



Nonlinear Structured Illumination Microscopy

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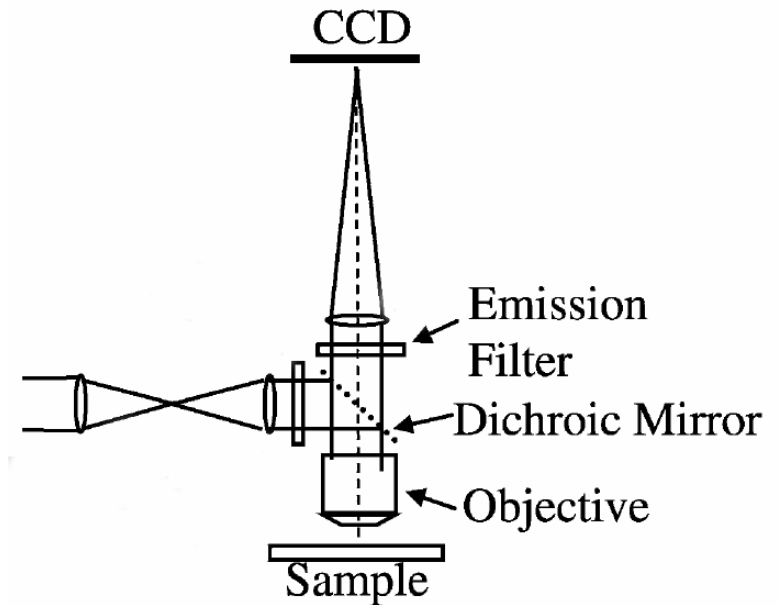
UCRL-PRES-208010

Fluorescence Microscopy

Image observed under the microscope be described by:

$$E(r) = H(r) \otimes \{I(r) \bullet D(r)\}$$

Image on CCD Point Spread Function Excitation Intensity Fluorophore Density



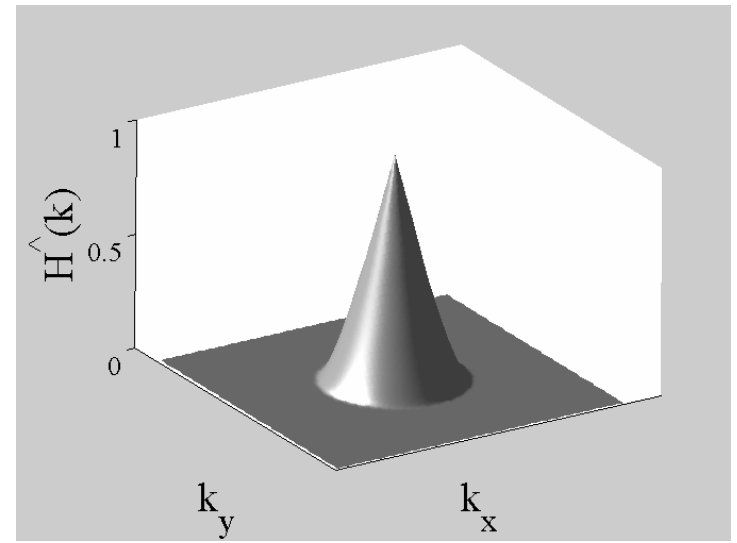
In reciprocal space the observed image is:

$$\hat{E}(k) = \hat{H}(k) \bullet \{\hat{I}(k) \otimes \hat{D}(k)\}$$

($\hat{E}(k)$ denotes F.T. of $E(r)$)

