Subspace Detection in Seismology

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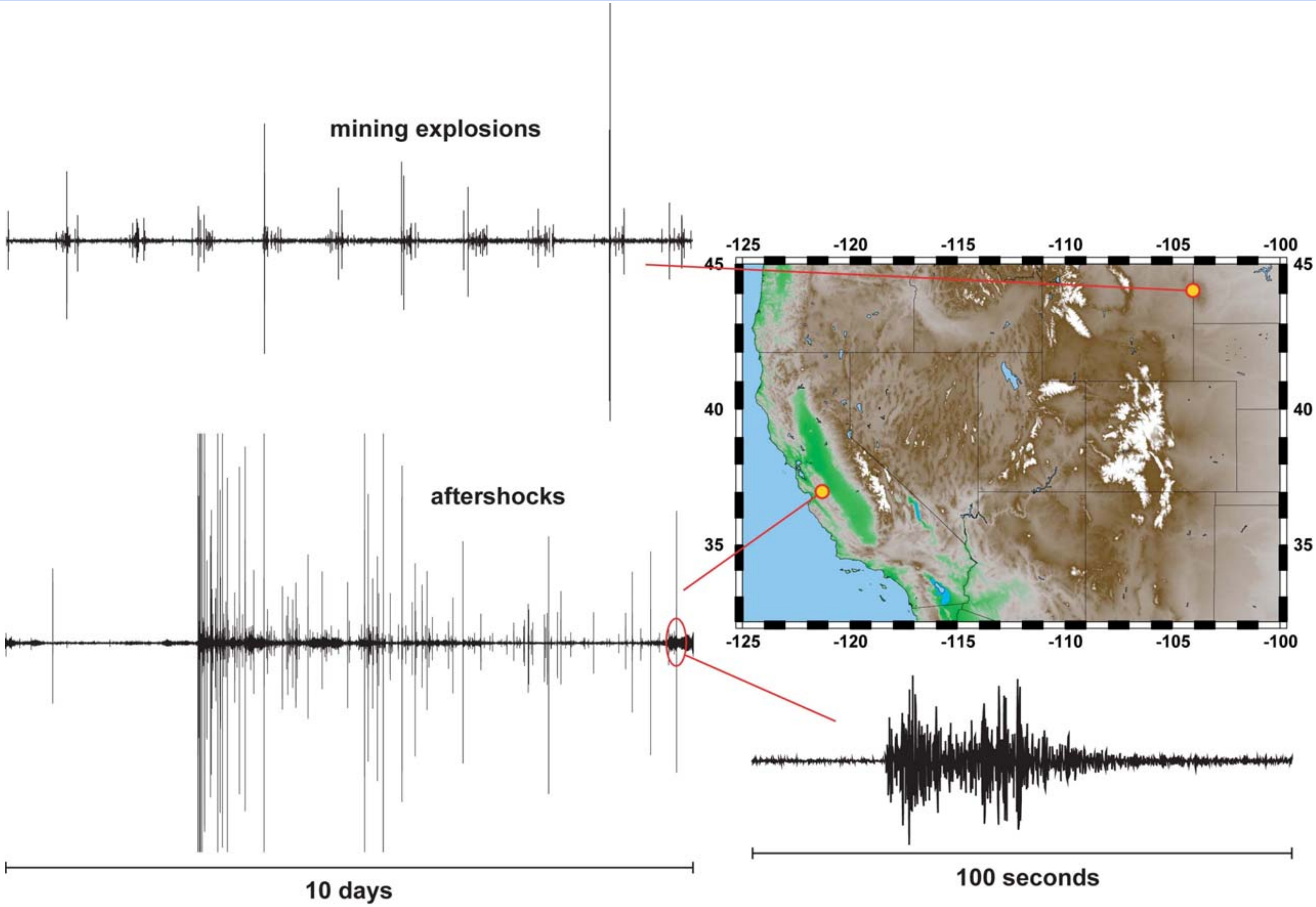
Seismic data are full of (nearly) repeating signals that must be detected, screened and characterized



