Optimal modal Fourier transform wave-front Control

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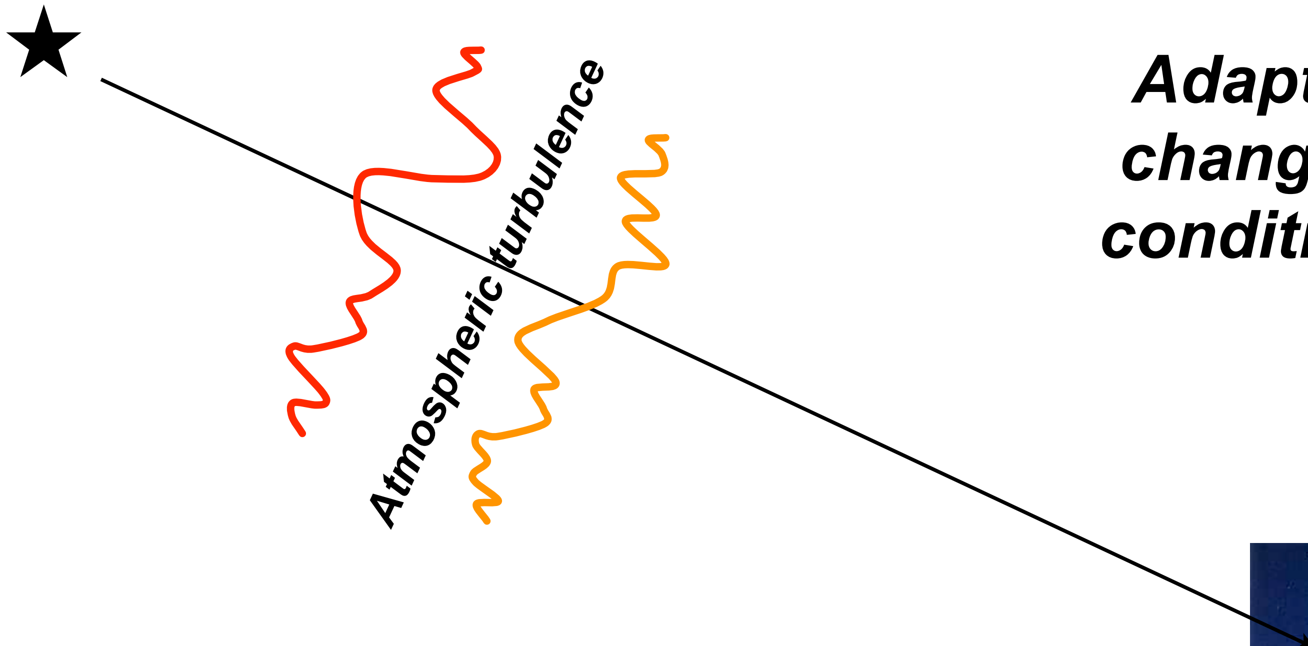
High-precision wave-front control