Resolution limit is given by the support of

$$\hat{H}(k) \text{ inside } k_{obs} = \frac{2NA}{\lambda}$$



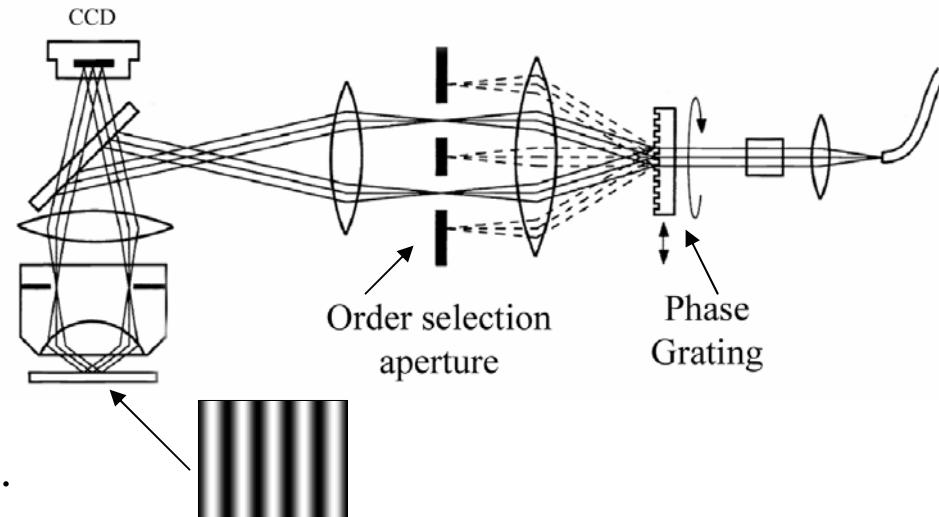
How can we overcome the resolution limit?

We can use spatially varying (structured) illumination $I(r)$ *!

Use sinusoidal pattern in the lateral direction:

$$I(r) = 1 + \cos(k_0 r + \varphi)$$

where k_0 is wave vector of illumination pattern.



* M.G.L Gustafsson, D.A. Agard, J.W. Sedat Proceedings SPIE v.3919(2000)

In Fourier Space:
$$\hat{I}(k) = \delta(k) + \frac{1}{2} \left[\delta(k + k_0) e^{i\phi} + \delta(k - k_0) e^{-i\phi} \right]$$

Using
$$\hat{E}(k) = \hat{H}(k) \bullet \left\{ \hat{I}(k) \otimes \hat{D}(k) \right\}$$

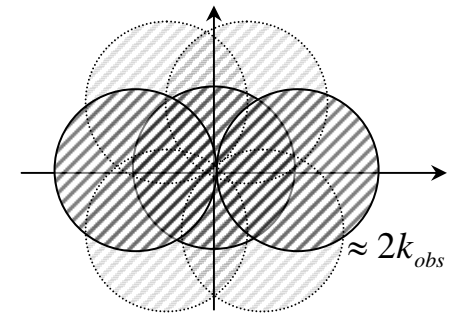
We now observe:

$$\hat{E}(k) = \hat{H}(k) \bullet \left\{ \hat{D}(k) + \frac{1}{2} \hat{D}(k - k_0) e^{-i\phi} + \frac{1}{2} \hat{D}(k + k_0) e^{i\phi} \right\}$$

Image reconstruction:

1. Separate $\hat{D}(k)$, $\hat{D}(k - k_0)$, $\hat{D}(k + k_0)$ from three images with different ϕ by solving a linear equation.
2. Put \hat{D} in correct place in the k-space.
3. Inverse Fourier transform to obtain $D(r)$.

We can almost double the resolution with structured illumination.



Effective observable region in reciprocal space

Can we obtain even higher resolution?

Suppose fluorescent emission depends non-linearly on the excitation:

$$E(r) = H(r) \otimes \{F[I(r)] \cdot D(r)\}$$

Expand $F[I(r)] = \sum_n a_n (I(r))^n$

If $I(r)$ is sinusoidal then in reciprocal space: $\hat{F}[I(k)] = \sum_n c_n \delta(k + n \cdot k_0) e^{in\varphi}$

Then we obtain

$$\hat{E}(k) = \hat{H}(k) \{ \hat{F}[I(k)] \otimes \hat{D}(k) \} = \hat{H}(k) \left\{ \sum_n c_n \hat{D}(k + n \cdot k_0) e^{in\varphi} \right\}$$