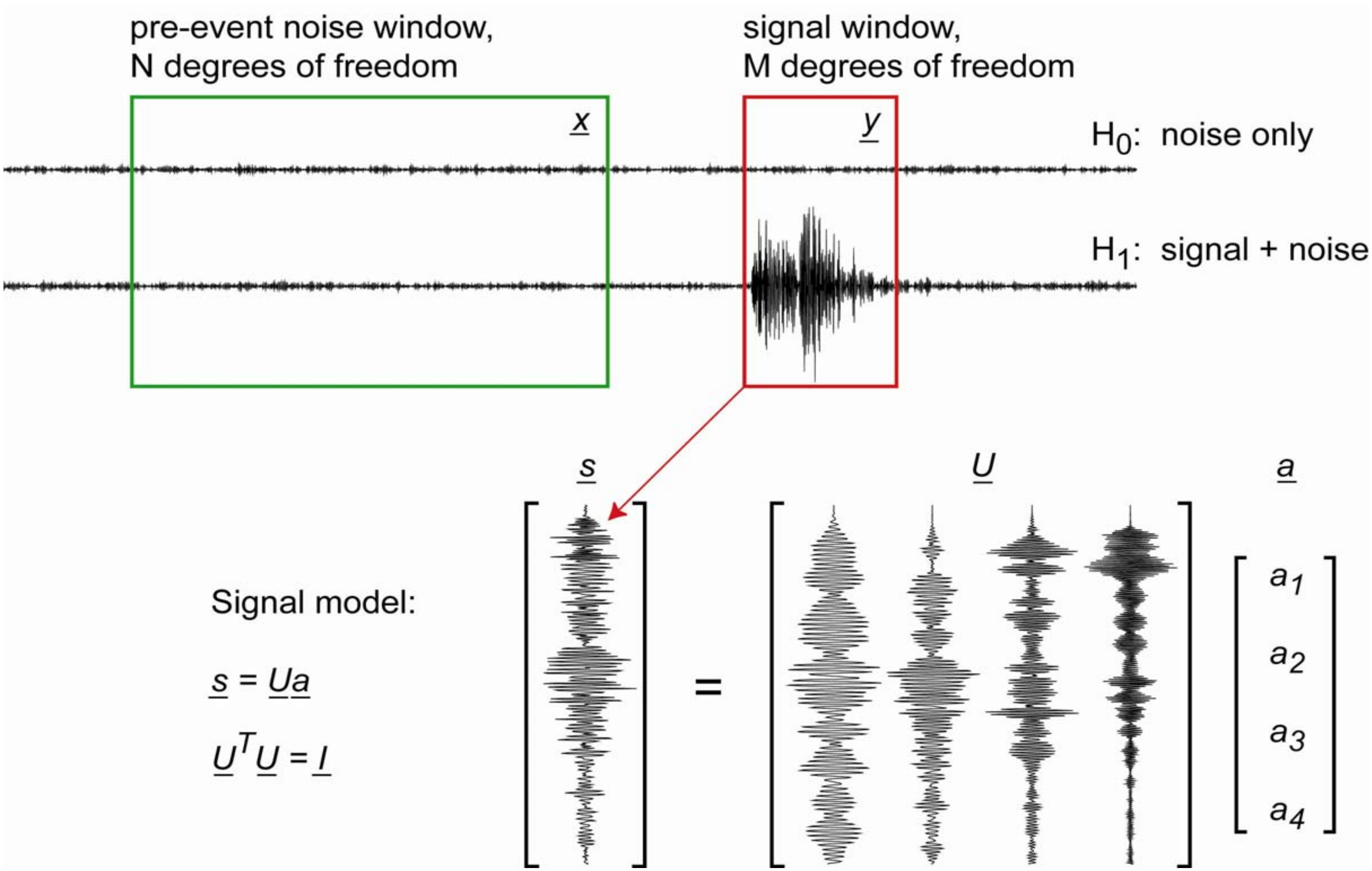
Current seismic detection practice is concentrated at the extremes of a spectrum of possibilities

Knowledge of the Signal

Signal Unknown		Signal Known
Simple Energy Detectors (STA/LTA)	Subspace Detectors	Correlators (Matched Filters)

- **Correlation detectors are very sensitive**
- **Energy detectors are broadly applicable**
- **An intermediate option is desirable**

Subspace detectors add an uncertain signal model to the usual formulation of the detection problem

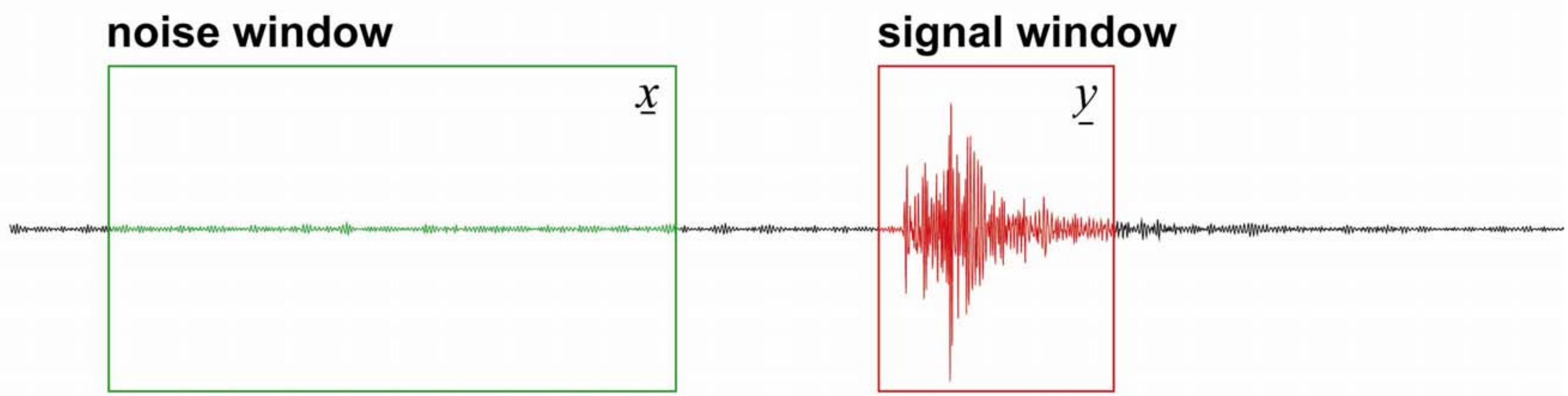


Signal model:

