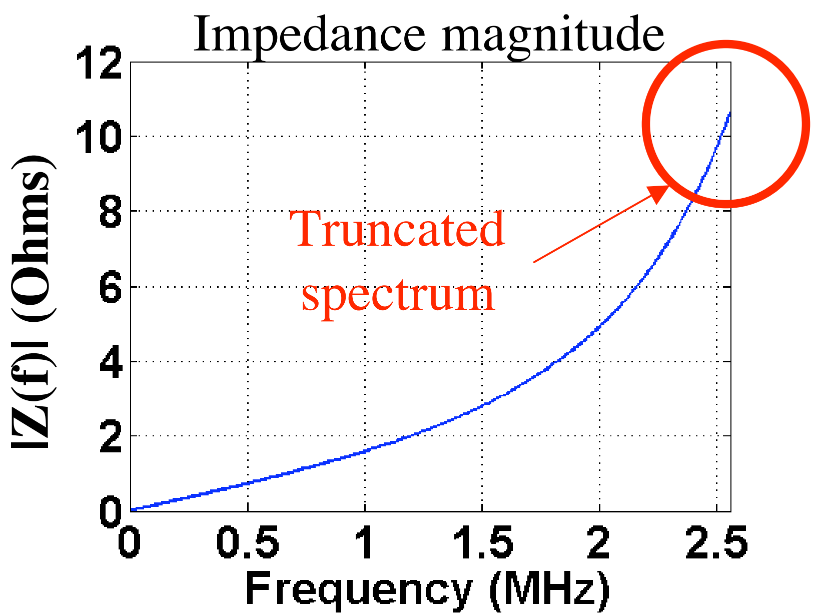
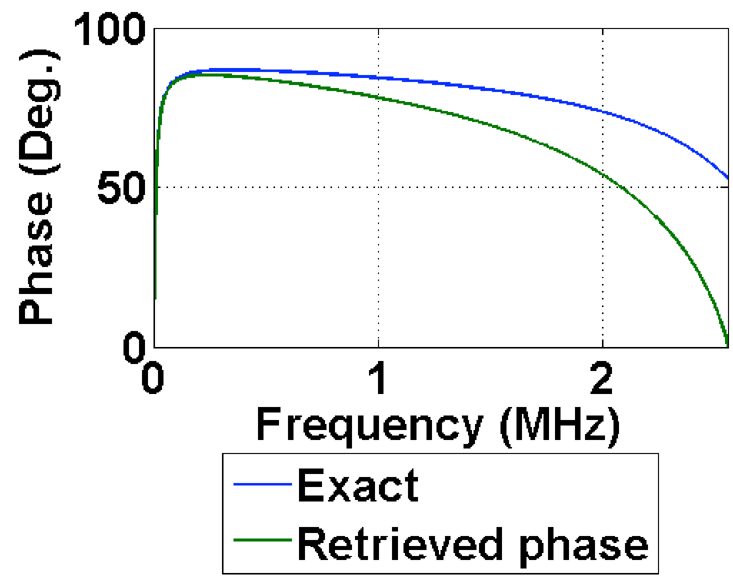
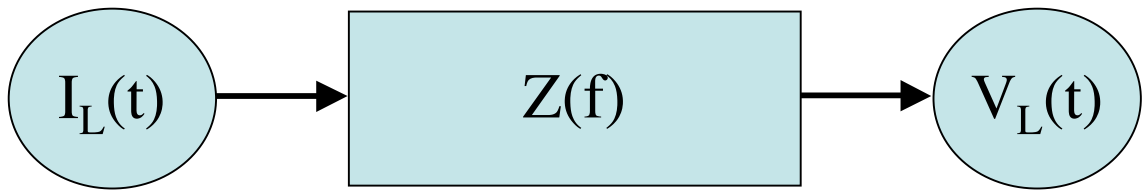
Cepstrum-based phase retrieval simulation: high resonant frequency model