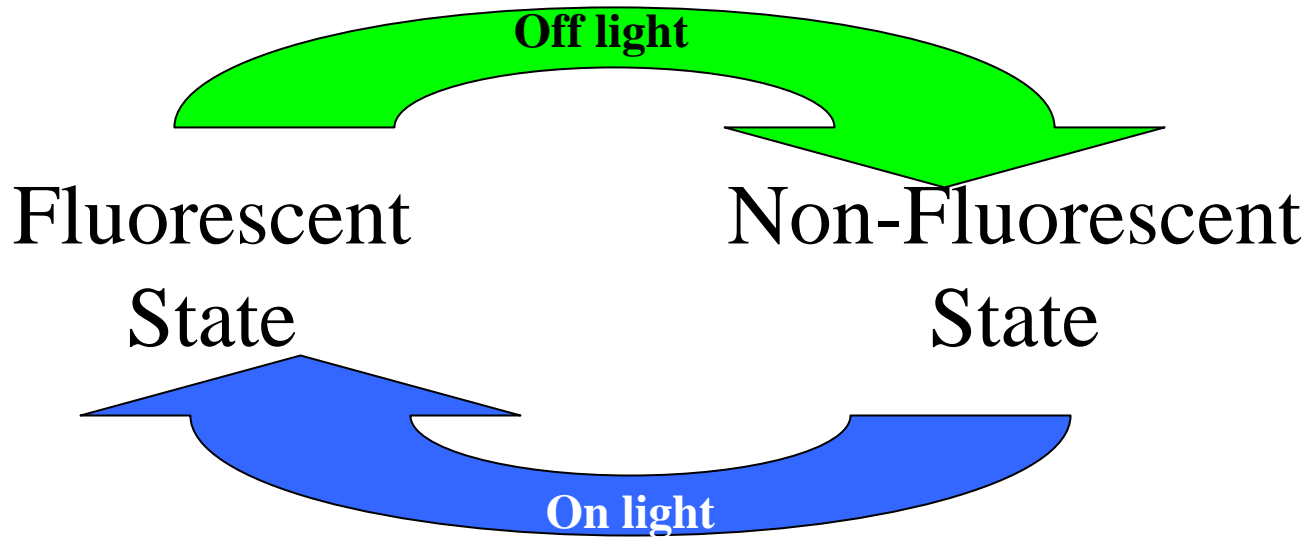
No hard limit on the obtainable resolution.

In reality, $c_n \rightarrow 0$ for large n . In the presence of noise this limits the resolution.

Possible non-linear processes:

- Two-photon fluorescence
- STED
- Saturation
- Switchable fluorescent molecules

Switchable protein markers



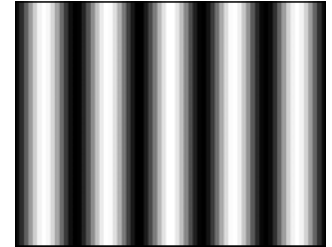
Residual fluorescence: $e^{-bTI_{off}(r)} D(r)$

where b = extinction coefficient, T = exposure time.

Possible illumination patterns

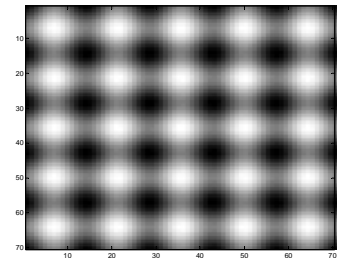
1D pattern $I(x) = 1 + \cos(kx + \varphi)$

Has to be rotated and shifted to obtain full resolution.



2D pattern $I(x, y) = 2 + \cos(kx + \varphi_x) + \cos(ky + \varphi_y)$

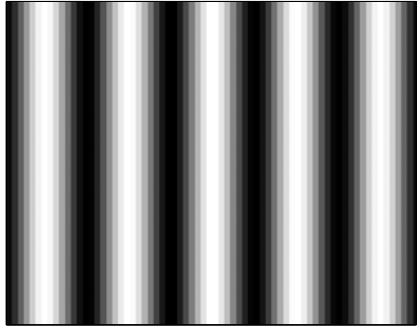
Only need to translate in x and y directions for full resolution.



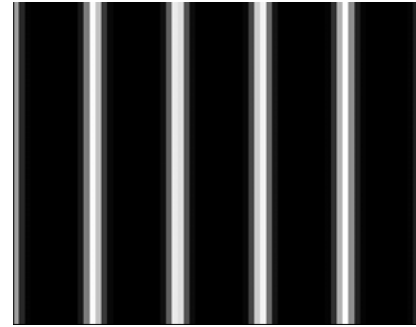
Which pattern is better?