$\underline{s} = \underline{U}\underline{a}$

$\underline{U}^T\underline{U} = \underline{I}$

A single detection framework can span detectors ranging from simple energy detectors to correlators



detection rule

$$\frac{\underline{y}_p^T \underline{y}_p}{\underline{x}^T \underline{x} + \underline{y}^T \underline{y} - \underline{y}_p^T \underline{y}_p} \gtrless \textit{threshold}$$

$$\underline{y}_p = \underline{U}\,\underline{U}^T \underline{y}$$

→

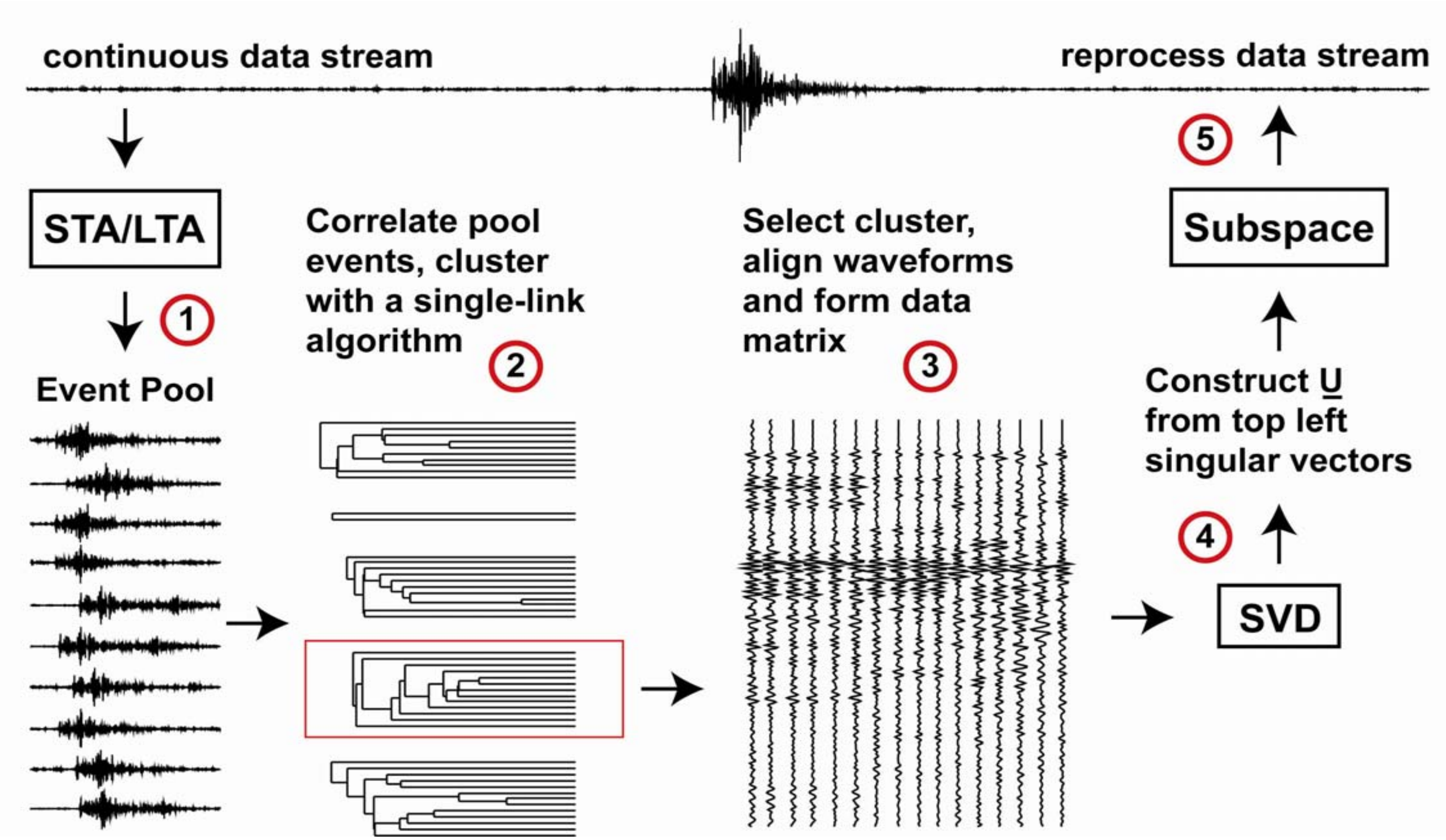
energy detector

$$\underline{U} = \underline{I} \qquad z = \frac{\underline{y}^T \underline{y}}{\underline{x}^T \underline{x}}$$