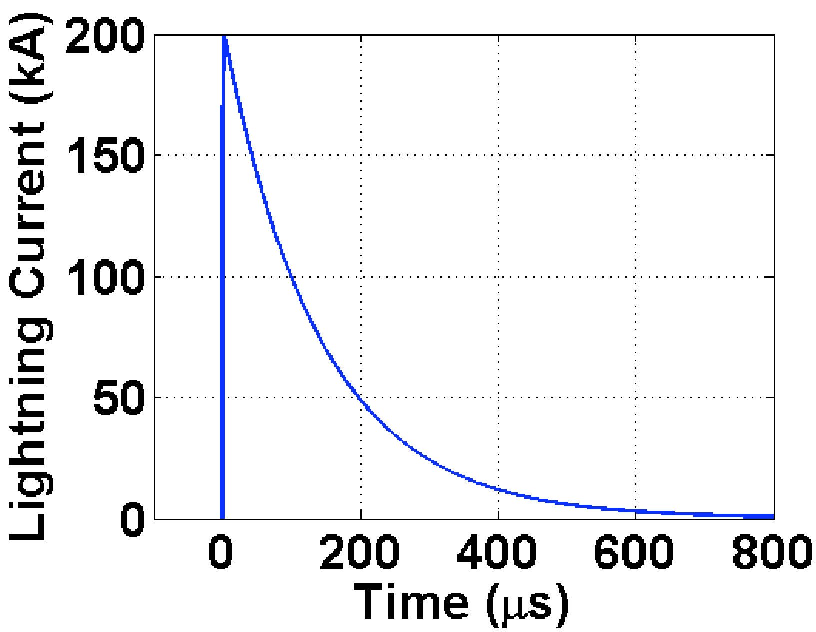


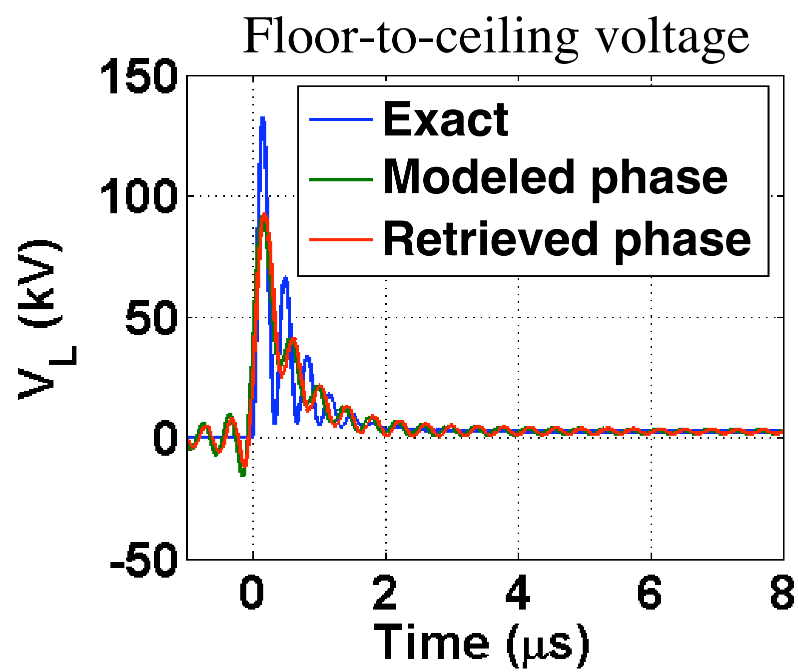




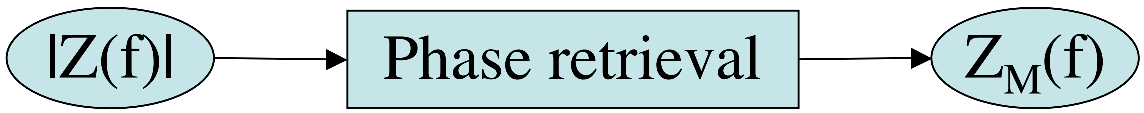
Floor-to-ceiling voltage comparison: high resonant frequency model simulation