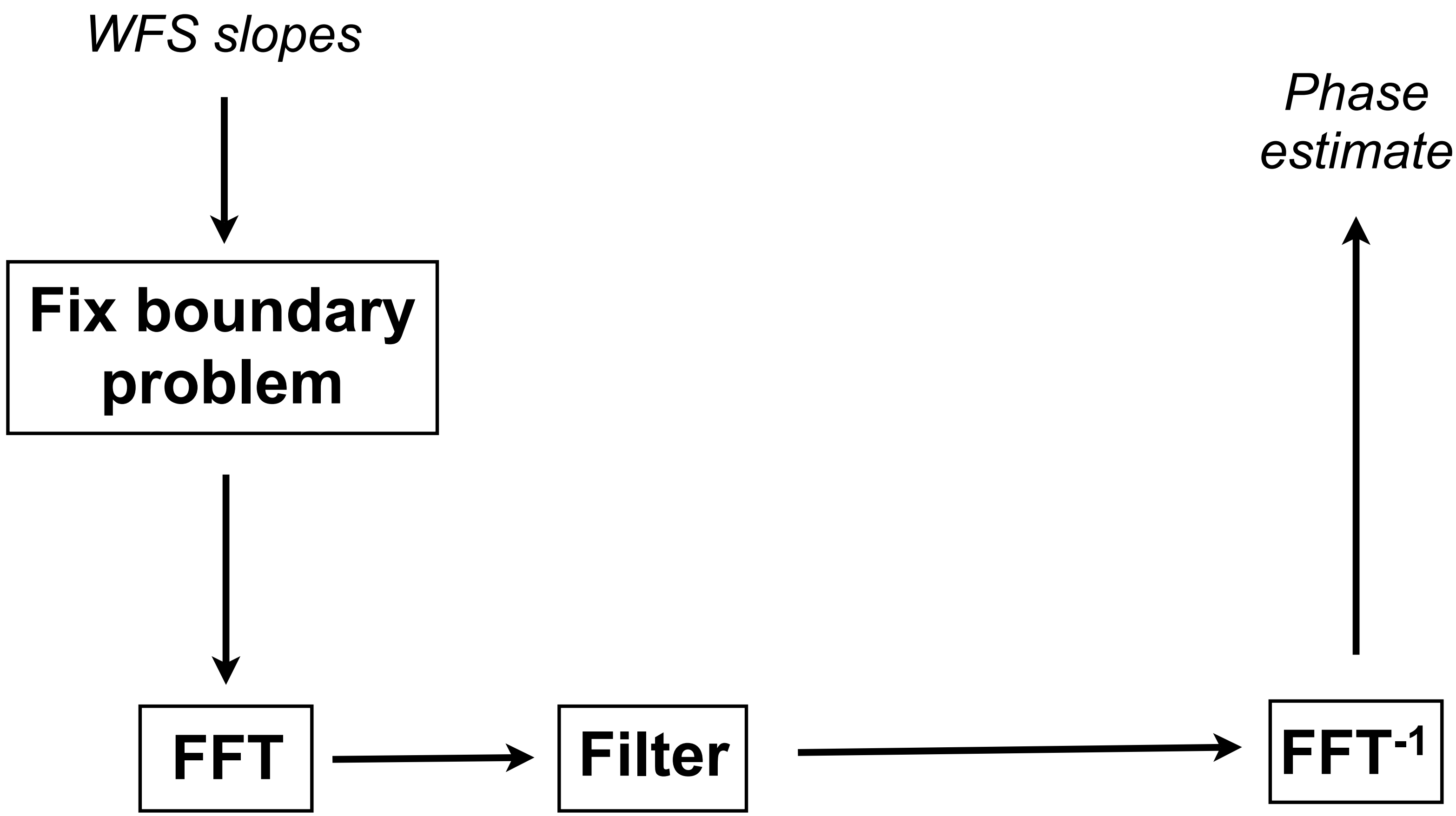


Adapt to changing conditions

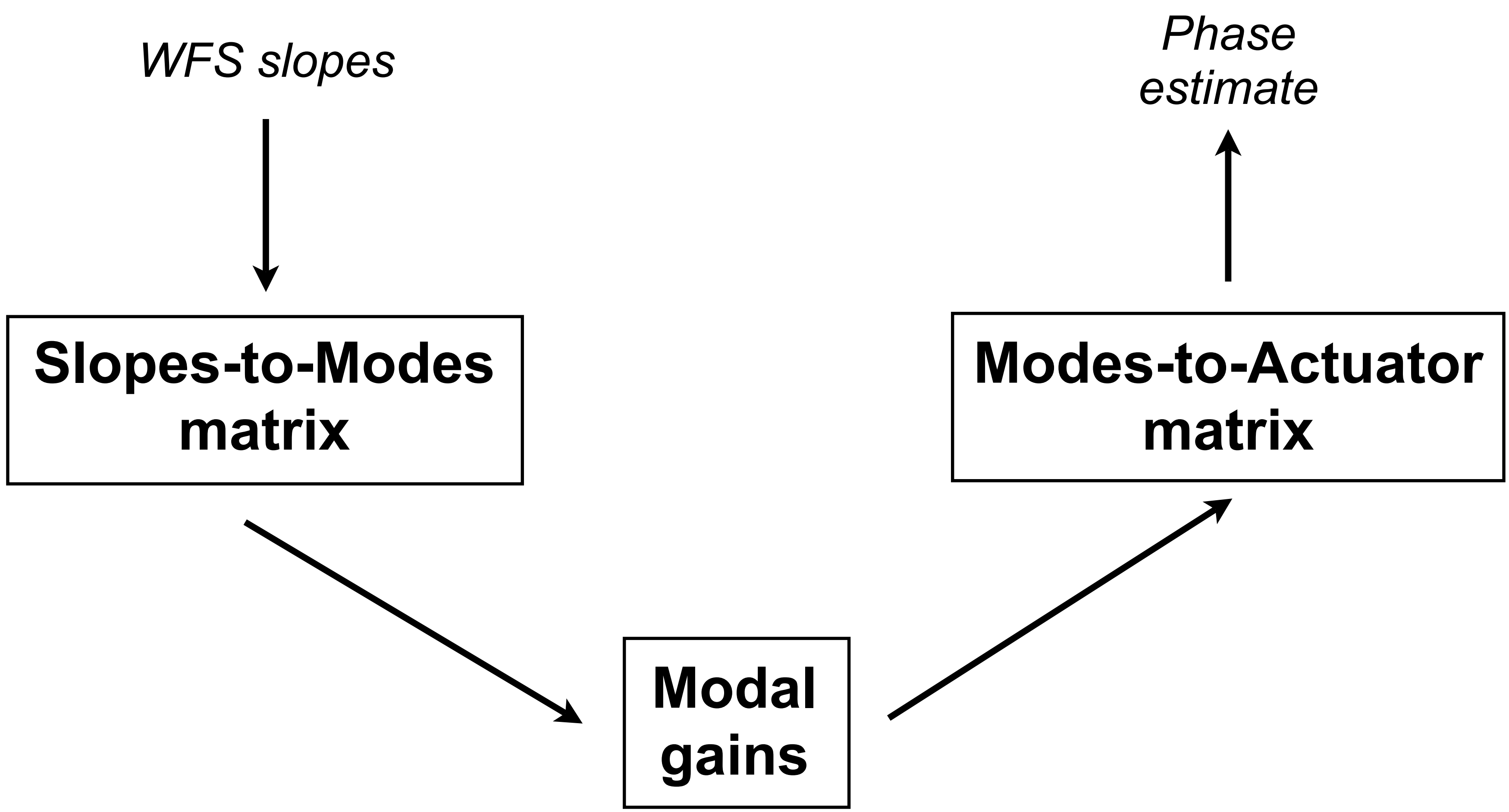
High spatial resolution

High frame-rates for control system

FTR works by filtering the slopes



Modal control uses a basis set



FTR modes are sines and cosines

§ FTR uses the DFT in the filtering process

$$X[k,l] = \frac{1}{N} \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} x[m,n] \exp\left(\frac{-j2\pi(km+ln)}{N} \right)$$

§ Modal coefficients are obtainable directly from the DFT values

$$< x[m,n], \mathcal{C}_{k,l}[m,n] >= \frac{1}{D_{k,l}} \mathrm{Re}\left\{X[k,l]\right\}$$

$$< x[m,n], \mathcal{S}_{k,l}[m,n] >= \frac{-1}{D_{k,l}} \mathrm{Im}\left\{X[k,l]\right\}$$