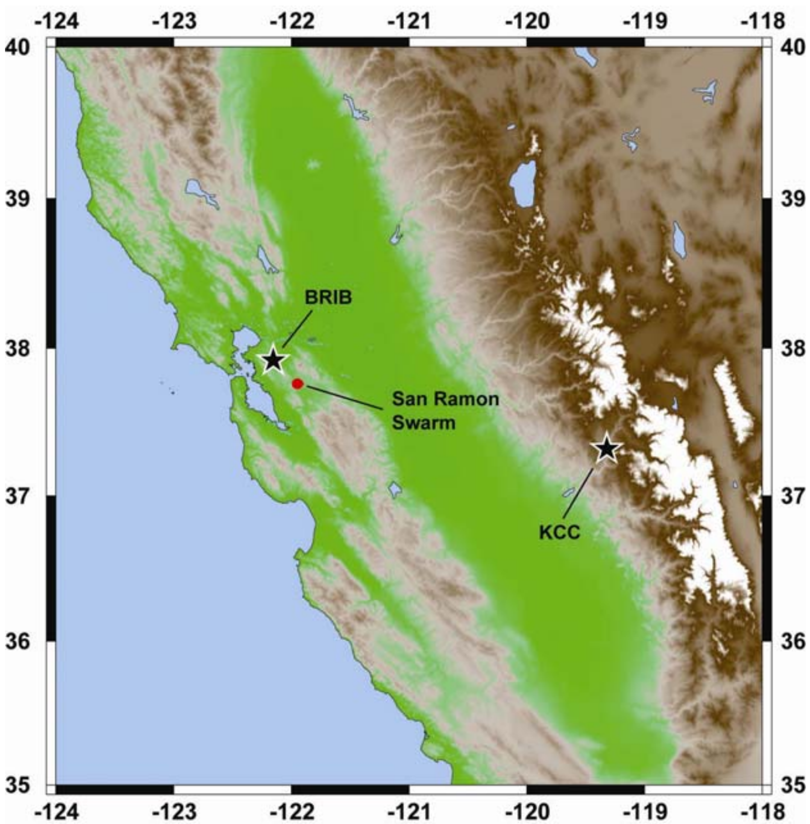
subspace detector

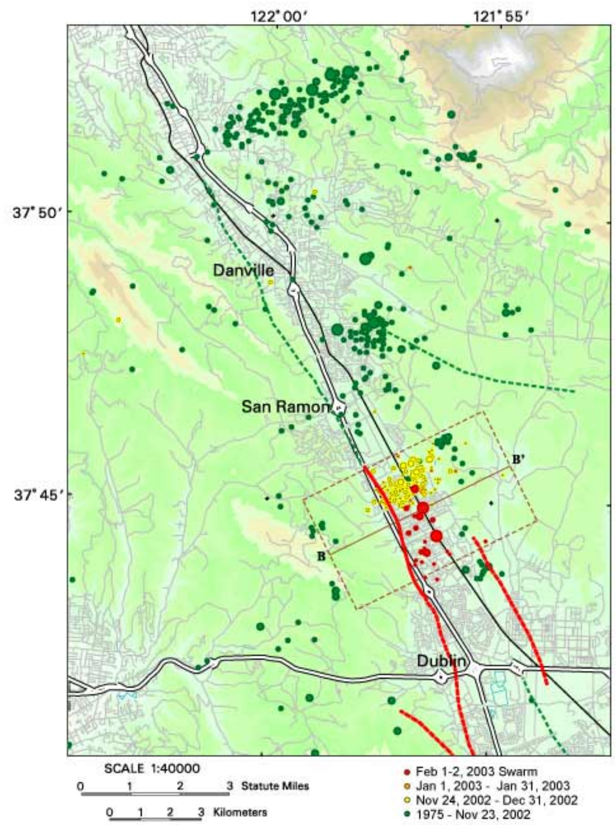
$$\underline{x}^T \underline{x} \rightarrow 0 \qquad z = \frac{\underline{y}_p^T \underline{y}_p}{\underline{y}^T \underline{y}}$$

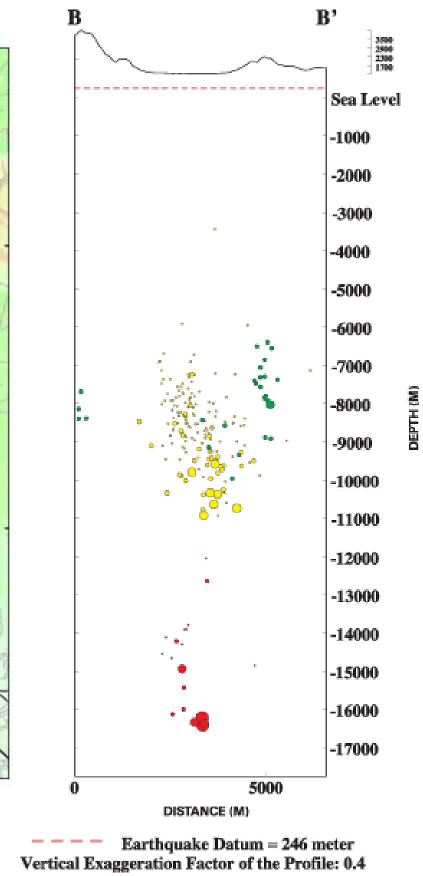
Processing sequence for detecting swarm events