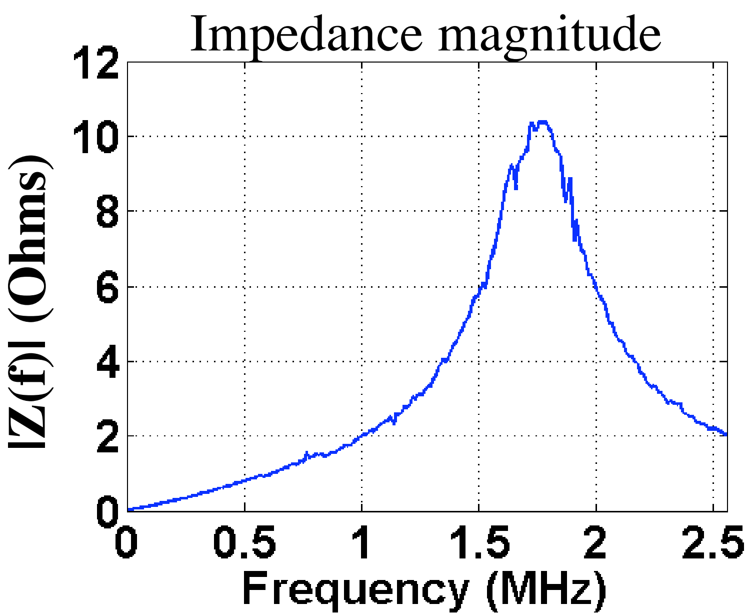


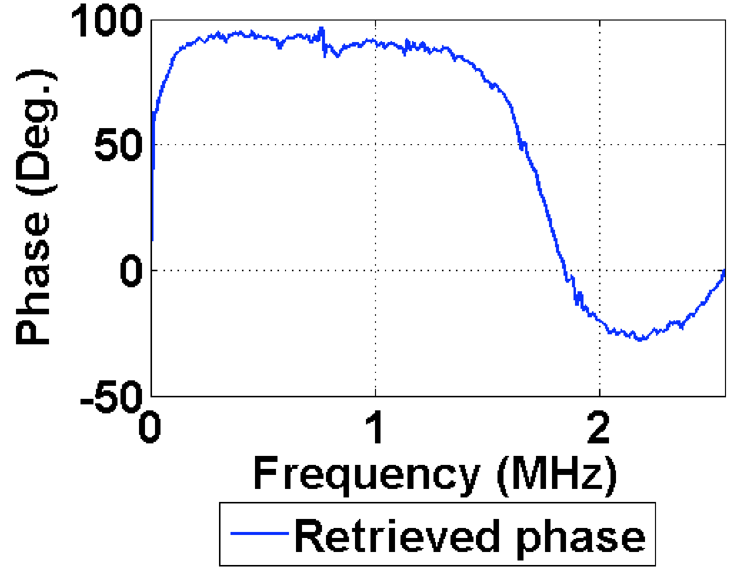




Cepstrum-based phase retrieval simulation: actual Site 300 data







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