Modes are eigenfunctions

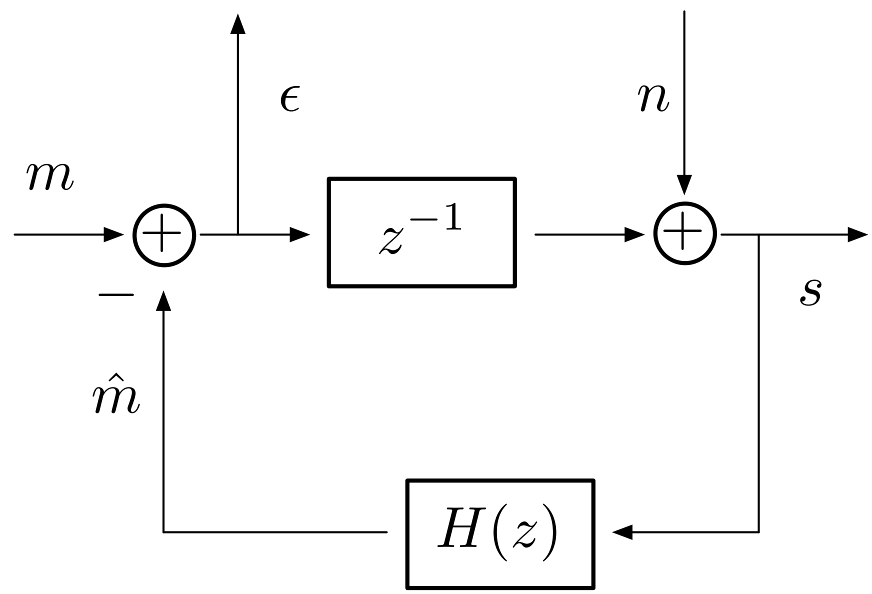
- Fourier modes are eigenfunctions of linear, shift-invariant (LSI) systems
  - *The modes for the slopes (on a square aperture) are the same as the modes for the phase*
  - *A cosine of phase at frequency [k,l] produces x- and y-slopes only at the cosine and sine of that frequency [k,l]*
  - *Where Mx[k,l] describes the filter which measures the x-slopes from actuator commands*

phase	**x-slope**
$\mathcal{C}_{k,l}[m,n]$	$A\mathcal{C}_{k,l}[m,n] + B\mathcal{S}_{k,l}[m,n]$
$S_{k,l}[m,n]$	$-B\mathcal{C}_{k,l}[m,n] + A\mathcal{S}_{k,l}[m,n]$

$$A = \mathrm{Re}\{M_x[k,l]\},\ B = -\mathrm{Im}\{M_x[k,l]\}$$

Optimal modal control scheme

- We follow Altair's implementation and assume an approximate model of control system (exact in simulation case) for each of the independent modes.
- We control a mode with feedback in the presence of noise.



Block diagram of control loop for a modal coefficient

Optimize the squared-residual error

§ Since the noise at any step is independent of past errors, if we minimize on the measurement s, we minimize on the residual error.

