Subspace Detection in Seismology

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Seismic data are full of (nearly) repeating signals that must be detected, screened and characterized.
Current seismic detection practice is concentrated at the extremes of a spectrum of possibilities.

Knowledge of the Signal

- Signal Unknown
  - Simple Energy Detectors (STA/LTA)
- Signal Known
  - Correlators (Matched Filters)

- Subspace Detectors

- Correlation detectors are very sensitive
- Energy detectors are broadly applicable
- An intermediate option is desirable
Subspace detectors add an uncertain signal model to the usual formulation of the detection problem.

Signal model:

\[ s = Ua \]

\[ U^T U = I \]
A single detection framework can span detectors ranging from simple energy detectors to correlators.

**Detection Rule**

\[
\frac{y^T y - y_p^T y_p}{x^T x + y^T y - y_p^T y_p} > \text{threshold}
\]

**Energy Detector**

\[
U = I \quad \Rightarrow \quad z = \frac{y^T y}{x^T x}
\]

**Subspace Detector**

\[
x^T x \rightarrow 0 \quad \Rightarrow \quad z = \frac{y^T y - y_p^T y_p}{y^T y}
\]
Processing sequence for detecting swarm events

1. STA/LTA
2. Correlate pool events, cluster with a single-link algorithm
3. Select cluster, align waveforms and form data matrix
4. Construct \( U \) from top left singular vectors
5. Subspace

- continuous data stream
- reprocess data stream
Example: Nov-Dec 2002 San Ramon, California Swarm

Data credit: NCEDC, Berkeley Seismological Laboratory
Detector dimension and threshold should be set to assure detection of the design events
The subspace dimension is chosen to optimize the probability of detection

- Maximize $P_D$ at a fixed $P_F$
- Minimize cost of computation where $P_D$ is essentially constant

$P_F = 10^{-6}$
The subspace detector has a higher noise floor, but significantly better processing gain.
The subspace detector captures twice as many events as the correlator at the same theoretical $P_F$.

Detection threshold: $\sim 1.5$ @ 240 km
The subspace detector has broader coverage in the source region.
Under plausibly achievable circumstances, subspace detectors may provide as much gain as arrays.

\[ P_f = 10^{-6} \]

- STA: 4 sec
- LTA: 40 sec
- B: 2 Hz
- 9-channel beam

Correlation window: 100 sec
Noise window: 200 sec
B: 2 Hz
9 channels
Summary: subspace detectors are a promising approach to detecting uncertain seismic signals

- They wrap event detection, location and characterization into a single operation

- They allow systematic exploitation of information about the range of variation in a signal
  - A rigorous statistical design approach is available
  - Theoretical prospect of detectors “dialable” from simple energy detectors to correlators

- Very sensitive detection capability has been demonstrated on an earthquake swarm