1D Pattern

Off light pattern $I_{off}(r)$

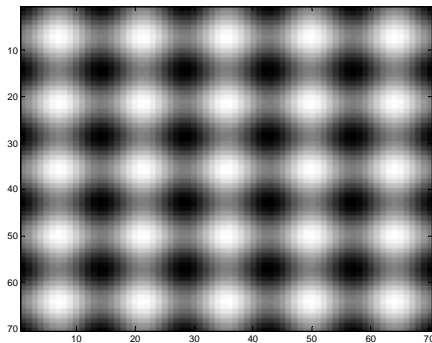


Residual fluorescence $e^{-bTI_{off}(r)}$



2D pattern

Off light pattern $I_{off}(r)$



Residual fluorescence $e^{-bTI_{off}(r)}$

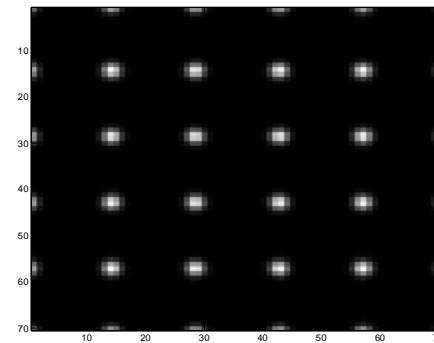


Image formation

We must consider Sources of “noise”:

- Detector noise
- Fluorophore degradation – permanently on fraction

We model image formation by:

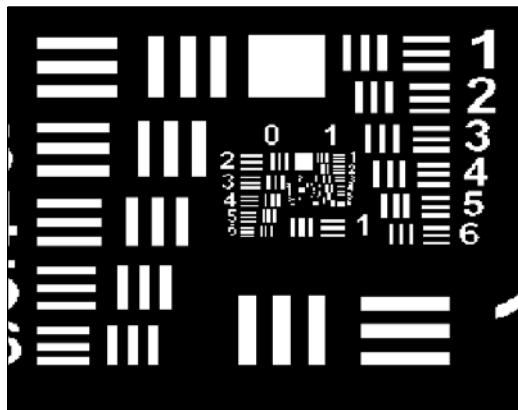
$$E(r) = (1 - \alpha)H(r) \otimes \{e^{-bTI_{off}(r)} \cdot D(r)\} + \alpha H(r) \otimes D(r) + \eta(r)$$