Example: Nov-Dec 2002 San Ramon, California Swarm

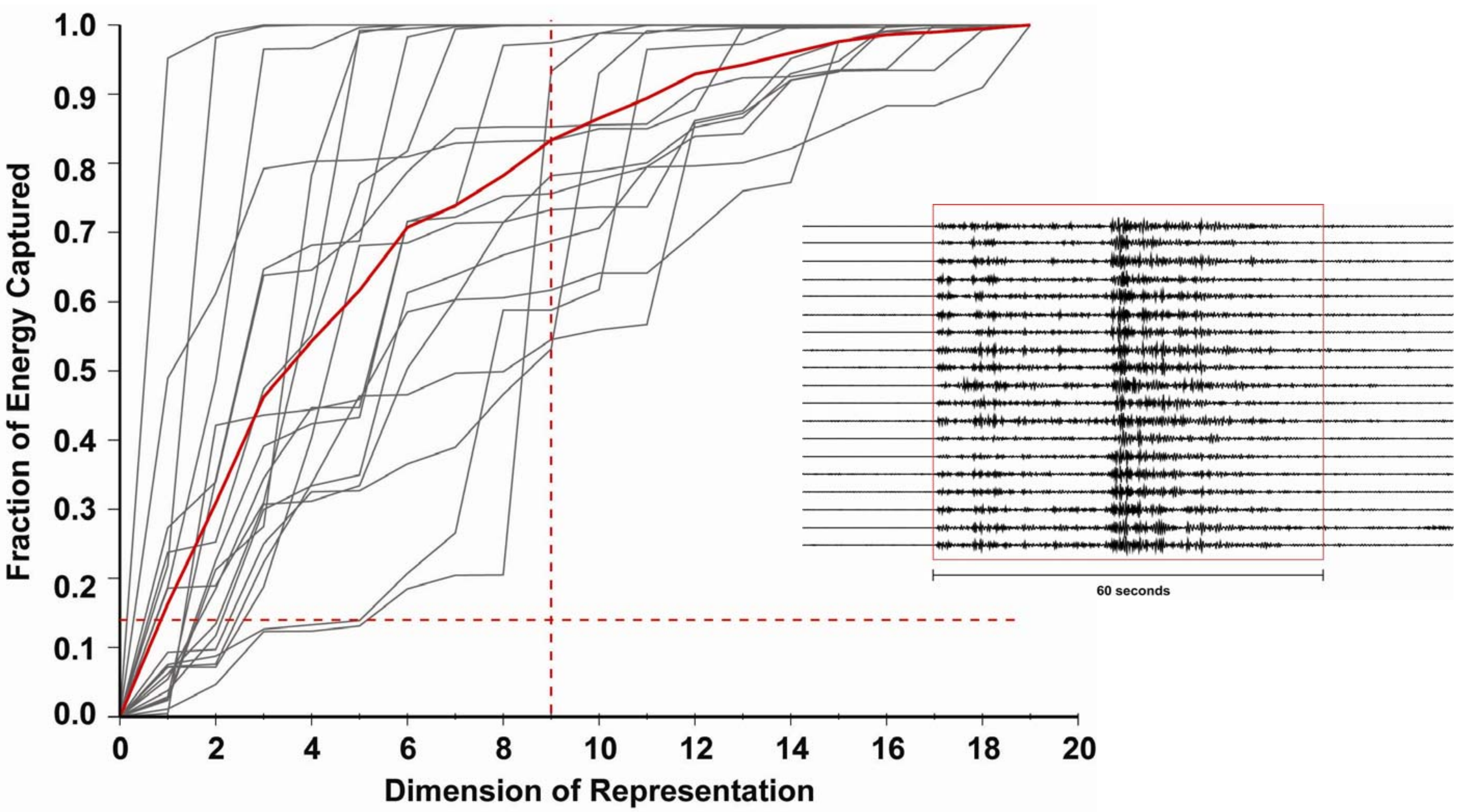






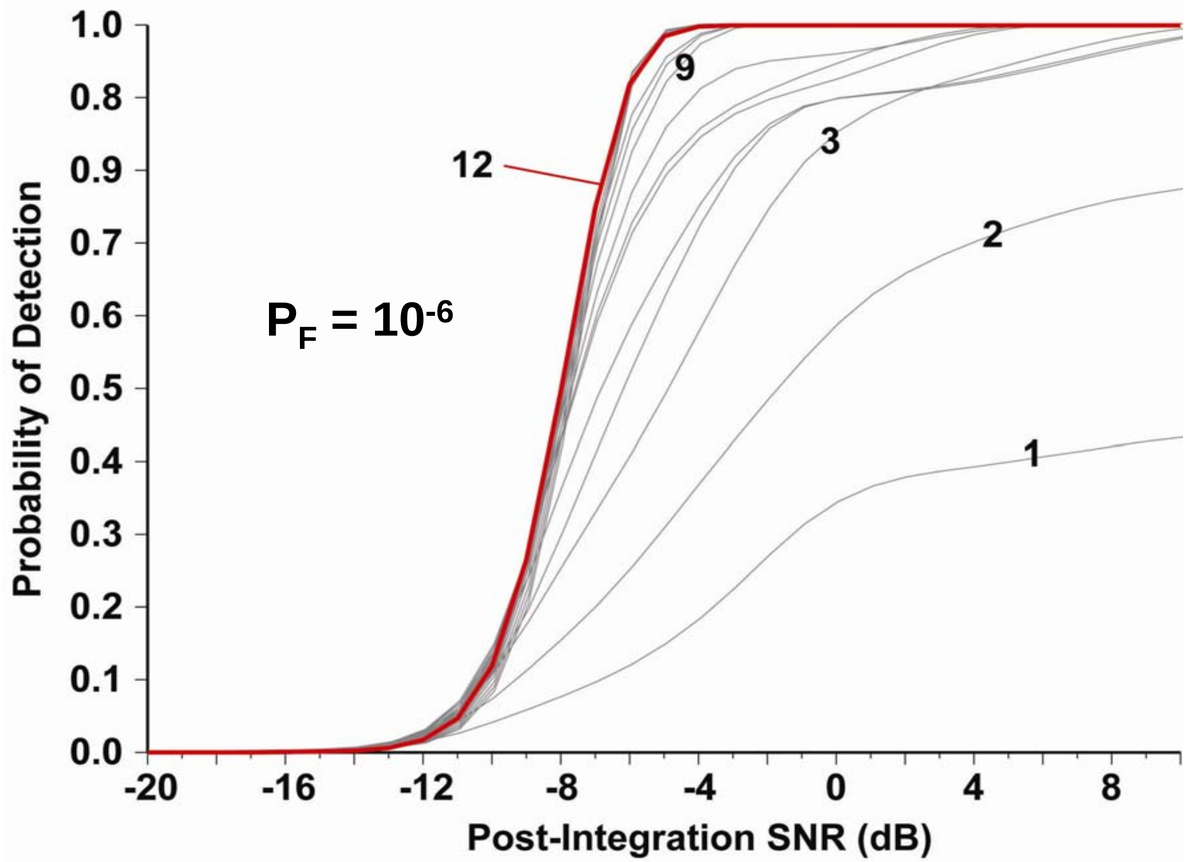
Data credit: NCEDC, Berkeley Seismological Laboratory