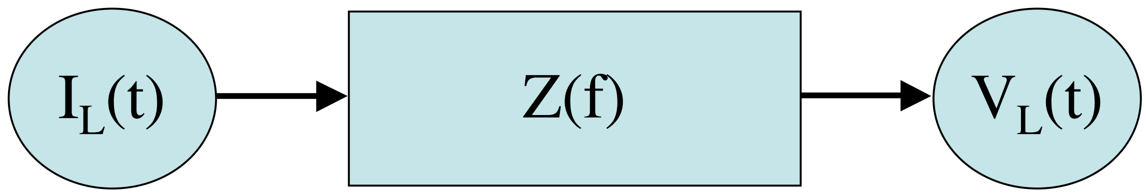
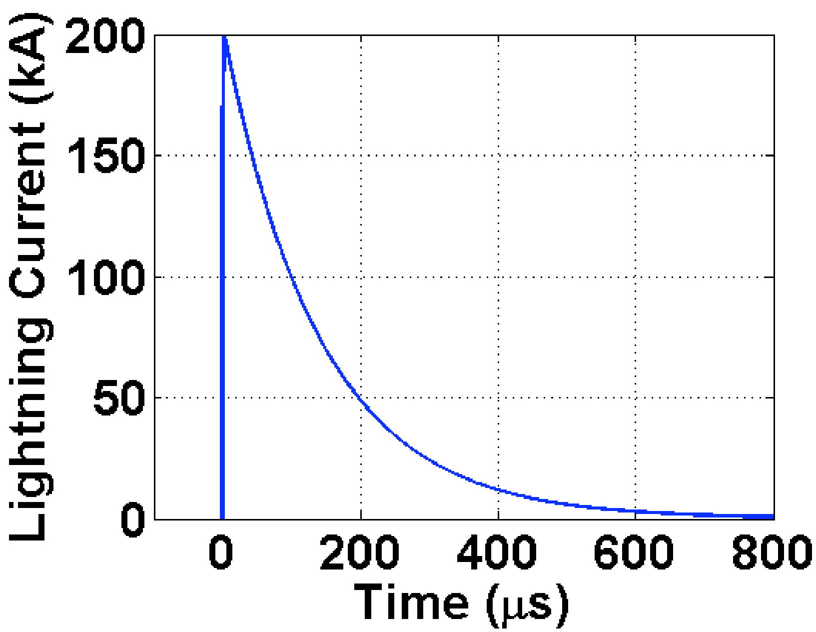
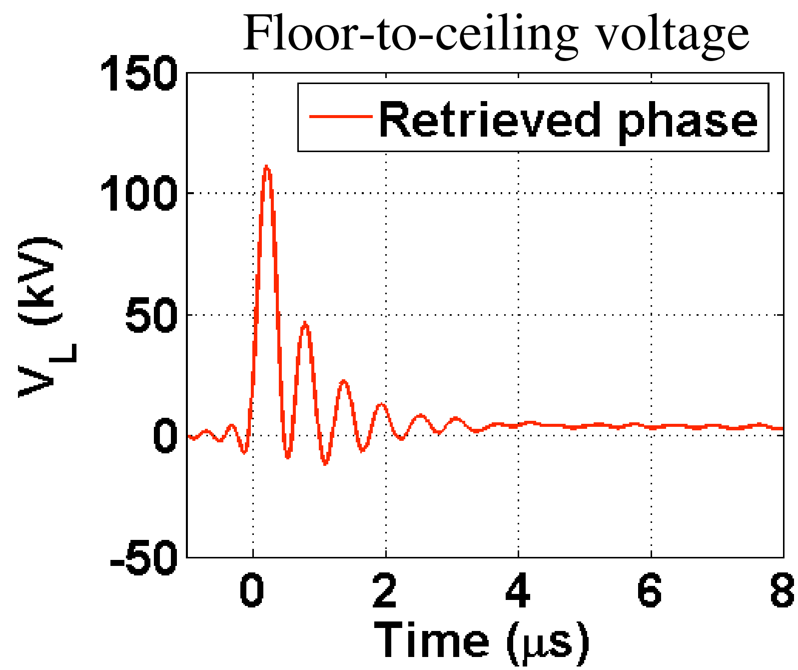
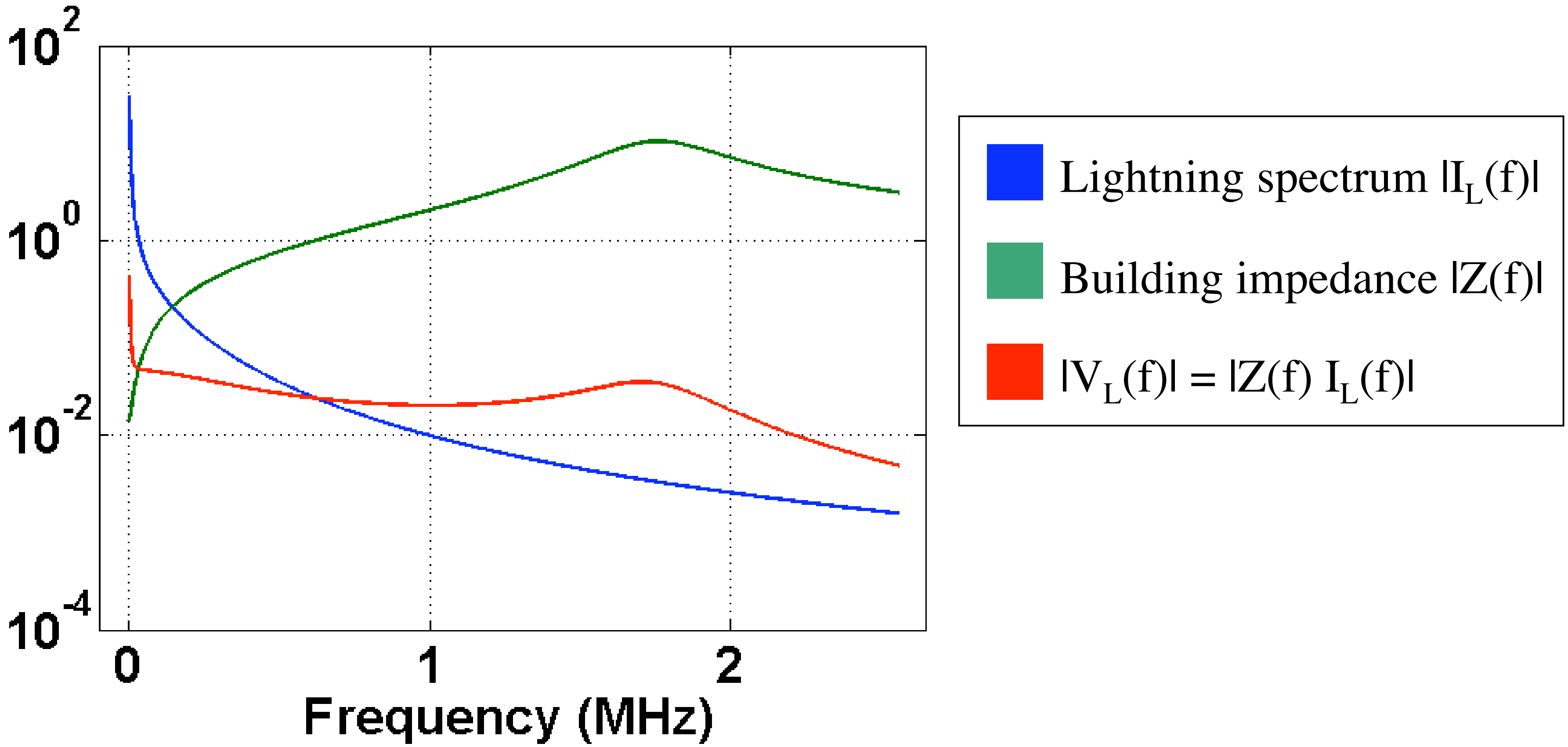
Floor-to-ceiling voltage: actual Site 300 data