α - permanently on fraction of fluorophore

η - Poisson noise

Simulation of the Non-linear Structured Illumination

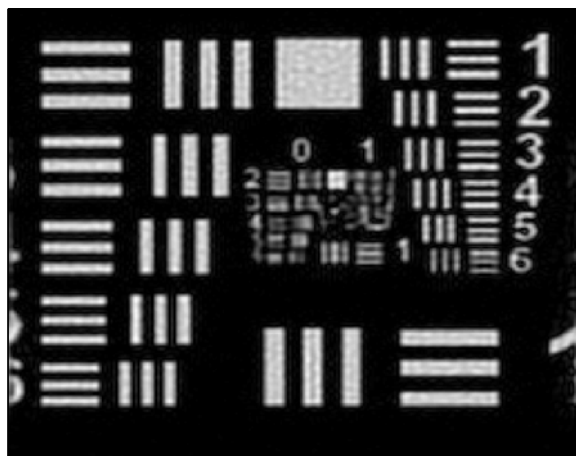
Original object



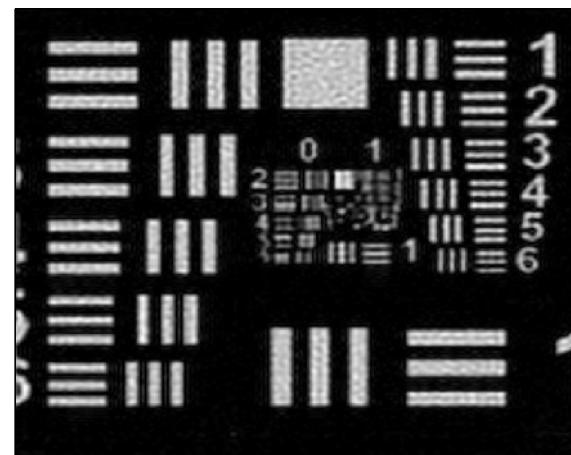
No resolution enhancement



With non-linear structured illumination



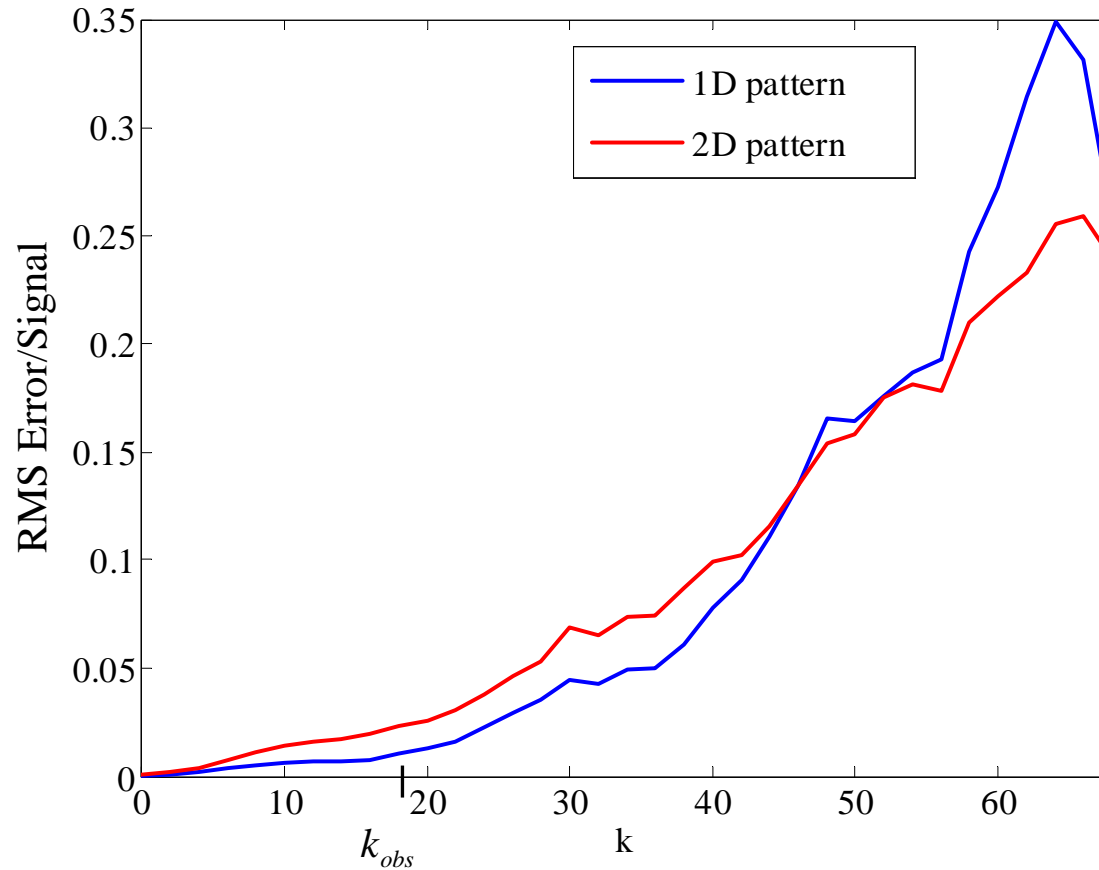
1D pattern



2D pattern

Reconstructed from 49 images

What is the resolution limit?



Error in reciprocal space as a function of wavenumber for 1D and 2D illumination. Assumed 4% fluorophore degradation and ~ 6000 expected photons in the brightest pixel.

Conclusions

- Noise limits the resolution in non-linear SI
- Resolution limits are similar with 1D and 2D patterns
- Future work will consider resolution dependence on the experimental parameters

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