Detector dimension and threshold should be set to assure detection of the design events

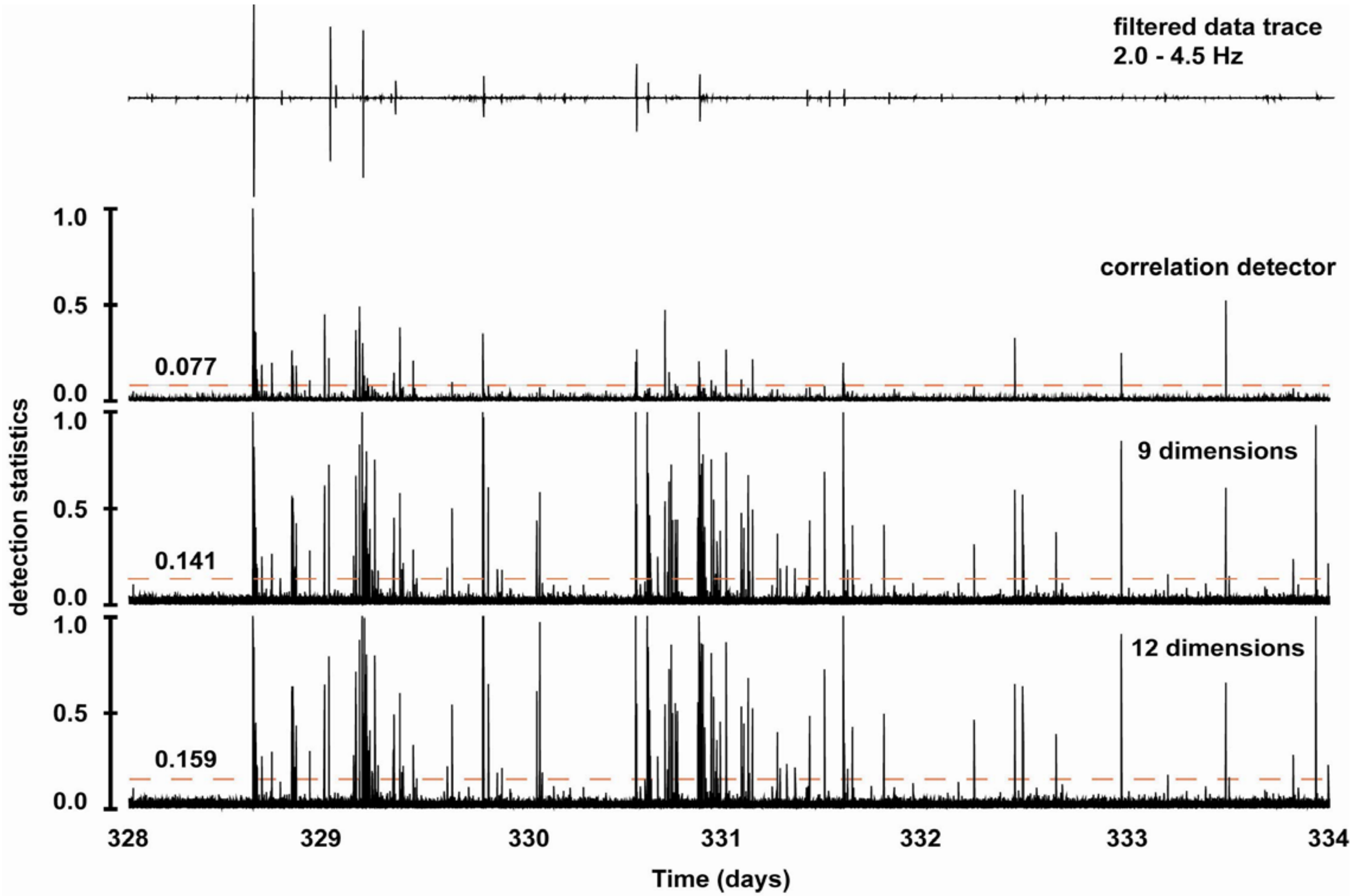


The subspace dimension is chosen to optimize the probability of detection

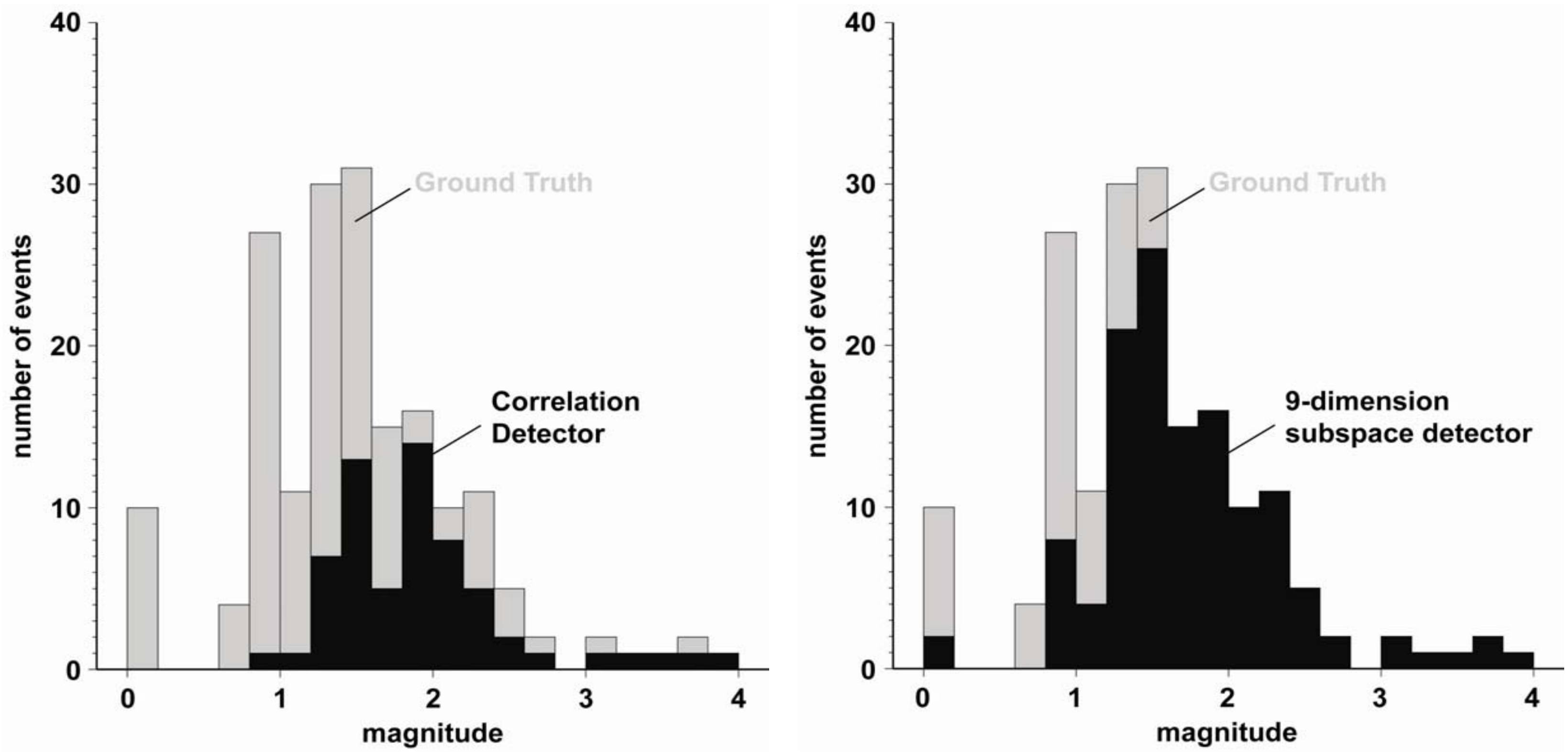
- Maximize P_D at a fixed P_F
- Minimize cost of computation where P_D is essentially constant



The subspace detector has a higher noise floor, but significantly better processing gain