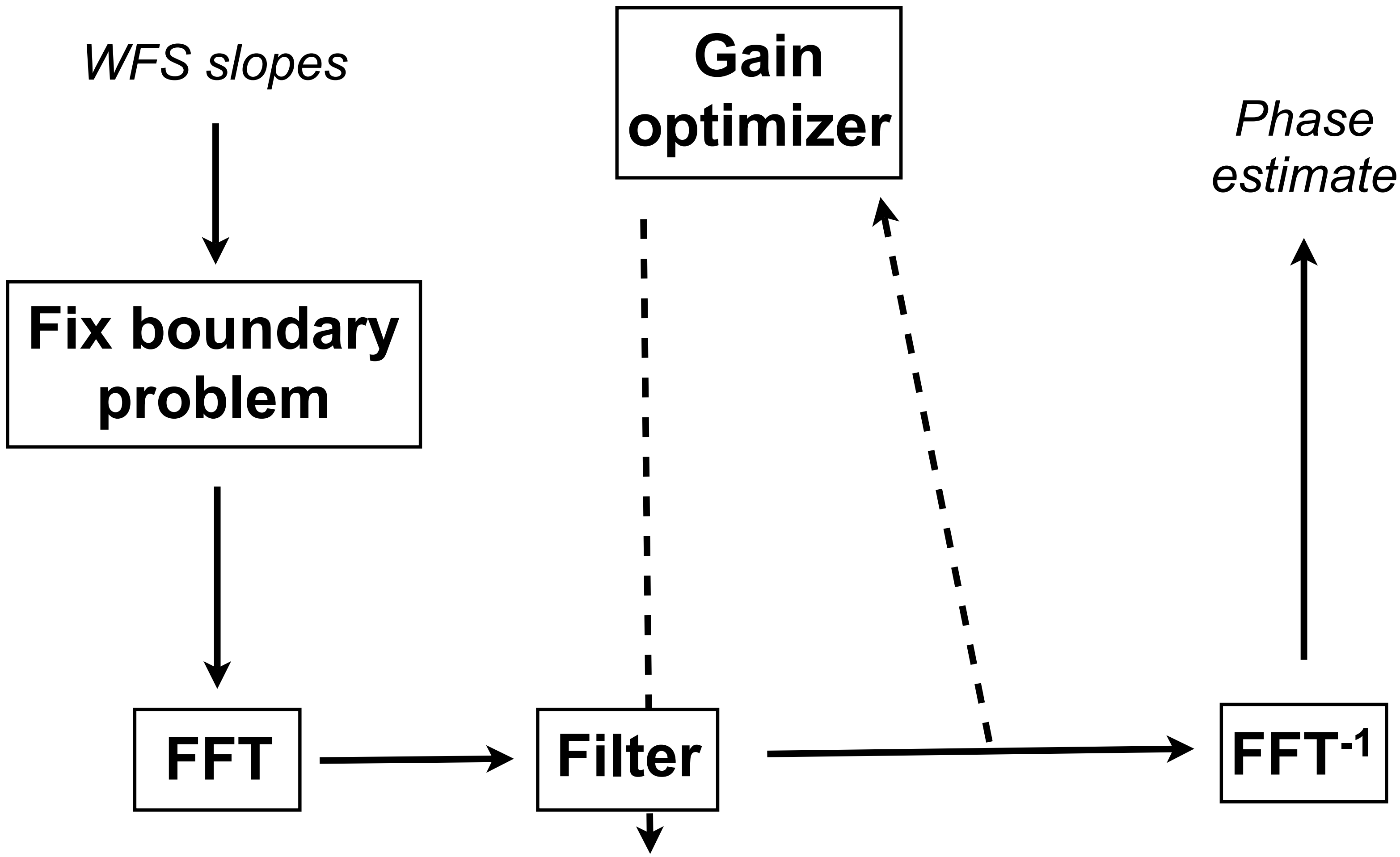
§ If we had perfect knowledge we would minimize

$$\mathcal{J} = \int \left| \frac{1}{1 + \exp(-j\omega) H(\omega)} \right|^2 [M(\omega) + N(\omega)] \; d\omega$$

§ But we don't... so we have to estimate the open-loop PSD from the closed-loop measurements using

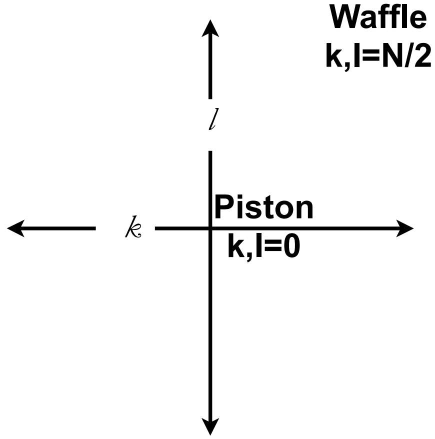
$$\hat{M}(\omega) + \hat{N}(\omega) = |1 + \exp(-j\omega) H_0(\omega)|^2 \, \hat{S}(\omega)$$

Gains are incorporated into filter



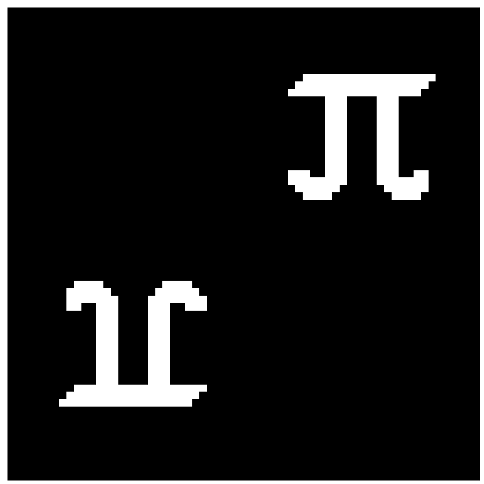
Modes correspond to PSF locations

- Each Fourier mode lives at a specific spatial frequency pair [k,l]
- Because the PSF is approximately the PSD of the residual phase (to second order), each Fourier mode appears at a specific location in the PSF