The lightning spectrum acts as a low-pass filter



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Summary and conclusions

- Measurements pose two challenges:
  - Truncated spectrum
  - Lack of phase
- The cepstrum-based method used shows promise in reconstructing the phase of the building transfer impedance
- Need wider-bandwidth measurements in some cases
- Future work
  - Forward modeling: simulate the building transfer function and measurement system
  - Advanced system identification algorithms
  - Deal more effectively with truncated spectrum
  - Application and comparison of other phase retrieval methods

Extra Viewgraphs

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No. W-7405-ENG-48.

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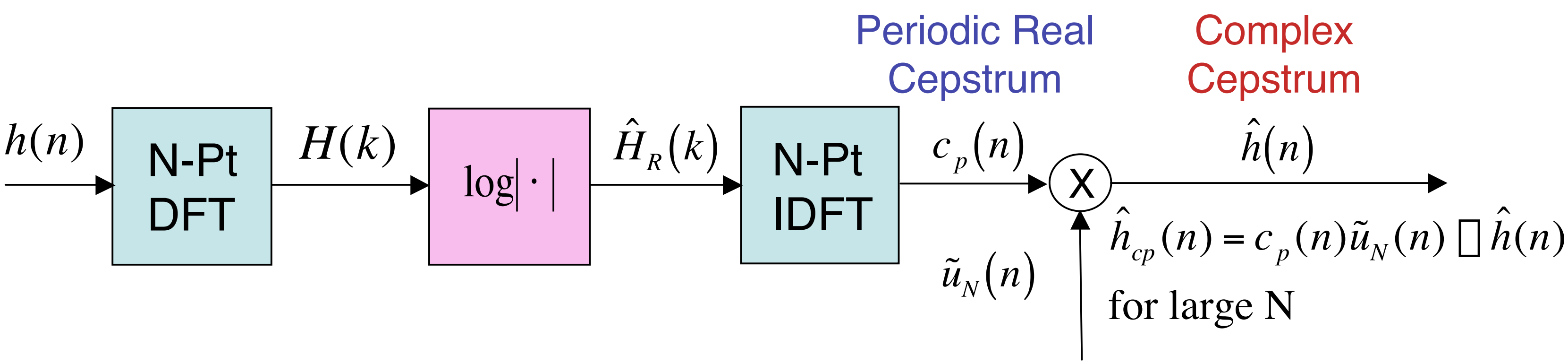
Given a Finite-Length, Real, Causal, Stable Sequence h(n), We Can Construct a Minimum Phase Realization $\hat{h}(n)$ of h(n)