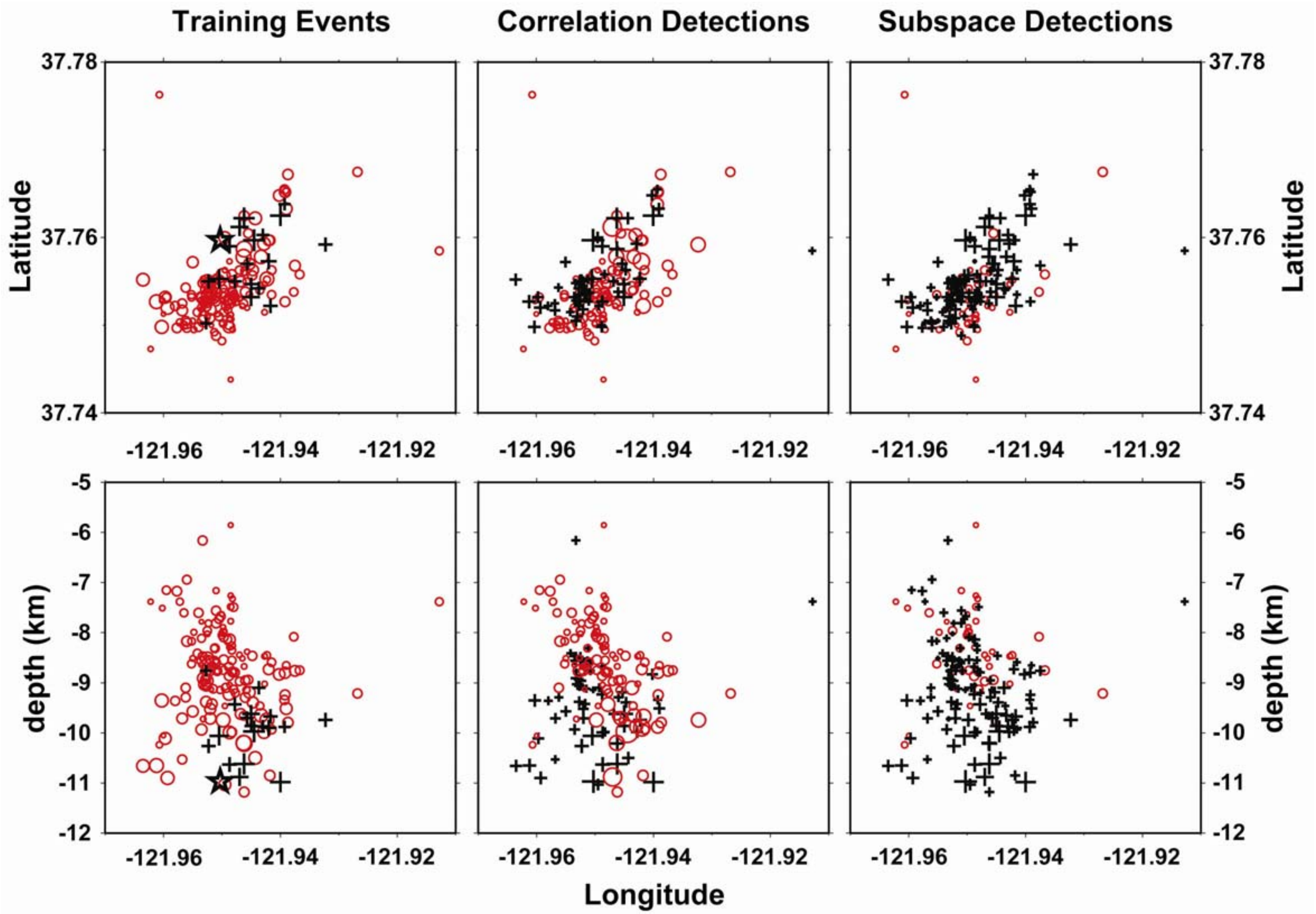


The subspace detector captures twice as many events as the correlator at the same theoretical P_F

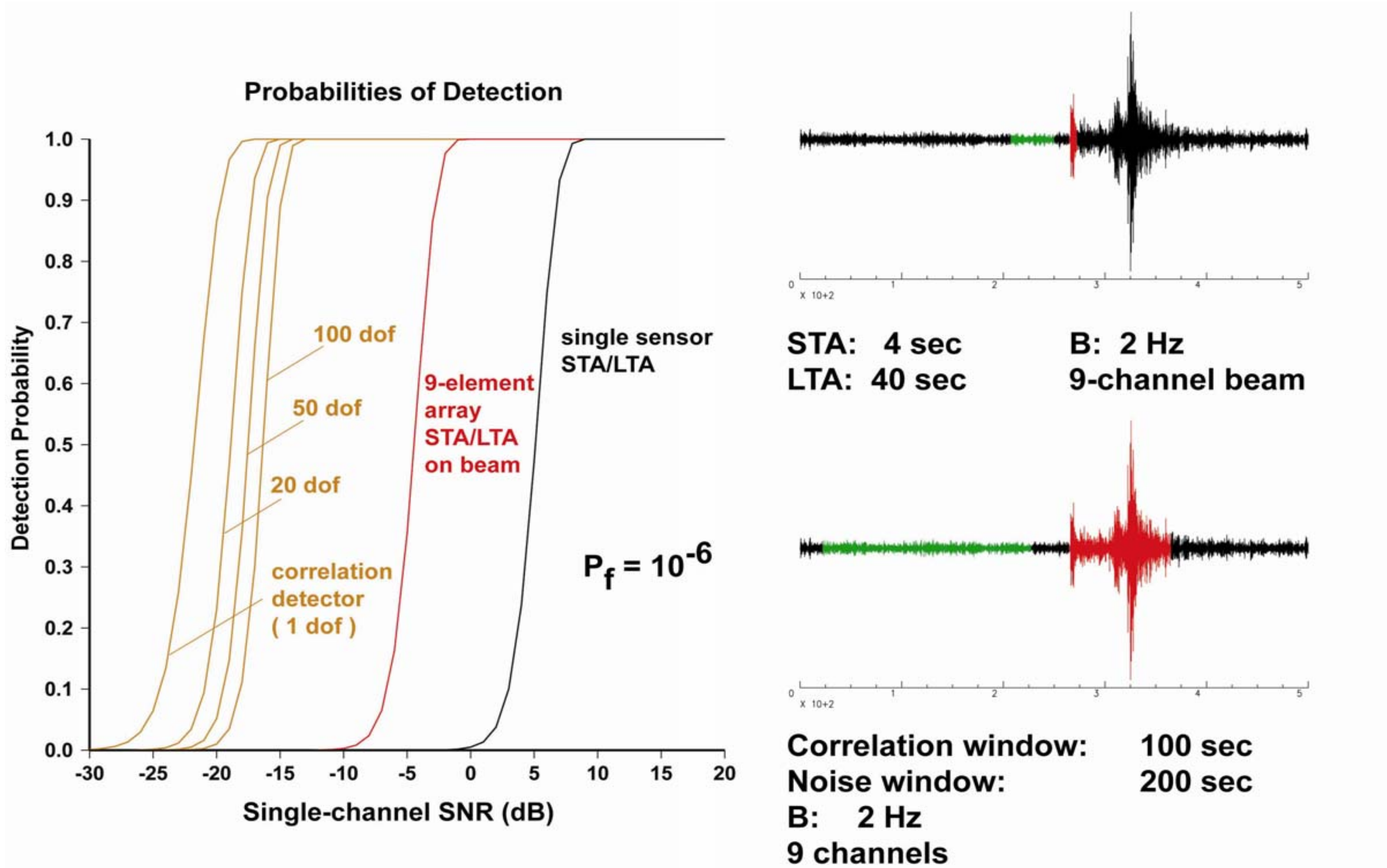


Detection threshold: ~1.5 @ 240 km

The subspace detector has broader coverage in the source region



Under plausibly achievable circumstances, subspace detectors may provide as much gain as arrays



Summary: subspace detectors are a promising approach to detecting uncertain seismic signals

- They wrap event detection, location and characterization into a single operation

- They allow systematic exploitation of information about the range of variation in a signal
  - A rigorous statistical design approach is available
  - Theoretical prospect of detectors “dialable” from simple energy detectors to correlators

- Very sensitive detection capability has been demonstrated on an earthquake swarm