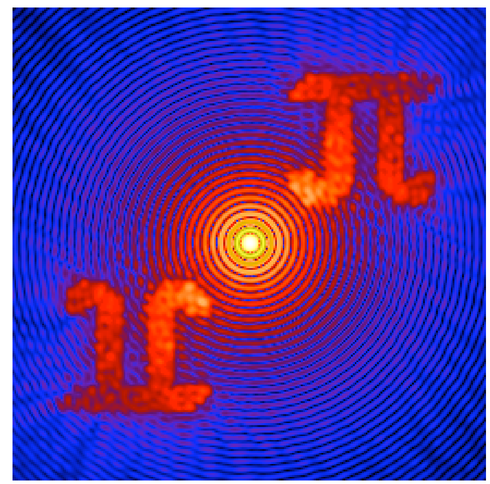


Frequency domain

Black = 0, White = 1

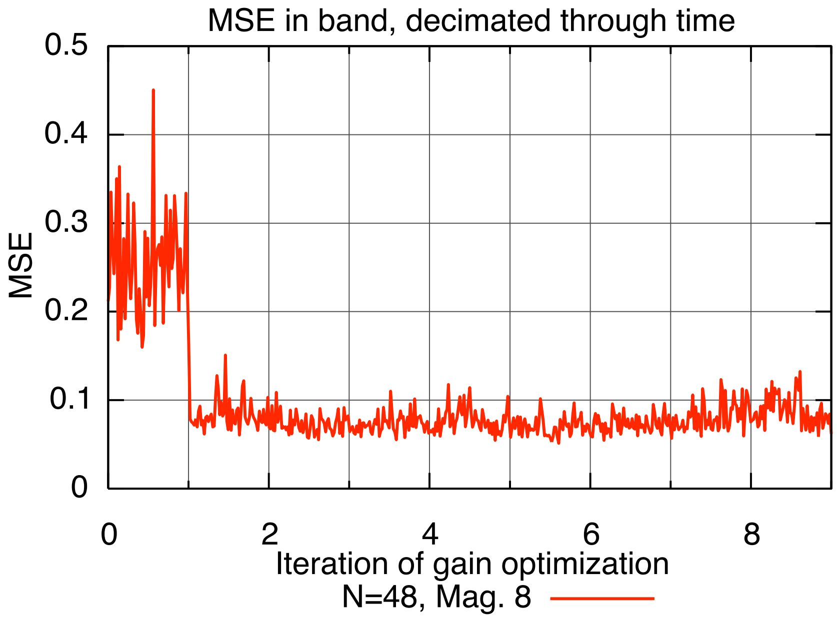


PSD/power at each mode



PSF

Significant reduction in residual error

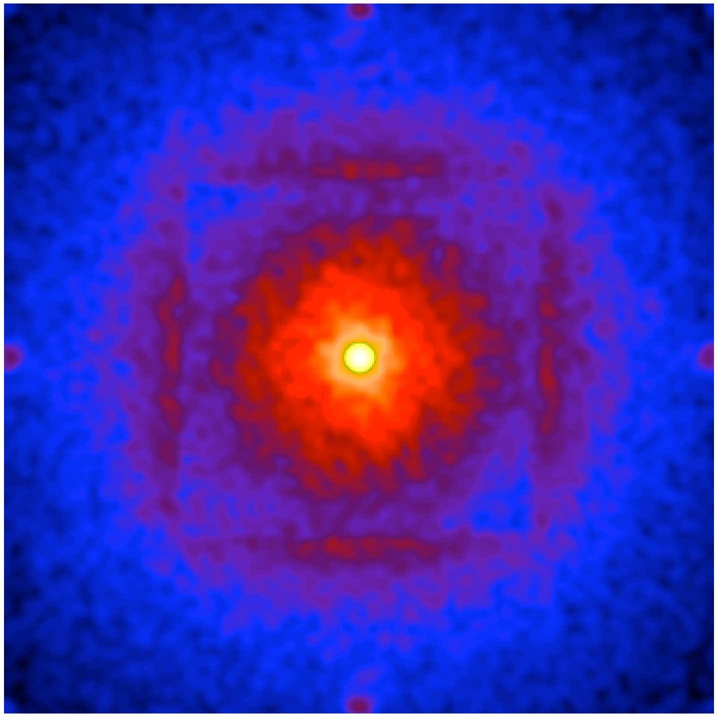


N=48, NGS Mag 8 example for 8 iterations of gain optimization

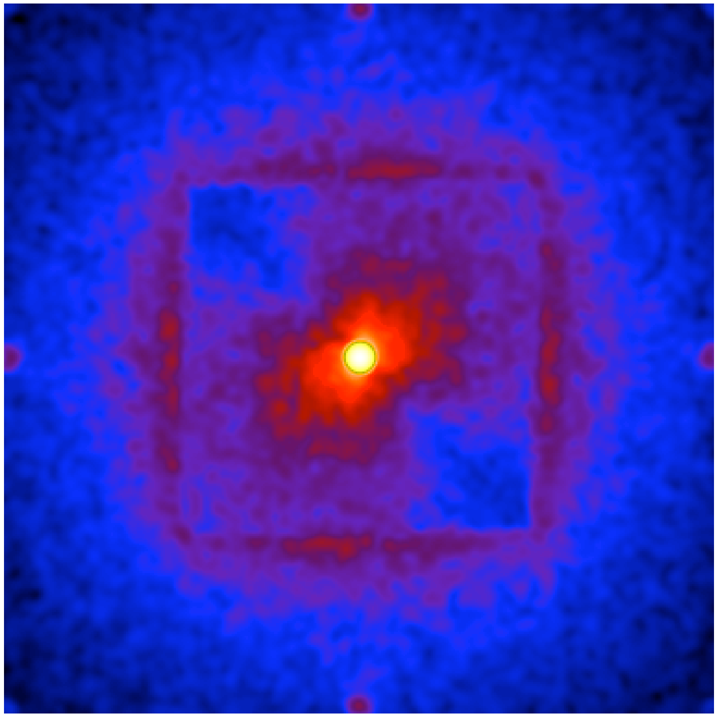
- Use of optimal gains improves performance
  - *significant reduction in residual MSE at each timestep*
  - *less variation in MSE at each timestep*

Contrast improved in PSF

- N=48 case with WFS SNR of 2.16
- Strehl increased from 0.75 to 0.87 (+12%)
- MSE in band reduced from 0.224 to 0.074 (3 times less)



