Optimal modal Fourier transform wave-front Control

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High-precision wave-front control

Adapt to changing conditions

High spatial resolution

High frame-rates for control system
FTR works by filtering the slopes

WFS slopes

Fix boundary problem

FFT → Filter → FFT⁻¹

Phase estimate

Lisa A. Poyneer's presentation on Optimal Fourier Control
Modal control uses a basis set

WFS slopes

Slopes-to-Modes matrix

Modes-to-Actuator matrix

Phase estimate

Modal gains
FTR modes are sines and cosines

FTR uses the DFT in the filtering process

\[
X[k, l] = \frac{1}{N} \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} x[m, n] \exp \left( -j2\pi \frac{(km + ln)}{N} \right)
\]

Modal coefficients are obtainable directly from the DFT values

\[
<x[m, n], C_{k,l}[m, n]> = \frac{1}{D_{k,l}} \text{Re} \{X[k, l]\}
\]

\[
<x[m, n], S_{k,l}[m, n]> = -\frac{1}{D_{k,l}} \text{Im} \{X[k, l]\}
\]
Modes are eigenfunctions

§ Fourier modes are eigenfunctions of linear, shift-invariant (LSI) systems

§ The modes for the slopes (on a square aperture) are the same as the modes for the phase

§ A cosine of phase at frequency \([k,l]\) produces x- and y-slopes only at the cosine and sine of that frequency \([k,l]\)

§ Where \(M_x[k,l]\) describes the filter which measures the x-slopes from actuator commands

\[
\begin{align*}
\text{phase} & \quad \text{x-slope} \\
C_{k,l}[m,n] & \quad AC_{k,l}[m,n] + BS_{k,l}[m,n] \\
S_{k,l}[m,n] & \quad -BC_{k,l}[m,n] + AS_{k,l}[m,n]
\end{align*}
\]

\[A = \text{Re}\{M_x[k,l]\}, \quad B = -\text{Im}\{M_x[k,l]\}\]
§ We follow Altair’s implementation and assume an approximate model of control system (exact in simulation case) for each of the independent modes.

§ We control a mode with feedback in the presence of noise.

Block diagram of control loop for a modal coefficient
Optimize the squared-residual error

§ Since the noise at any step is independent of past errors, if we minimize on the measurement $s$, we minimize on the residual error.

§ If we had perfect knowledge we would minimize

$$J = \int \left| \frac{1}{1 + \exp(-j\omega)H(\omega)} \right|^2 [M(\omega) + N(\omega)] \, d\omega$$

§ But we don’t... so we have to estimate the open-loop PSD from the closed-loop measurements using

$$\hat{M}(\omega) + \hat{N}(\omega) = |1 + \exp(-j\omega)H_0(\omega)|^2 \hat{S}(\omega)$$
Gains are incorporated into filter

\[ \text{WFS slopes} \rightarrow \text{Fix boundary problem} \rightarrow \text{FFT} \rightarrow \text{Filter} \rightarrow \text{Gain optimizer} \rightarrow \text{FFT}^{-1} \]

Phase estimate
Modes correspond to PSF locations

§ Each Fourier mode lives at a specific spatial frequency pair \([k,l]\)

§ Because the PSF is approximately the PSD of the residual phase (to second order), each Fourier mode appears at a specific location in the PSF
Significant reduction in residual error

Use of optimal gains improves performance

§ significant reduction in residual MSE at each timestep

§ less variation in MSE at each timestep

N=48, NGS Mag 8 example for 8 iterations of gain optimization
Contrast improved in PSF

- N=48 case with WFS SNR of 2.16
- Strehl increased from 0.75 to 0.87 (+12%)
- MSE in band reduced from 0.224 to 0.074 (3 times less)

Before

After
Optimal filter reflects conditions

§ Input phase aberration is a frozen sheet of phase moving across the aperture

§ Deformable Mirror (DM) has unknown low-pass response which attenuates high spatial frequencies

§ Optimal gains compensate for both

Example filter, N=64
Trade bandwidth and sensor errors

At high SNRs, optimal gains produce equivalent or more measurement error but less temporal error than before.

At low SNRs, optimal gains produce less measurement error but more temporal error than before.

Data for N=48, median over a set of 25 random phase screens.
Computational load is satisfiable today

- FTR each timestep: $15N^2 \log N + 20N^2$
- Estimating periodograms for $t$ steps of telemetry:
  $$N^2(5 + 2.5 \log t)$$
- Averaging the periodograms and finding the optimal gain ($k$ is for evaluations in root-finding):
  $$N^2(1 + k) + 4k$$

Assuming $k = 10$ (using fast method), a 64x64 system at 2.5k kHz has a maximum load of 1.43 GFLOPs/sec.
OFC is both fast and smart

§ Optimal Fourier Control combines the best of state-of-the art approaches:
  § computationally efficient
  § adaptive control optimizes performance given current observing conditions

§ Further research areas
  § sensitivity analysis to determine performance across a wide range of conditions
  § exploration of more complex control laws or predictive control