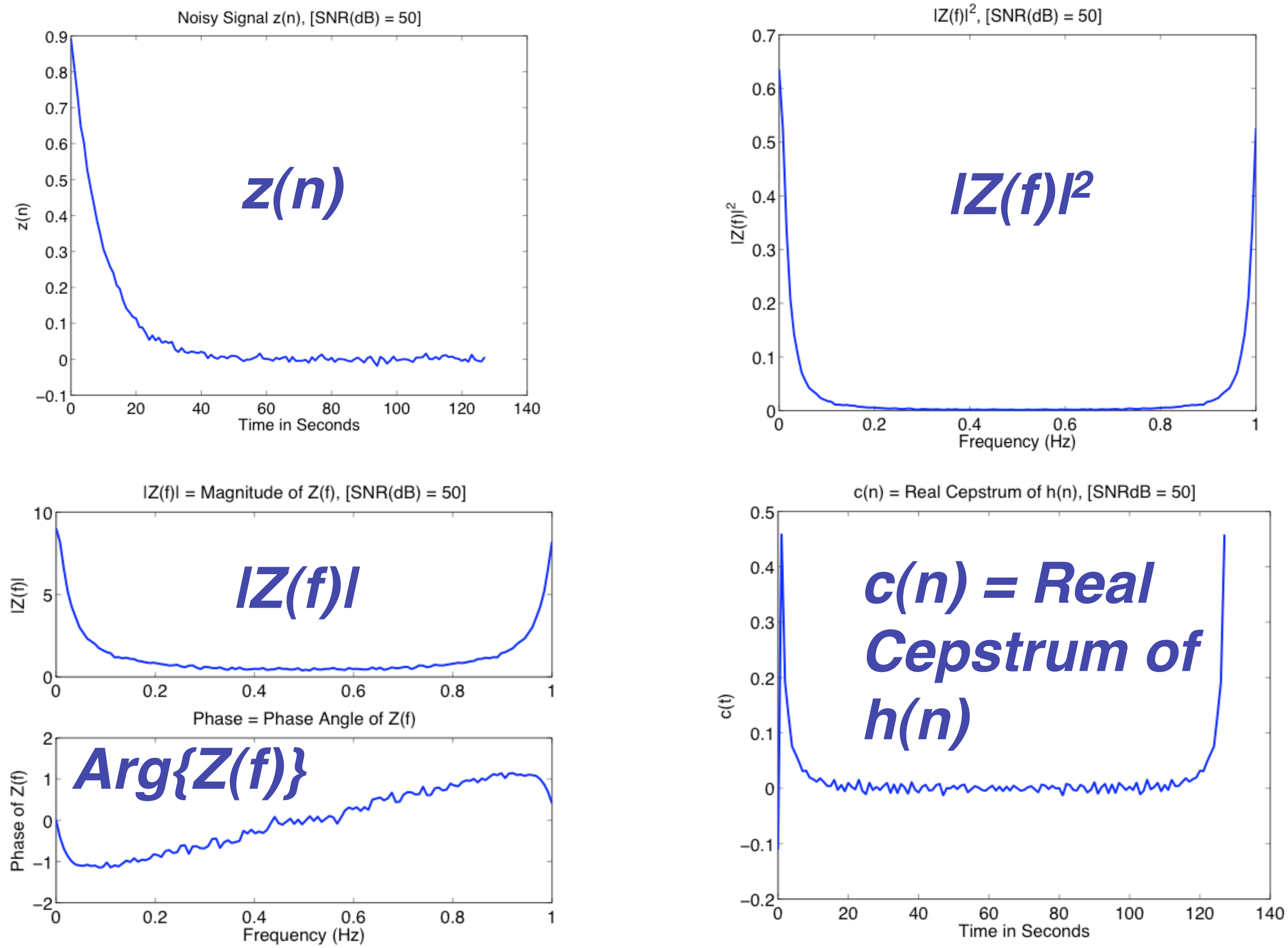


Cepstral Window $\tilde{u}_N(n) = \begin{cases} 1, & n = 0, N/2 \\ 2, & n = 1, 2, \ldots, (N/2) - 1 \\ 0, & n = (N/2 + 1), \ldots, N - 1 \end{cases}$

$\hat{h}(n)$ = A *minimum phase* reconstruction of the finite-length, real, causal, stable sequence h(n)