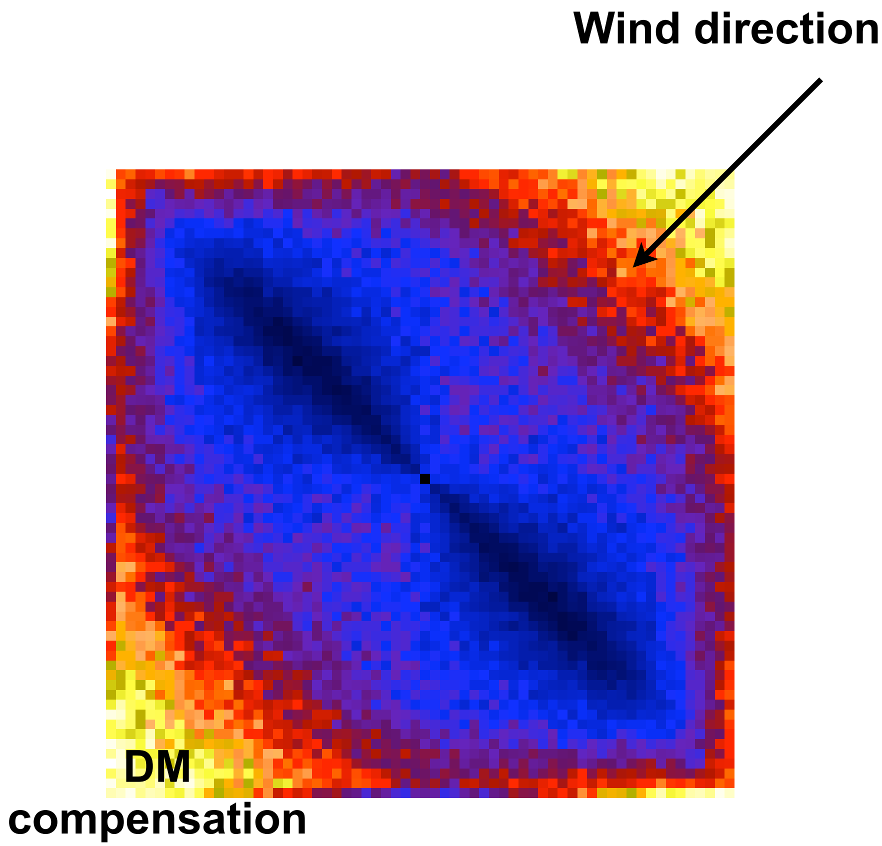
Before



After

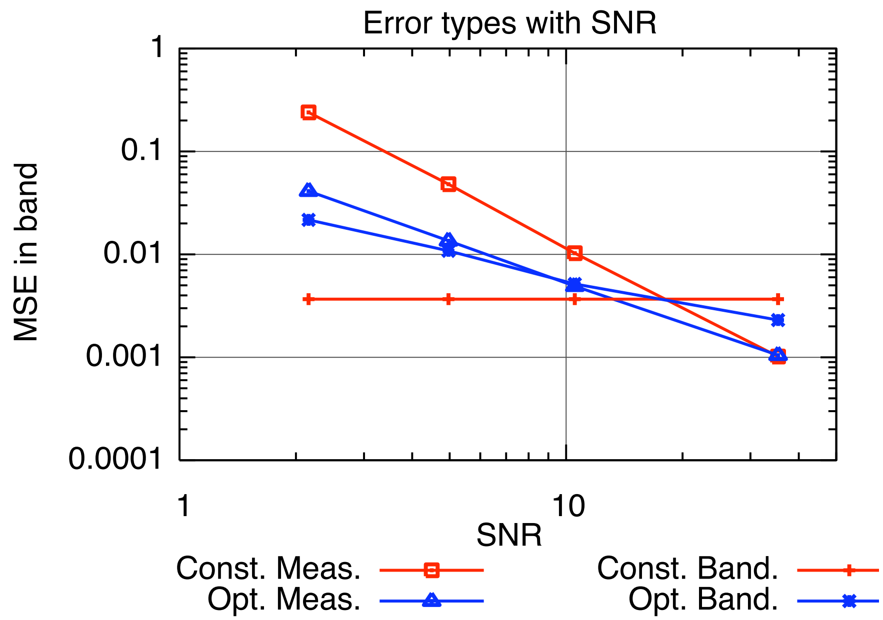
Optimal filter reflects conditions

- Input phase aberration is a frozen sheet of phase moving across the aperture
- Deformable Mirror (DM) has unknown low-pass response which attenuates high spatial frequencies
- Optimal gains compensate for both



Example filter, N=64

Trade bandwidth and sensor errors



Data for N=48, median over a set of 25 random phase screens

- At high SNRs, optimal gains produce equivalent or more measurement error but less temporal error than before
- At low SNRs, optimal gains produce less measurement error but more temporal error than before

Computational load is satisfiable today

- FTR each timestep: $15N^2 \lg N + 20N^2$
- Estimating periodograms for t steps of telemetry:

$$N^2(5 + 2.5 \lg t)$$

- Averaging the periodograms and finding the optimal gain (k is for evaluations in root-finding):

$$N^2(1 + k) + 4k$$

- Assuming k = 10 (using fast method), a 64x64 system at 2.5k kHz has a maximum load of 1.43 GFLOPs/sec.

OFC is both fast and smart

- Optimal Fourier Control combines the best of state-of-the art approaches:
  - *computationally efficient*
  - *adaptive control optimizes performance given current observing conditions*

- Further research areas
  - *sensitivity analysis to determine performance across a wide range of conditions*
  - *exploration of more complex control laws or predictive control*