= An estimate of the *complex cepstrum* of h(n) for large N

Example B: SNR = 50 dB:
z(n) = x(n) + v(n), where *v(n) ~ N[0, 4.26e-5]*



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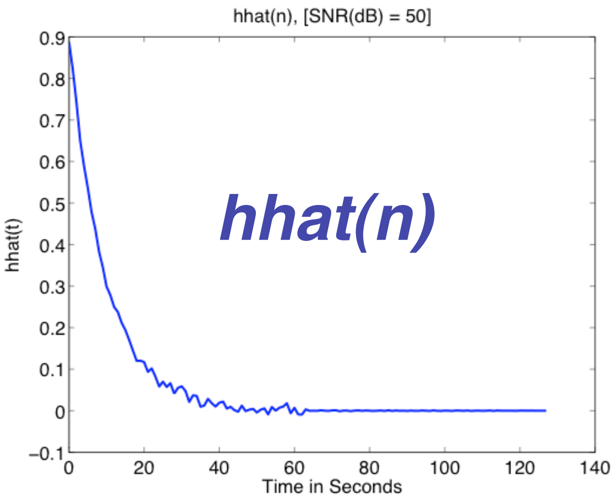
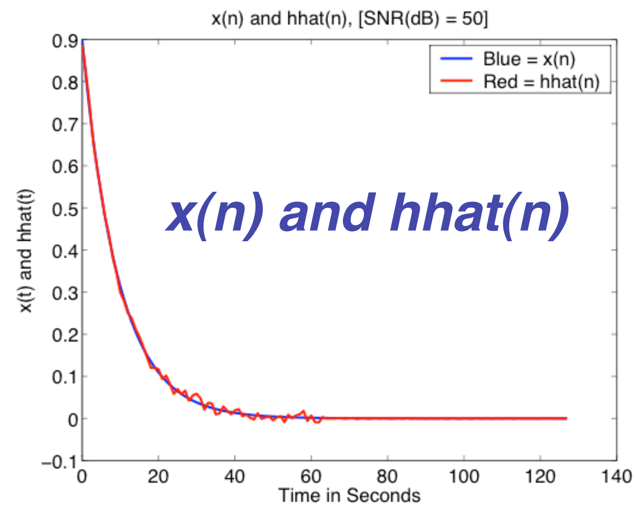
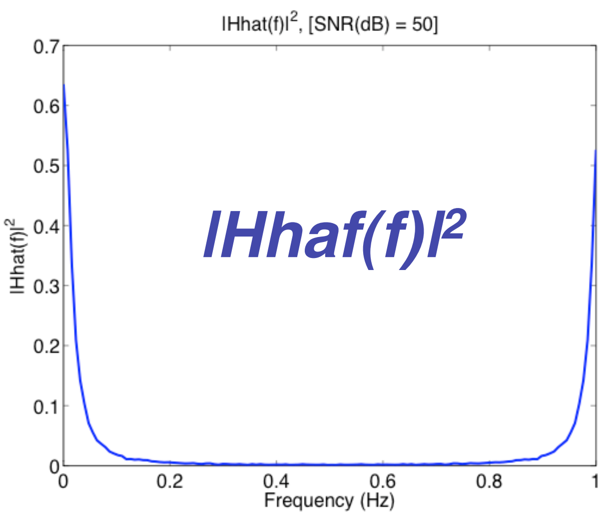
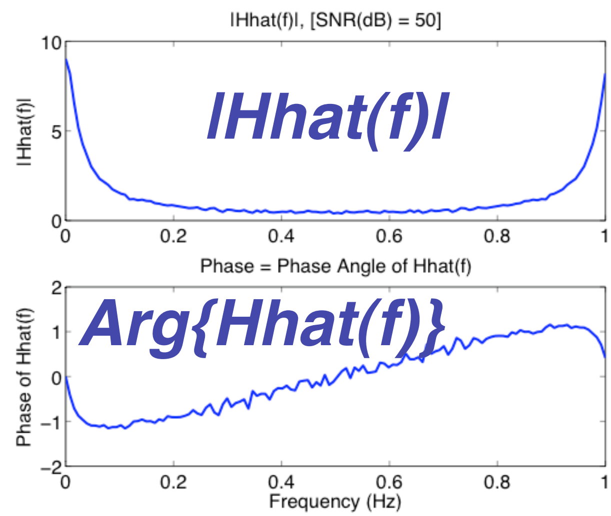
Example B: SNR = 50 dB:
z(n) = x(n) + v(n), where *v(n) ~ N[0, 4.26e-5]*









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Example B: SNR = 50 dB: z(n) = x(n) + v(n), where *v(n) ~ N[0, 4.26e-5]*